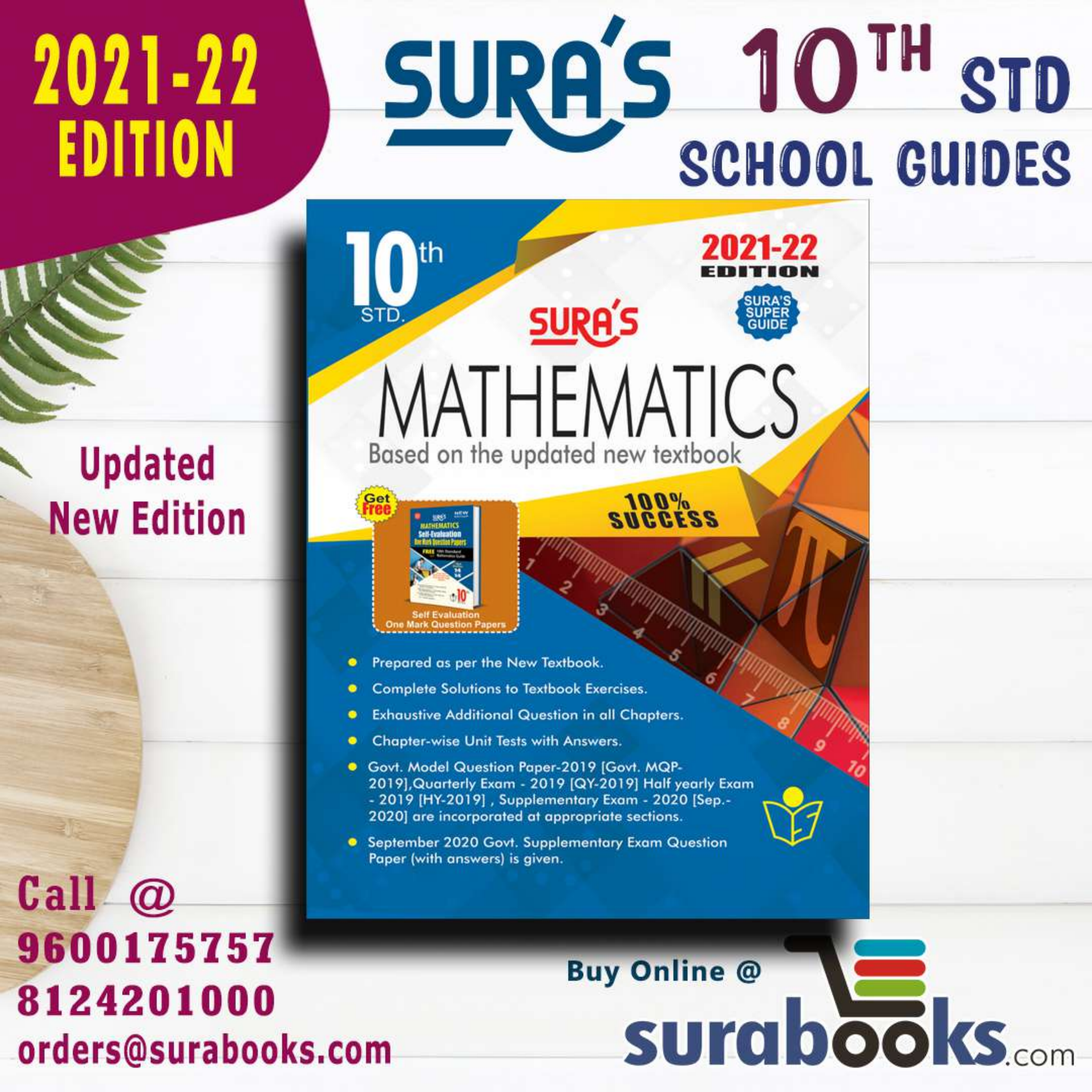

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PREFACE

The woods are lovely, dark and deep.
But I have promises to keep, and
miles to go before I sleep - Robert Frost

Respected Principals, Correspondents, Head Masters / Head Mistresses, Teachers, and dear Students.

From the bottom of our heart, we at SURA Publications sincerely thank you for the support and patronage that you have extended to us for more than a decade.

It is in our sincerest effort we take the pride of releasing **SURA'S Mathematics** for 10th Standard - Edition 2021-22. This guide has been authored and edited by qualified teachers having teaching experience for over a decade in their respective subject fields. This Guide has been reviewed by reputed Professors who are currently serving as Head of the Department in esteemed Universities and Colleges.

With due respect to Teachers, I would like to mention that this guide will serve as a teaching companion to qualified teachers. Also, this guide will be an excellent learning companion to students with exhaustive exercises, additional problems and 1 marks as per new model in addition to precise answers for exercise problems.

In complete cognizance of the dedicated role of Teachers, I completely believe that our students will learn the subject effectively with this guide and prove their excellence in Board Examinations.

I once again sincerely thank the Teachers, Parents and Students for supporting and valuing our efforts.

God Bless all.

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Chapter 1

RELATIONS AND FUNCTIONS

FORMULAE TO REMEMBER

- **Vertical line test :**

  A curve drawn in a graph represents a functions, if every vertical line intersects the curve in at most one point.

- **Horizontal line test :**

  A function represented in a graph is one - one, if every horizontal line intersect the curve in at most one point.

- Linear functions has applications in Cryptography as well as in several branches of Science and Technology.

EXERCISE 1.1

1. Find A× B, A×A and B ×A
(i) A = {2,−2,3} and B ={1,−4} (ii) A= B ={p,q}
(iii) A= {m,n} ; B = ϕ **[PTA - 1]**

Sol. (i) A = {2, –2, 3}, B = {1, –4}

$A \times B = \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$

$A \times A = \{(2, 2), (2, -2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3, 3)\}$

$B \times A = \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$

(ii) $A = B = \{(p,q)\}$

$A \times B = \{(p, p), (p, q), (q, p), (q, q)\}$

$A \times A = \{(p, p), (p, q), (q, p), (q, q)\}$

$B \times A = \{(p, p), (p, q), (q, p), (q, q)\}$

(iii) $A = \{m,n\}$, $B = \phi$

$A \times B = \{\ \}$

$A \times A = \{(m,m), (m,n), (n, m), (n, n)\}$

$B \times A = \{\ \}$

2. Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A× B and B × A.

Sol. $A = \{1, 2, 3\}$, $B = \{2, 3, 5, 7\}$

$A \times B = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$

$B \times A = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$

3. If B ×A={(−2, 3),(−2, 4),(0, 3),(0, 4),(3, 3), (3, 4)} find A and B. **[Qy - 2019]**

Sol. Given $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$

Here $B = \{-2, 0, 3\}$

[All the first elements of the order pair]

and $A = \{3, 4\}$

[All the second elements of the order pair]

4. If A ={5, 6}, B = {4, 5, 6} , C ={5, 6, 7}, Show that A×A = (B × B)∩(C ×C).

Sol. $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$

$A \times A = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(1)

$B \times B = \{(4, 4), (4, 5), (4, 6), (5, 4), \underline{(5, 5)}, \underline{(5, 6)}, (6, 4), \underline{(6, 5)}, \underline{(6, 6)}\}$ ...(2)

$C \times C = \{\underline{(5, 5)}, \underline{(5, 6)}, (5, 7), \underline{(6, 5)}, \underline{(6, 6)}, (6, 7), (7, 5), (7, 6), (7, 7)\}$ ...(3)

$(B \times B) \cap (C \times C) = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(4)

(1) = (4)

$A \times A = (B \times B) \cap (C \times C)$. It is proved.

5. Given A ={1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if(A∩C) × (B∩D) = (A×B)∩(C × D) is true? **[Qy - 2019]**

Sol. LHS $= \{(A \cap C) \times (B \cap D)\}$

$A \cap C = \{3\}$

$B \cap D = \{3, 5\}$

$(A \cap C) \times (B \cap D) = \{(3, 3), (3, 5)\}$ ...(1)

RHS $= (A \times B) \cap (C \times D)$

$A \times B = \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), \underline{(3, 3)}, \underline{(3, 5)}\}$

$C \times D = \{(3, 1), \underline{(3, 3)}, \underline{(3, 5)}, (4, 1), (4, 3), (4, 5)\}$

$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ ...(2)

$\therefore$ (1) = (2) $\therefore$ It is true.

6. Let A={$x \in \mathbb{W}$ |x < 2}, B = {$x \in \mathbb{N}$ |1 < $x \leq 4$} and C = {3, 5} . Verify that

(i) A × (B∪C) =(A × B)∪(A × C) **[PTA - 2]**

(ii) A×(B∩C) = (A × B)∩(A × C) **[PTA - 5]**

(iii) (A∪B) × C = (A×C)∪(B × C)

Sol. **(i) A × (B ∪ C) = (A × B) ∪ (A × C)**

$A = \{x \in \mathbb{W} \mid x < 2\} = \{0, 1\}$

[Whole numbers less than 2]

$B = \{x \in \mathbb{N} \mid 1 < x \leq 4\} = \{2, 3, 4\}$

$C = \{3, 5\}$

[Natural numbers from 2 to 4]

LHS $= A \times (B \cup C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$

$= \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(1)

RHS $= (A \times B) \cup (A \times C)$

$(A \times B) = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1,5)\}$

$(A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(2)

(1) = (2), LHS = RHS Hence it is proved.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

LHS $= A \times (B \cap C)$

$(B \cap C) = \{3\}$

$A \times (B \cap C) = \{(0, 3), (1, 3)\}$ ...(1)

RHS $= (A \times B) \cap (A \times C)$

$(A \times B) = \{(0, 2), \underline{(0, 3)}, (0, 4), (1, 2), \underline{(1, 3)}, (1, 4)\}$

$(A \times C) = \{\underline{(0, 3)}, (0, 5), \underline{(1, 3)}, (1, 5)\}$

$(A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ...(2)

$(1) = (2) \Rightarrow$ LHS = RHS.

Hence it is verified.

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

LHS $= (A \cup B) \times C$

$A \cup B = \{0, 1, 2, 3, 4\}$

$(A \cup B) \times C = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$ ...(1)

RHS $= (A \times C) \cup (B \times C)$

$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$(B \times C) = \{(2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$

$(A \times C) \cup (B \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$ ...(2)

$(1) = (2)$

$\therefore$ LHS = RHS. Hence it is verified.

7. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ [Sep. - 2020]

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ [PTA - 1]

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$

$C = \{2\}$

[$\because$ 2 is the only even prime number]

Sol. **(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$**

LHS $= (A \cap B) \times C$

$A \cap B = \{2, 3, 5, 7\}$

$(A \cap B) \times C = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ...(1)

RHS $= (A \times C) \cap (B \times C)$

$(A \times C) = \{(1, 2), \underline{(2, 2)}, \underline{(3, 2)}, (4, 2), \underline{(5, 2)}, (6, 2), \underline{(7, 2)}\}$

$(B \times C) = \{\underline{(2, 2)}, \underline{(3, 2)}, \underline{(5, 2)}, \underline{(7, 2)}\}$

$(A \times C) \cap (B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ...(2)

$(1) = (2)$

$\therefore$ LHS = RHS. Hence it is verified.

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

LHS $= A \times (B - C)$

$(B - C) = \{3, 5, 7\}$

$A \times (B - C) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\}$ ...(1)

RHS $= (A \times B) - (A \times C)$

$(A \times B) = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7), (7, 2), (7, 3), (7, 5), (7, 7)\}$

$(A \times C) = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$

$(A \times B) - (A \times C) = (1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\}$ ...(2)

$(1) = (2) \Rightarrow$ LHS = RHS. Hence it is verified.

EXERCISE 1.2

1. Let $A = \{1,2,3,7\}$ and $B = \{3,0,-1,7\}$, which of the following are relation from A to B ?

(i) $\mathbb{R}_1 = \{(2,1), (7,1)\}$

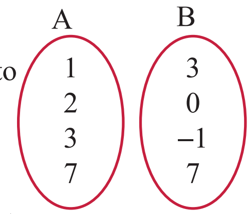
(ii) $\mathbb{R}_2 = \{(-1,1)\}$

(iii) $\mathbb{R}_3 = \{(2,-1), (7,7), (1,3)\}$

(iv) $\mathbb{R}_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

Sol. Given $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$

(i) $R_1 = \{(2, 1), (7, 1)\}$



2 and 7 cannot be related to 1 since $1 \notin B$

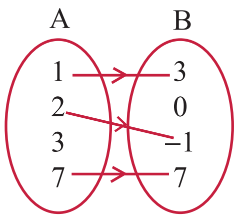
$\therefore R_1$ is not a relation.

(ii) $R_2 = \{(-1, 1)\}$

-1 cannot be related to 1 since $-1 \notin A$ and $1 \notin B$

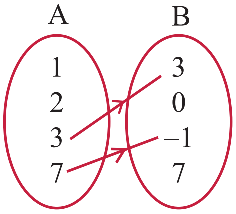
$\therefore R_2$ is not a relation.

(iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$



R_3 is a relation since 2 is related to -1, 7 is related to 7 and 1 is related to 3.

(iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$



7 is related to –1
3 is related to 3
Since $0 \notin A$, 0 cannot be related to 3 and 7.
$\therefore R_4$ is not a relation.

2. Let A={1, 2, 3, 4,...,45} and R be the relation defined as "is square of " on A. Write $\mathbb{R}$ as a subset of A × A. Also, find the domain and range of $\mathbb{R}$.

Sol. Given A = {1, 2, 3, 4, . . . 45}

$\therefore A \times A = \{(1, 1)\ (1, 2)\ (1,3) \ldots (1, 45)$
$(2, 1)\ (2, 2) \ldots (2, 45)\ (45, 1)\ (45, 2)$
$(45, 3) \ldots (45, 45)\}$ ... (1)

R is defined as "is square of "

$\therefore R = \{(1,1)\ (2,4)\ (3,9)\ (4,16)\ (5, 25)\ (6,36)\}$ ... (2)

[$\because$ 1 is the square of 1, 2 is the square of 4 and so on]

From (1) and (2), R is the subset of A × A

$\therefore R \subset A \times A$

Domain of R = {1, 2, 3, 4, 5, 6}

[All the first elements of the order pair in (2)]

Range of R = {1, 4, 9, 16, 25, 36}

[All the second elements of the order pair in (2)]

3. A Relation $\mathbb{R}$ is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range. **[PTA - 5]**

Sol. Given $\mathbb{R} = \{(x, y) / y = x + 3\}$ and $x \in \{0, 1, 2, 3, 4, 5\}$

When $x = 0$, $y = 0 + 3 = 3$ [$\because y = x + 3$]
When $x = 1$, $y = 1 + 3 = 4$
When $x = 2$, $y = 2 + 3 = 5$
When $x = 3$, $y = 3 + 3 = 6$
When $x = 4$, $y = 4 + 3 = 7$
When $x = 5$, $y = 5 + 3 = 8$

$\therefore \mathbb{R} = \{(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)\}$

$\therefore$ Domain of $\mathbb{R}$ = {0, 1, 2, 3, 4, 5}

[All the first element in $\mathbb{R}$]

Range of $\mathbb{R}$ = {3, 4, 5, 6, 7, 8}

[All the second element in $\mathbb{R}$]

4. Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$

(ii) $\{(x, y) | y = x + 3, x, y$ are natural numbers $< 10\}$

Sol. (i) $R = \{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}$ and $y \in \{1, 2, 3, 4\}\}$

When $x = 2$, $y = \frac{x}{2} = \frac{2}{2} = 1$

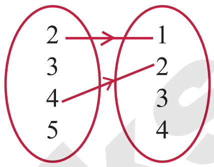
[$\because x = 2y \Rightarrow y = \frac{x}{2}$]

When $x = 3$, $y = \frac{3}{2}$

When $x = 4$, $y = \frac{4}{2} = 2$

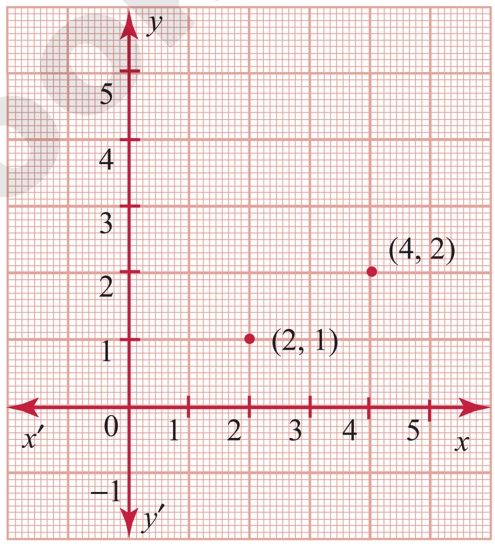
When $x = 5$, $y = \frac{5}{2}$

(a) an arrow diagram



3 cannot be related to $\frac{3}{2}$ and 5 cannot be related to $\frac{5}{2}$.

(b) a graph



(c) Roster form : R = {(2, 1), (4, 2)}

(ii) $R = \{(x, y) | y = x + 3$, x and y are natural numbers <10}

$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9$

$y = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

[$\because x$ and y are natural numbers less than 10]

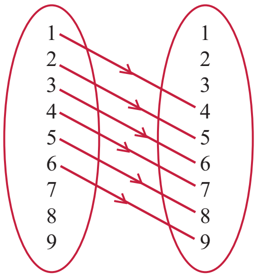
Given $y = x + 3$

When $x = 1$, $y = 1 + 3 = 4$
When $x = 2$, $y = 2 + 3 = 5$
When $x = 3$, $y = 3 + 3 = 6$
When $x = 4$, $y = 4 + 3 = 7$
When $x = 5$, $y = 5 + 3 = 8$
When $x = 6$, $y = 6 + 3 = 9$

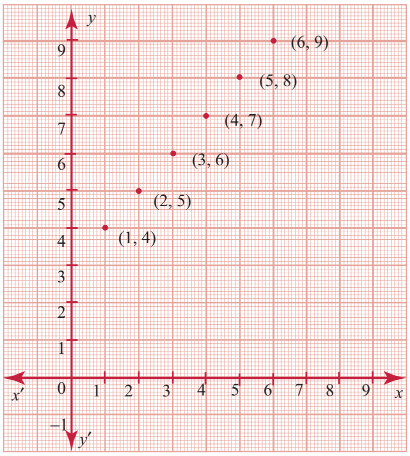
$$\left.\begin{array}{ll} \text{When } x = 7, & y = 7 + 3 = 10 \\ \text{When } x = 8, & y = 8 + 3 = 11 \\ \text{When } x = 9, & y = 9 + 3 = 12 \end{array}\right\} [10, 11, 12 \notin y]$$

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

(a) an arrow diagram



(b) a graph



(c) Roster form :

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation $\mathbb{R}$ is defined by $x\mathbb{R}y$, where x is the salary given to person y, express the relation $\mathbb{R}$ through an ordered pair and an arrow diagram.

Sol.

A – Assistants $\to$ A_1, A_2, A_3, A_4, A_5

C – Clerks $\to$ C_1, C_2, C_3, C_4

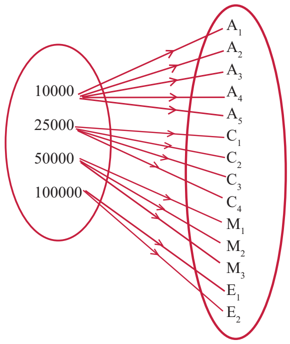
M – Managers $\to$ M_1, M_2, M_3

E – Executive officer $\to$ E_1, E_2

xRy is defined as x is the salary for assistants is ₹10,000, clerks is ₹25,000, Manger is ₹50,000 and for the executing officer ₹1,00,000.

(a) $\therefore$ R = {(10,000, A_1), (10,000, A_2), (10,000, A_3), (10,000, A_4), (10,000, A_5), (25,000, C_1), (25,000, C_2), (25,000,C_3), (25,000, C_4) (50,000, M_1), (50,000, M_2), (50,000, M_3), (1,00,000, E_1), (1,00,000, E_2)}

(b)



EXERCISE 1.3

1. Let f = {$(x, y)|x, y \in \mathbb{N}$ and $y = 2x$} be a relation on $\mathbb{N}$. Find the domain, co-domain and range. Is this relation a function?

Sol. Given f = {$(x, y) \mid x, y \in \mathbb{N}$ and $y = 2x$}

When $x = 1$, $y = 2(1) = 2$

When $x = 2$, $y = 2(2) = 4$

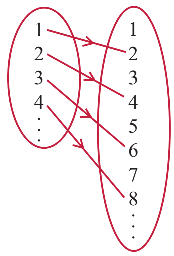
When $x = 3$, $y = 2(3) = 6$

When $x = 4$, $y = 2(4) = 8$ and so on.

R = {(1, 2), (2, 4), (3, 6), (4, 8), (5, 10),...}

Domain of R = {1, 2, 3, 4,...},

Range of R = {2, 4, 6, 8,...}



Since all the elements of domain are related to some elements of co-domain, this relation f is a function.

2. Let X = {3, 4, 6, 8}. Determine whether the relation $\mathbb{R}$ = {$(x, f(x)) \mid x \in X, f(x) = x^2 + 1$} is a function from X to $\mathbb{N}$?

Sol. x = {3, 4, 6, 8}

R = $((x, f(x))|x \in X, f(x) = x^2 + 1)$

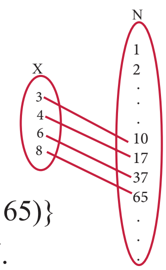
$f(x) = x^2 + 1$

$f(3) = 3^2 + 1 = 10$

$f(4) = 4^2 + 1 = 17$

$f(6) = 6^2 + 1 = 37$

$f(8) = 8^2 + 1 = 65$



R = {(3, 10), (4, 17), (6, 37), (8, 65)}

Yes, R is a function from X to $\mathbb{N}$.

Since all the elements of X are related to some elements of $\mathbb{N}$.

3. Given the function $f: x \to x^2 - 5x + 6$, evaluate
(i) $f(-1)$ (ii) $f(2a)$
(iii) $f(2)$ (iv) $f(x-1)$

Give the function $f: x \to x^2 - 5x + 6$.

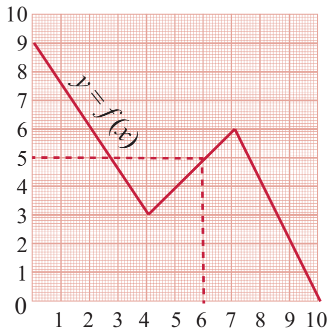
(i) $f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$

(ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$

(iii) $f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$

(iv) $f(x-1) = (x-1)^2 - 5(x-1) + 6$
$= x^2 - 2x + 1 - 5x + 5 + 6$
$= x^2 - 7x + 12$

4. A graph representing the function $f(x)$ is given in figure it is clear that $f(9) = 2$.



(i) Find the following values of the function
(a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$
(ii) For what value of x is $f(x) = 1$?
(iii) Describe the following (i) Domain (ii) Range.
(iv) What is the image of 6 under f?

Sol. (i) From the graph

(a) $f(0) = 9$ (c) $f(2) = 6$

(b) $f(7) = 6$ (d) $f(10) = 0$

(ii) At $x = 9.5, f(x) = 1$

(iii) Domain $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$= \{x \mid 0 \le x \le 10, x \in \mathbb{R}\}$

Range $= \{x \mid 0 \le x \le 9, x \in \mathbb{R}\}$
$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(iv) The image of 6 under f is 5. Since when you draw a line at $x = 6$, it meets the graph at 5.

5. Let $f(x) = 2x + 5$. If $x \neq 0$ then find $\dfrac{f(x+2) - f(2)}{x}$.

Sol. Given $f(x) = 2x + 5, x \neq 0$. $\dfrac{f(x+2) - f(2)}{x}$

$f(x) = 2x + 5$

$\Rightarrow f(x+2) = 2(x+2) + 5$
$= 2x + 4 + 5 = 2x + 9$

$\Rightarrow f(2) = 2(2) + 5 = 4 + 5 = 9$

$\therefore \dfrac{f(x+2) - f(2)}{x} = \dfrac{2x + 9 - 9}{x} = \dfrac{2x}{x} = 2$

6. A function f is defined by $f(x) = 2x - 3$
(i) find $\dfrac{f(0) + f(1)}{2}$
(ii) find x such that $f(x) = 0$.
(iii) find x such that $f(x) = x$.
(iv) find x such that $f(x) = f(1 - x)$.

Sol. Given $f(x) = 2x - 3$

(i) $\dfrac{f(0) + f(1)}{2}$

$f(0) = 2(0) - 3 = -3$

$f(1) = 2(1) - 3 = -1$

$\therefore \dfrac{f(0) + f(1)}{2} = \dfrac{-3-1}{2} = \dfrac{-4}{2} = -2$

(ii) $f(x) = 0 \Rightarrow 2x - 3 = 0$

$2x = 3$

$x = \dfrac{3}{2}$

(iii) $f(x) = x \Rightarrow 2x - 3 = x \Rightarrow 2x - x = 3$

$x = 3$

(iv) $f(x) = f(1 - x)$

$2x - 3 = 2(1 - x) - 3$

$2x - 3 = 2 - 2x - 3$

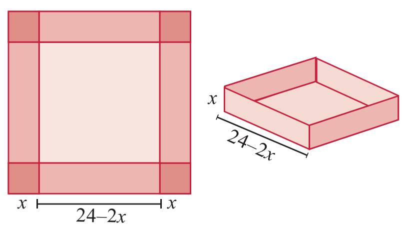
$2x + 2x = 2 - \cancel{3} + \cancel{3}$

$4x = 2$

$x = \dfrac{\cancel{2}}{\cancel{4}\,2}$

$x = \dfrac{1}{2}$

7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown in figure. Express the volume V of the box as a function of x.



Sol. Volume of the box = Volume of the cuboid
$= l \times b \times h$ cu. units

Here $l = 24 - 2x$

$b = 24 - 2x$

$h = x$

$\therefore V = (24 - 2x)(24 - 2x) \times x$
$= (576 - 48x - 48x + 4x^2)x$

$V = 4x^3 - 96x^2 + 576x$

8. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Sol. Given $f(x) = 3 - 2x$

Also, it is given that $f(x^2) = [f(x)]^2$

$$f(x^2) = 3 - 2x^2 \text{[Replacing } x \text{ by } x^2] \quad \dots (1)$$

$$[f(x)]^2 = (3 - 2x)^2 = 9 - 12x + 4x^2 \quad \dots (2)$$

$$[\because (a - b)^2 = a^2 - 2ab + b^2]$$

From (1) and (2),

$\Rightarrow \quad 9 - 12x + 4x^2 = 3 - 2x^2$

$\Rightarrow \quad 9 - 12x + 4x^2 - 3 - 2x^2 = 0$

$\Rightarrow \quad 6x^2 - 12x + 6 = 0$

1, -2, -1 -1

Dividing by 6, we get $x^2 - 2x + 1 = 0$

On factorizing we get, $(x - 1)(x - 1) = 0$

$\Rightarrow \quad x = 1$

9. A plane is flying at a speed of 500 km per hour. Express the distance d travelled by the plane as function of time t in hours.

Sol. $$\text{Speed} = \frac{\text{distance covered}}{\text{time taken}}$$

$\Rightarrow \quad$ distance = Speed × time

$\Rightarrow \quad d = 500 \times t \; [\because \text{time} = t \text{ hrs}]$

$\Rightarrow \quad d = 500\,t$

10. The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length(x) as $y = ax + b$, where a, b are constants. **[PTA - 4]**

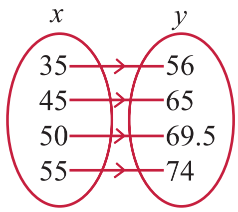
Length 'x' of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

(i) Check if this relation is a function.
(ii) Find a and b.
(iii) Find the height of a person whose forehand length is 40 cm.
(iv) Find the length of forehand of a person if the height is 53.3 inches.

Sol. Given relation is $y = ax + b$ ...(1)

(i) The given ordered pairs are

R = { (35, 56) (45, 65) (50, 69.5) (55, 74)}



Since all the elements of x are related to some elements of y, the given relation is a function.

(ii) Consider any two ordered pairs (35, 56) and (45, 65)

Substitute $\overset{x}{(35,}\ \overset{y}{56)}$ in $y = ax + b$ we get,

$$56 = a(35) + b \quad \dots (1)$$

Similarly substitute (45, 65) in $y = ax + b$, we get

$$65 = a(45) + b \quad \dots(2)$$

$(2) \rightarrow \quad 65 = 45a + \cancel{b} \quad \dots\dots(2)$

$(1) \rightarrow \quad \underline{\overset{-}{56} = \overset{-}{35a} + \overset{-}{\cancel{b}}} \quad \dots(3)$

Substituting, $9 = 10a$

$\Rightarrow \quad a = \dfrac{9}{10} = 0.9$

Substituting $a = 0.9$ in (1) we get

$56 = 35(0.9) + b$

$\Rightarrow \quad 56 = 31.5 + b$

$\Rightarrow \quad b = 56 - 31.5 = 24.5$

Since $y = ax + b$

We get $y = 0.9x + 24.5$

(iii) When the length of the forehand $x = 40$ cm,

$y = 0.9(40) + 24.5$

$\Rightarrow \quad y = 36 + 24.5 = 60.5$ inches

$\therefore$ The required height of the person is 60.5 inches.

(iv) When the length of the forehand $y = 53.3$ inches,

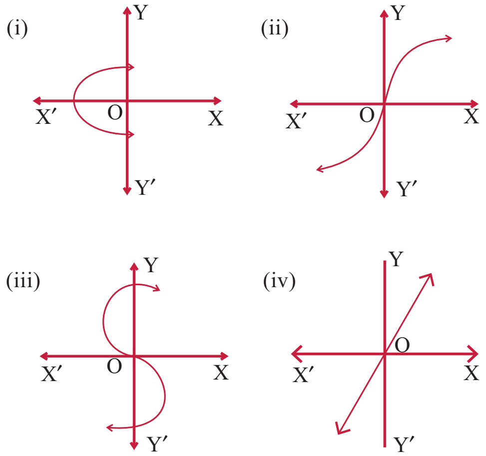
$$53.3 = 0.9x + 24.5 \quad [\because y = 0.9x + 24.5]$$

$\Rightarrow \quad 53.3 - 24.5 = 0.9x \quad \Rightarrow 28.8 = 0.9x$

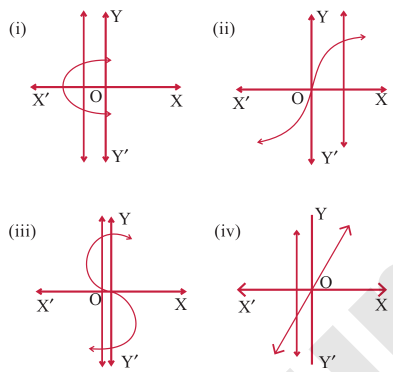
$\Rightarrow \quad x = \dfrac{28.8 \times 10}{0.9 \times 10} \Rightarrow x = \dfrac{288}{9} = 32$ cm

EXERCISE 1.4

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.



Sol.



(i) It is not a function. The graph meets the vertical line at more than one points.

(ii) It is a function as the curve meets the vertical line at only one point.

(iii) It is not a function as it meets the vertical line at more than one points.

(iv) It is a function as it meets the vertical line at only one point.

2. Let f : A $\to$ B be a function defined by $f(x) = \dfrac{x}{2} - 1$, where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}.

Represent f by [Govt. MQP - 2019]

(i) set of ordered pairs;

(ii) a table;

(iii) an arrow diagram;

(iv) a graph

Sol. $f : A \to B$

A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}

$$f(x) = \frac{x}{2} - 1, \quad f(2) = \frac{2}{2} - 1 = 0$$

$$f(4) = \frac{4}{2} - 1 = 1 \quad f(6) = \frac{6}{2} - 1 = 2$$

$$f(10) = \frac{10}{2} - 1 = 4 \quad f(12) = \frac{12}{2} - 1 = 5$$

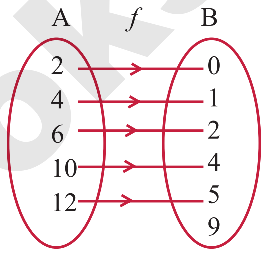
(i) Set of ordered pairs

={(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)}

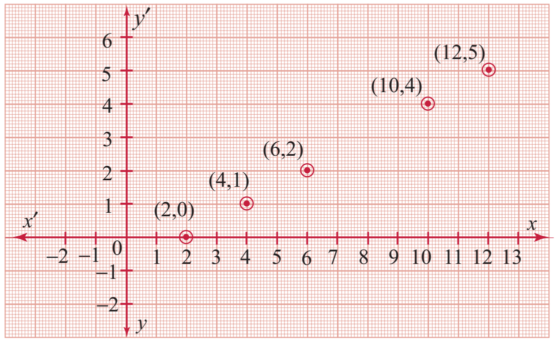
(ii) a table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iii) an arrow diagram;



(iv) a graph



3. Represent the function f ={(1, 2),(2, 2),(3, 2), (4, 3), (5, 4)} through

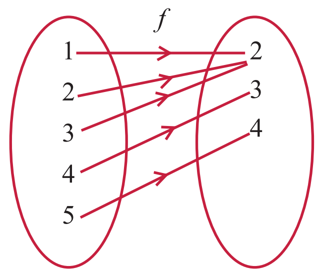
(i) an arrow diagram

(ii) a table form

(iii) a graph

Sol. f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)}

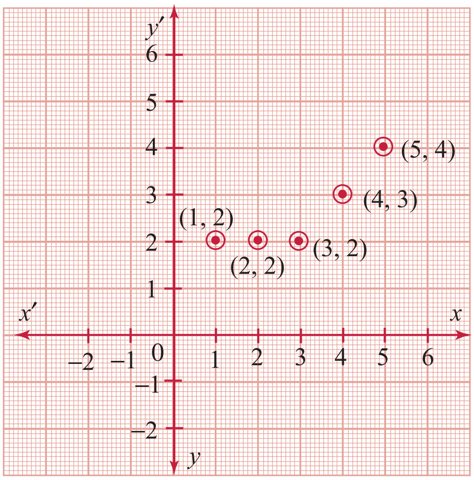
(i) An arrow diagram.



(ii) a table form

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

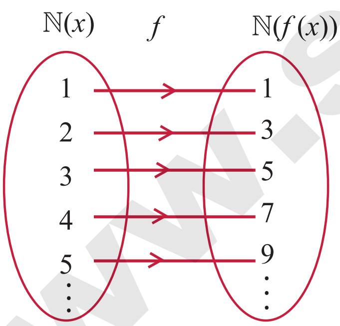
(iii) A graph representation.



4. Show that the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x - 1$ is one - one but not onto.

Sol. $f : \mathbb{N} \to \mathbb{N}$

$$f(x) = 2x - 1$$
$$\mathbb{N} = \{1, 2, 3, 4, 5, ...\}$$
$$f(1) = 2(1) - 1 = 1$$
$$f(2) = 2(2) - 1 = 3$$
$$f(3) = 2(3) - 1 = 5$$
$$f(4) = 2(4) - 1 = 7$$
$$f(5) = 2(5) - 1 = 9$$



In the figure, for different elements in x, there are different images in $f(x)$.

Hence $f : \mathbb{N} \to \mathbb{N}$ is a one-one function.

A function $f : \mathbb{N} \to \mathbb{N}$ is said to be onto function if the range of f is equal to the co-domain of f.

Range = $\{1, 3, 5, 7, 9, ...\}$

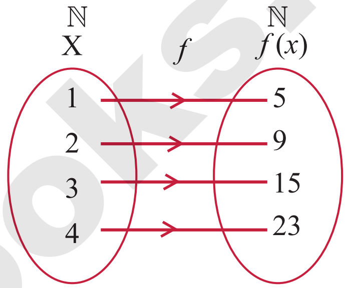
Co-domain = $\{1, 2, 3, ..\}$

But here the range is not equal to co-domain. Therefore it is one-one but not onto function.

5. Show that the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one - one function.

Sol. $f : \mathbb{N} \to \mathbb{N}$

$$f(m) = m^2 + m + 3$$
$$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}, m \in \mathbb{N}$$
$$f(m) = m^2 + m + 3$$
$$f(1) = 1^2 + 1 + 3 = 5$$
$$f(2) = 2^2 + 2 + 3 = 9$$
$$f(3) = 3^2 + 3 + 3 = 15$$
$$f(4) = 4^2 + 4 + 3 = 23$$



In the figure, for different elements in the (X) domain, there are different images in $f(x)$. Hence $f : \mathbb{N} \to \mathbb{N}$ is a one to one but not onto function as the range of f is not equal to co-domain.

Co-domain = $\mathbb{N}$

Range = $\{5, 9, 15, 23\}$

Hence it is proved.

6. Let A = $\{1, 2, 3, 4\}$ and B = $\mathbb{N}$. Let $f : A \to B$ be defined by $f(x) = x^3$ then, **[Hy - 2019]**

(i) find the range of f

(ii) identify the type of function

Sol.
$$A = \{1, 2, 3, 4\}$$
$$B = \mathbb{N}$$

$f : A \to B, f(x) = x^3$

(i) $f(1) = 1^3 = 1$

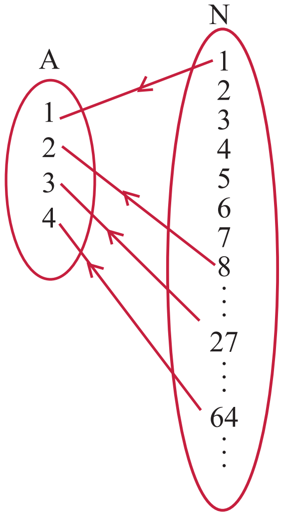
$f(2) = 2^3 = 8$

$f(3) = 3^3 = 27$

$f(4) = 4^3 = 64$

(ii) The range of $f = \{1, 8, 27, 64, ...\}$

(iii)



Here co-domain = N = {1, 2, 3 ...}
Range = {1, 8, 27, 64}
Different elements have different images and co-domain $\neq$ Range.
$\therefore$ The given function is one - one into function.

7. In each of the following cases state whether the function is bijective or not. Justify your answer.

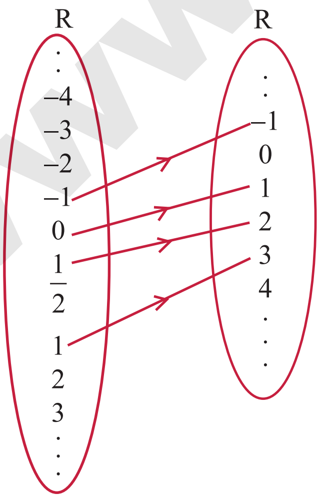
(i) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 1$

(ii) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3 - 4x^2$

Sol. Given $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2x + 1$

(i) When $x = 1$,

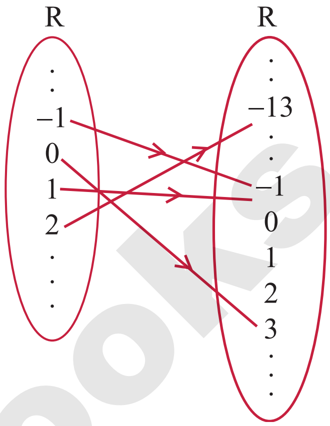
$$f(1) = 2(1) + 1 = 3$$
$$f(2) = 2(2) + 1 = 5$$
$$f(0) = 2(0) + 1 = 1$$
$$f(-1) = 2(-1) + 1 = -2 + 1 = -1$$
$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2 \text{ and so on}$$



Here, different element in domain have different images in B and Co-domain = Range = R.
$\therefore$ f is a bijective function.

(ii) Given $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3 - 4x^2$

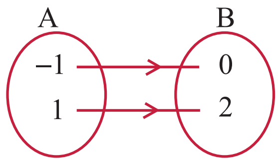
$$f(1) = 3 - 4(1^2) = 3 - 4(1) = 3 - 4 = -1$$
$$f(2) = 3 - 4(2^2) = 3 - 4(4) = 3 - 16 = -13$$
$$f(0) = 3 - 4(0)^2 = 3 - 0 = 3$$
$$f(-1) = 3 - 4(-1)^2 = 3 - 4(1) = 3 - 4 = -1$$



Here, different element in domain do not have different images in B. Since 1 and –1 are related to –1.
$\therefore f$ is not one - one.
Hence, f is not a bijective function.

8. Let A = {–1, 1} and B = {0, 2}. If the function f : A $\to$ B defined by $f(x) = ax + b$ is an onto function? Find a and b.

Sol. Given A = {–1, 1}, B = {0, 2} and f : A $\to$ B is defined by $f(x) = ax + b$ is an onto function.



$$f(-1) = 0$$
$$\Rightarrow \quad a(-1) + b = 0 \quad [\because \text{Sub } x = -1, y = 0 \text{ in } y = ax + b]$$
$$\Rightarrow \quad -a + b = 0 \quad \text{... (1)}$$
$$\text{Also } f(1) = 2$$
$$\Rightarrow \quad a(1) + b = 2 \quad [\because \text{Sub } x = 1, y = 2 \text{ in } y = ax + b] \quad \text{...(2)}$$
$$\Rightarrow \quad \cancel{a} + b = 2$$
$$(1) \Rightarrow \quad -\cancel{a} + b = 0$$
$$\text{Adding,} \quad 2b = 2$$
$$\Rightarrow \quad b = \frac{2}{2} = 1$$

Substituting $b = 1$ in (2) we get
$$a + 1 = 2 \quad \Rightarrow a = 2 - 1 = 1$$
$$\therefore a = 1, b = 1$$

9. **If the function f is defined by**

$$f(x) = \begin{cases} x+2; & x>1 \\ 2; & -1 \le x \le 1 \\ x-1; & -3 < x < -1 \end{cases}$$ **find the values of**

(i) $f(3)$ **(ii)** $f(0)$

(iii) $f(-1.5)$ **(iv)** $f(2) + f(-2)$

Sol. (i) $f(3) \Rightarrow f(x) = x + 2 \Rightarrow 3 + 2 = 5$ $[\because x = 3]$

(ii) $f(0) \Rightarrow 2$ $[\because 0 \in -1 \le x \le 1]$

(iii) $f(-1.5) = x - 1 = -1.5 - 1 = -2.5$

(iv) $f(2) + f(-2)$

$$f(2) = 2 + 2 = 4 \quad [\because f(x) = x + 2]$$
$$f(-2) = -2 - 1 = -3 \quad [\because f(x) = x - 1]$$
$$f(2) + f(-2) = 4 - 3 = 1$$

10. **A function $f : [-5, 9] \rightarrow \mathbb{R}$ is defined as follows:**

$$f(x) = \begin{cases} 6x+1; & -5 \le x < 2 \\ 5x^2-1; & 2 \le x < 6 \\ 3x-4; & 6 \le x \le 9 \end{cases}$$

Find (i) $f(-3) + f(2)$ (ii) $f(7) - f(1)$ [PTA - 4]

(iii) $2f(4) + f(8)$ (iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$ [PTA- 4]

Sol. $f : [-5, 9] \rightarrow \mathbb{R}$

(i) $f(-3) + f(2)$

$$f(-3) = 6x + 1 = 6(-3) + 1 = -17$$
$$f(2) = 5x^2 - 1 = 5(2^2) - 1 = 19$$
$$\therefore f(-3) + f(2) = -17 + 19 = 2$$

(ii) $f(7) - f(1)$

$$f(7) = 3x - 4 = 3(7) - 4 = 17$$
$$f(1) = 6x + 1 = 6(1) + 1 = 7$$
$$f(7) - f(1) = 17 - 7 = 10$$

(iii) $2f(4) + f(8)$

$$f(4) = 5x^2 - 1 = 5 \times 4^2 - 1 = 79$$
$$f(8) = 3x - 4 = 3 \times 8 - 4 = 20$$
$$\therefore 2f(4) + f(8) = 2 \times 79 + 20 = 178$$

(iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$

$$f(-2) = 6x + 1 = 6(-2) + 1 = -11$$
$$f(6) = 3x - 4 = 3(6) - 4 = 14$$
$$f(4) = 5x^2 - 1 = 5(4^2) - 1 = 79$$
$$f(-2) = 6x + 1 = 6(-2) + 1 = -11$$
$$\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{2(-11) - 14}{79 + (-11)} = \frac{-22 - 14}{68}$$
$$= \frac{-36}{68} = \frac{-9}{17}$$

11. **The distance S an object travels under the influence of gravity in time t seconds is given by $S(t) = \frac{1}{2}gt^2 + at + b$ where, (g is the acceleration due to gravity), a, b are constants. Verify whether the function S (t) is one-one or not.** [PTA - 3]

Sol. $S(t) = \frac{1}{2}gt^2 + at + b$

Let t be 1, 2, 3, . . ., seconds.

$$S(1) = \frac{1}{2}g(1^2) + a(1) + b = \frac{1}{2}g + a + b$$
$$S(2) = \frac{1}{2}g(2^2) + a(2) + b$$
$$= 2g + 2a + b$$

Yes, for every different values of t, there will be different values as images. And there will be different pre-images for the different values of the range. Therefore it is one-one function.

12. **The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by t(C)= F where F = $\frac{9}{5}$ C +32. Find,**

(i) $t(0)$ [PTA - 1]

(ii) $t(28)$

(iii) $t(-10)$

(iv) **the value of C when t(C) = 212** [PTA - 1]

(v) **the temperature when the Celsius value is equal to the Fahrenheit value.** [PTA - 1]

Sol. (i) $t(0) = F$

$$F = \frac{9}{5}(C) + 32 = \frac{9}{5}(0) + 32 = 32°F$$

(ii) $t(28) = F = \frac{9}{5}(28) + 32 = \frac{252}{5} + 32$

$$= 50.4 + 32 = 82.4°F$$

(iii) $t(-10) = F = \frac{9}{5}(-10) + 32 = 14°F$

(iv) $t(C) = 212$

i.e $\frac{9}{5}(C) + 32 = 212 \Rightarrow \frac{9}{5}C = 212 - 32 = 180$

$$\frac{9}{5}C = 180 \Rightarrow C = \frac{\overset{20}{\cancel{180}} \times 5}{\cancel{9}} = 100°C$$
$$C = 100°C.$$

(v) when C = F

$$\frac{9}{5}C + 32 = C$$
$$32 = C - \frac{9}{5}C$$
$$32 = C\left(1 - \frac{9}{5}\right)$$

$$32 = C\left(\frac{5-9}{5}\right)$$

$$32 = C\left(\frac{-4}{5}\right)$$

$$C = \overset{8}{\cancel{32}} \times \frac{-5}{\cancel{4}}$$

$$C = -40°$$

EXERCISE 1.5

1. Using the functions f and g given below, find fog and gof. Check whether $fog = gof$.

(i) $f(x) = x - 6$, $g(x) = x^2$

(ii) $f(x) = \frac{2}{x}$, $g(x) = 2x^2 - 1$

(iii) $f(x) = \frac{x+6}{3}$, $g(x) = 3 - x$

(iv) $f(x) = 3 + x$, $g(x) = x - 4$ **[Govt. MQP - 2019]**

(v) $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

Sol. (i) Given $f(x) = x - 6$, $g(x) = x^2$

$fog(x) = f(g(x)) = f(x^2)$ $[\because g(x) = x^2]$

$= x^2 - 6$

[In $f(x) = x - 6$, Replace x by x^2] ...(1)

$gof(x) = g(f(x)) = g(x-6)$ $[\because f(x) = x - 6]$

$= (x-6)^2$

[In $g(x) = x^2$, Replace x by $x - 6$]

$= x^2 - 12x + 36$

$[\because (a-b)^2 = a^2 - 2ab + b^2]$ ... (2)

From (1) and (2),

$fog(x) \neq gof(x)$

(ii) Given $f(x) = \frac{2}{x}$, $g(x) = 2x^2 - 1$

$fog(x) = f(g(x)) = f(2x^2 - 1)$ $[\because g(x) = 2x^2 - 1]$

$= \frac{2}{2x^2 - 1}$

[In $f(x) = \frac{2}{x}$. Replace x by $2x^2 - 1$] ...(1)

$gof(x) = g(f(x)) = g\left(\frac{2}{x}\right)$ $[\because f(x) = \frac{2}{x}]$

$= 2\left(\frac{2}{x}\right)^2 - 1$

[In $g(x) = 2x^2 - 1$, Replace x by $\frac{2}{x}$]

$= 2\left(\frac{4}{x^2}\right) - 1 = \frac{8}{x^2} - 1$ ...(2)

From (1) and (2),

$fog(x) \neq gof(x)$

(iii) Given $f(x) = \frac{x+6}{3}$, $g(x) = 3 - x$

$fog(x) = f(g(x)) = f(3-x)$ $[\because g(x) = 3 - x]$

$= \frac{3 - x + 6}{3}$

[In $f(x) = \frac{x+6}{3}$, Replace x by $3 - x$]

$= \frac{9-x}{3}$ ...(1)

$gof(x) = g(f(x)) = g\left(\frac{x+6}{3}\right)$ $[\because f(x) = \frac{x+6}{3}]$

$= 3 - \left(\frac{x+6}{3}\right)$

[In $g(x) = 3 - x$, Replace x by $\frac{x+6}{3}$]

$= \frac{9 - x - 6}{3} = \frac{3-x}{3}$ ...(2)

From (1) and (2),

$fog(x) \neq gof(x)$

(iv) Given $f(x) = 3 + x$, $g(x) = x - 4$

$fog(x) = f(g(x)) = f(x-4)$ $[\because g(x) = x - 4]$

$= 3 + (x - 4)$

[In $f(x) = 3 + x$, Replace x by $x - 4$]

$= 3 + x - 4 = x - 1$ ... (1)

$gof(x) = g(f(x)) = g(3 + x)$

$= 3 + x - 4$ $[\because f(x) = 3 + x]$

[In $g(x) = x - 4$, Replace x by $3 + x$]

$= x - 1$ ...(2)

From (1) and (2),

$fog(x) = gof(x)$

(v) Given $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

$fog(x) = f(g(x)) = f(1 + x)$ $[\because g(x) = 1 + x]$

$= 4(1 + x)^2 - 1$

[In $f(x) = 4x^2 - 1$, Replace x by $1 + x$]

$= 4(1 + x^2 + 2x) - 1 = 4 + 4x^2 + 8x - 1$

$= 4x^2 + 8x + 3$ ...(1)

$gof(x) = g(f(x)) = g(4x^2 - 1)$ $[\because f(x) = 4x^2 - 1]$

$= 1 + 4x^2 - 1$

[In $g(x) = 1 + x$, Replace x by $4x^2 - 1$]

$= 4x^2$ ...(2)

From (1) and (2),

$fog(x) \neq gof(x)$

2. Find the value of k, such that $fog = gof$

(i) $f(x) = 3x + 2$, $g(x) = 6x - k$ **[Hy - 2019]**

(ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

Sol. (i) Given $f(x) = 3x + 2$, $g(x) = 6x - k$ and

$fog = gof$

$fog(x) = f(g(x)) = f(6x-k)$

$[\because g(x) = 6x-k]$

$= 3(6x-k)+2$

[In $f(x) = 3x+2$, Replace x by $6x-k$]

$= 18x - 3k + 2$ ...(1)

Now $gof(x) = g(f(x)) = g(3x+2)$

$[\because f(x) = 3x+2]$

$= 6(3x+2) - k$

[In $g(x) = 6x-k$, Replace x by $3x+2$]

$= 18x + 12 - k$ ...(2)

Also it is given that $fog = gof$

$\Rightarrow \cancel{18x} - 3k + 2 = \cancel{18x} + 12 - k$ [Using (1) and (2)]

$\Rightarrow -3k+2 = 12-k$

$\Rightarrow -3k+k = 12-2$

$\Rightarrow -2k = 12-2$

$\Rightarrow -2k = 10 \Rightarrow k = \frac{10}{-2} = -5$

$\therefore k = -5$

(ii) $f(x) = 2x-k,\ g(x) = 4x+5$

$fog(x) = f(g(x)) = f(4x+5)$

$[\because g(x) = 4x+5]$

$= 2(4x+5) - k$

[$\because$ In $f(x) = 2x-k$, Replace x by $4x+5$]

$= 8x + 10 - k$ ...(1)

$gof(x) = g(f(x)) = g(2x-k)$

$[\because f(x) = 2x-k]$

$= 4(2x-k)+5$

[In $g(x) = 4x+5$, Replace x by $2x-k$]

$= 8x - 4k + 5$ ...(2)

Given that

$fog(x) = gof(x)$

$\Rightarrow \cancel{8x} + 10 - k = \cancel{8x} - 4k + 5$

[From (1) and (2)]

$\Rightarrow 10-k = -4k+5 \quad \Rightarrow -k+4k = 5-10$

$\Rightarrow 3k = -5 \quad \Rightarrow k = \frac{-5}{3}$

3. If $f(x) = 2x-1$, $g(x) = \frac{x+1}{2}$, show that $fog = gof = x$.

Sol. Given $f(x) = 2x-1$, $g(x) = \frac{x+1}{2}$

S.T $fog = gof = x$

$fog(x) = f(g(x)) = f\left(\frac{x+1}{2}\right)$

$[\because g(x) = \frac{x+1}{2}]$

$= \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1$

[$\because$ In $f(x) = 2x-1$, Replace x by $\frac{x+1}{2}$]

$= x+1-1 = x$ ...(1)

Now,

$gof(x) = g(f(x)) = g(2x-1)$

$[\because f(x) = 2x-1]$

$= \frac{2x-\cancel{1}+\cancel{1}}{2}$

[In $g(x) = \frac{x+1}{2}$, Replace x by $2x-1$]

$= \frac{2x}{2} = x$ ...(2)

From (1) and (2)

$fog(x) = gof(x) = x$ Hence proved.

4. If $f(x) = x^2-1$, $g(x) = x-2$ find a, if $gof(a) = 1$. [PTA - 2]

Sol. Given $f(x) = x^2-1,\ g(x) = x-2$

$gof(x) = g(f(x)) = g(x^2-1)$

$[\because f(x) = x^2-1]$

$= x^2 - 1 - 2$

[$\because$ In $g(x) = x-2$, Replace x by x^2-1]

$= x^2 - 3$

$\therefore gof(a) = a^2 - 3$ [Replacing x by a]

Given that $gof(a) = 1$

$\Rightarrow a^2 - 3 = 1 \Rightarrow a^2 = 4$

$\Rightarrow a = \pm\sqrt{4} \Rightarrow a = \pm 2$

5. Let A,B,C $\subseteq \mathbb{N}$ and a function f : A$\to$ B be defined by $f(x) = 2x+1$ and g : B $\to$ C be defined by $g(x) = x^2$. Find the range of fog and gof.

Sol. Given $f(x) = 2x+1$ and

$g(x) = x^2$

$fog(x) = f(g(x)) = f(x^2)$ $[\because g(x) = x^2]$

$= 2x^2 + 1$

[In $f(x) = 2x+1$, replace x by x^2]

$\therefore fog(x) = 2x^2+1$ ... (1)

$gof(x) = g(f(x)) = g(2x+1)$

$[\because f(x) = 2x+1]$

$= (2x+1)^2$

[In $g(x) = x^2$, replace x by $2x+1$]

$[\because (a+b)^2 = a^2+2ab+b^2]$

Now f : A $\to$ B, and g : B $\to$ C

$\therefore fog$: C $\to$ A and A, B, C $\subseteq$ N

$\therefore$ Range of fog is

$\{y/y = 2x^2+1, x \in C\}$ and

Range of gof is

$\{y/y = (2x+1)^2, x \in N\}$ $[\because gof : A \to C]$

6. If $f(x) = x^2 - 1$. Find (i) fof, (ii) $fofof$

Sol. (i) fof

$fof(x) = f(f(x)) = f(x^2 - 1)$ $[\because f(x) = x^2 - 1]$

$= (x^2 - 1)^2 - 1$ [In $f(x) = x^2 - 1$, replace x by $x^2 - 1$]

$= x^4 - 2x^2 + \cancel{1} - \cancel{1}$ $[\because (a-b)^2 = a^2 - 2ab + b^2$ Here $a = x^2, b = 1]$

$fof(x) = x^4 - 2x^2$ ... (1)

(ii) $fofof = fo[fof(x)]$

$= fo[x^4 - 2x^2]$ [Using (1)]

$= f(x^4 - 2x^2)$

$= (x^4 - 2x^2)^2 - 1$ [In $f(x) = x^2 - 1$, replace x by $x^4 - 2x^2$]

$\therefore fofof(x) = x^8 - 4x^6 + 4x^4 - 1$

$[\because (a-b)^2 = a^2 - 2ab + b^2$ Here $a = x^4, b = -2x^2]$

7. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and fog is one-one? **[PTA - 6]**

Sol. Given $f(x) = x^5$

$g(x) = x^4$

$fog(x) = f(g(x)) = f(x^4)$ $[\because g(x) = x^4]$

$= (x^4)^5$ [In $f(x) = x^5$, replace x by x^4]

$= x^{20}$

$\therefore fog(x) = x^{20}$

Now, $fog(1) = 1^{20} = 1$

and $fog(-1) = (-1)^{20} = 1$ $[\because 20$ is an even number$]$

$\therefore$ Two elements 1 and -1 have same image as 1.

$\therefore fog(x)$ is not one-one.

8. Consider the functions $f(x)$, $g(x)$, $h(x)$ as given below. Show that $(fog)oh = fo(goh)$ in each case.

(i) $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$

(ii) $f(x) = x^2$, $g(x) = 2x$ and $h(x) = x + 4$

(iii) $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 3x - 5$ **[PTA - 2]**

(i) Given $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$

Sol. Consider $fog(x) = f(g(x)) = f(3x + 1)$ $[\because g(x) = 3x + 1]$

$= 3x + \cancel{1} - \cancel{1}$ [In $f(x) = x - 1$, replace x by $3x + 1$]

$\therefore fog = 3x$

LHS $= (fog)h = fog(h(x))$

$= fog(x^2)$ $[\because h(x) = x^2]$

$= 3x^2$ [In $fog = 3x$, replace x by x^2]

RHS $= fo(goh) = f(g(h(x))$

$= f(g(x^2))$ $[\because h(x) = x^2]$

$= f(3x^2 + 1)$ [In $g(x) = x - 1$, replace x by x^2]

$= 3x + \cancel{1} - \cancel{1}$ [In $f(x) = 3x + 1$, replace x by $3x^2 + 1$]

$= 3x^2$ ... (2)

$\therefore$ LHS = RHS

[From (1) and (2)] Hence proved.

(ii) Given $f(x) = x^2$, $g(x) = 2x$, $h(x) = x + 4$

Consider $fog(x) = f(g(x)) = f(2x)$ $[\because g(x) = 2x]$

$= (2x^2)$ [In $f(x) = x^2$, replace x by $2x$]

$= 4x^2$

LHS $(fog)oh = fog(h(x))$

$= fog(x + 4)$ $[\because h(x) = x + 4]$

$= 4(x + 4)^2$ [In $fog = 4x^2$, replace x by $x + 4$]

$= 4(x^2 + 8x + 16)$ $[\because (a+b)^2 = a^2 + 2ab + b^2$. Here $a = x, b = 4]$

$= 4x^2 + 32x + 64$ ... (1)

RHS $= fo(goh) = f(g(h(x))$

$= f(g(x + 4))$ $[\because h(x) = x + 4]$

$= f(2(x + 4))$ [In $g(x) = 2x$, replace x by $x + 4$]

$= f(2x + 8) = (2x + 8)^2$ [In $f(x) = x^2$, replace x by $2x + 8$]

$= 4x^2 + 32x + 64$ ...(2)

$[\because (a+b)^2 = a^2 + 2ab + b^2$. Here $a = 2x, b = 8]$

LHS = RHS

From (1) and (2) Hence proved.

(iii) $f(x) = x - 4$, $g(x) = x^2$, $h(x) = 3x - 5$

Consider

$fog(x) = f(g(x)) = f(x^2)$ $[\because g(x) = x^2]$

$= x^2 - 4$ [In $f(x) = x - 4$, replace x by x^2]

LHS $= (fog)oh$

$= (fog)(h(x)) = fog(3x - 5)$ $[\because h(x) = 3x - 5]$

$= (3x - 5)^2 - 4$ [In $fog(x) = x^2 - 4$, replace x by $3x - 5$]

$= 9x^2 - 30x + 25 - 4$ $[\because (a-b)^2 = a^2 - 2ab + b^2$ Here $a = 3x, b = 5]$

$= 9x^2 - 30x + 21$ ...(1)

RHS $= fo(goh)(x) = f(g(h(x))$

$= f(g(3x - 5)$ $[\because h(x) = 3x - 5]$

$= f(3x - 5)^2$ [In $g(x) = x^2$, replace x by $3x - 5$]

$= (3x - 5)^2 - 4$

[In $f(x) = x - 4$, replace x by $3x - 5$]
$= 9x^2 - 30x + 25 - 4$
[$\because (a - b)^2 = a^2 - 2ab + b^2$ Here $a = 3x$, $b = 5$]
$= 9x^2 - 30x + 21$ ...(2)

From (1) and (2)

LHS = RHS Hence proved.

9. Let $f = \{(-1, 3),(0, -1),(2, -9)\}$ be a linear function from $\mathbb{Z}$ into $\mathbb{Z}$. Find $f(x)$.

Sol. Given $f = \{(-1, 3), (0, -1), (2, -9)\}$ is a linear function from $\mathbb{Z}$ into $\mathbb{Z}$.

Since f is a linear function, let $y = ax + b$ be the linear function which is of degree one.

Sub $(-1, 3)$ in $y = ax + b$, we get

$3 = a(-1) + b \Rightarrow -a + b = 3$ ... (1)

Sub $(0, -1)$ in $y = ax + b$, we get

$-1 = a(0) + b \Rightarrow -1 = b \Rightarrow b = -1$

Sub $b = -1$ in (1) we get,

$-a - 1 = 3 \Rightarrow -a = 3 + 1 = 4$

$\Rightarrow a = -4$

Sub $a = -4$, $b = -1$ in $y = ax + b$ we get,

$y = -4x - 1$

$\therefore$ The required linear function is $-4x - 1$.

10. In electrical circuit theory, a circuit $C(t)$ is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a,b are constants. Show that the circuit $C(t) = 3t$ is linear.

Sol. Given $C(at_1 + bt_2) = a.c(t_1) + b.c(t_2)$

Let $C(t) = 3t$

LHS $= C(at_1 + bt_2) = 3(at_1 + bt_2)$
$= 3at_1 + 3bt_2$ ... (1)

RHS $= a.c(t_1) + b.c(t_2) = a.3t_1 + b.3t_2$
[$\because c(t_1) = 3t_1$ and $c(t_2) = 3t_2$]
$= 3at_1 + 3bt_2$ ... (2)

LHS = RHS

From (1) and (2)

Hence $C(t) = 3t$ is linear function.

EXERCISE 1.6

Multiple choice questions.

1. If $n(A\times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is

(A) 1 (B) 2 (C) 3 (D) 6

[Ans. (C) 3]

Hint: If $n(A \times B) = 6$
$A = \{1, 1\}, n(A) = 2$
$n(B) = 3$

2. $A=\{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$ then $n[(A\cup C) \times B]$ is [PTA - 3]

(A) 8 (B) 20 (C) 12 (D) 16

[Ans. (C) 12]

Hint: $A = \{a, b, p\}, B = \{2, 3\}$,
$C = \{p, q, r, s\}$

$n(A \cup C) \times B$

$A \cup C = \{a, b, p, q, r, s\}$

$(A \cup C) \times B = \{(a,2),(a,3),(b,2),(b,3),(p,2),(p,3),(q,2),(q,3),(r,2),(r,3),(s, 2), (s, 3)\}$

$n[(A \cup C) \times B] = 12$

3. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ then state which of the following statement is true. [Sep.- 2020]

(A) $(A \times C) \subset (B \times D)$
(B) $(B \times D) \subset (A \times C)$
(C) $(A \times B) \subset (A \times D)$
(D) $(D \times A) \subset (B \times A)$

[Ans. (A) $(A \times C) \subset (B \times D)$]

Hint: $A = \{1, 2\}, B = \{1, 2, 3, 4\}$,
$C = \{5, 6\}, D = \{5, 6, 7, 8\}$
$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$
$B \times D = \{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2, 7), (2, 8), (3, 5), (3, 6), (3, 7),(3, 8)\}$

$\therefore (A \times C) \subset B \times D$ It is true.

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is [PTA - 2]

(A) 3 (B) 2 (C) 4 (D) 8

[Ans. (B) 2]

Hint: $n(A) = 5$
$n(B) = x$
$n(A \times B) = 1024 = 2^{10}$
$2^{5x} = 2^{10} \Rightarrow 5x = 10$
$\Rightarrow x = 2$

5. The range of the relation $R = \{(x, x^2) \mid x$ is a prime number less than 13$\}$ is [PTA - 4; Hy - 2019]

(A) {2,3,5,7} (B) {2,3,5,7,11}
(C) {4,9,25,49,121} (D) {1,4,9,25,49,121}

[Ans. (C) {4, 9, 25, 49, 121}]

Hint: $R = \{(x, x^2)/x$ is a prime number $<13\}$
The squares of 2, 3, 5, 7, 11 are
{4, 9, 25, 49, 121}

6. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is
(A) $(2, -2)$ (B) $(5,1)$
(C) $(2,3)$ (D) $(3, -2)$
[Ans. (D) $(3, -2)$]

Hint: $(a + 2, 4), (5, 2a + b) \Rightarrow a + 2 = 5$
$$a = 3 \Rightarrow 2a + b = 4$$
$$6 + b = 4 \Rightarrow b = -2$$

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is
(A) m^n (B) n^m (C) $2^{mn} - 1$ (D) 2^{mn}
[Ans. (C) $2^{mn}-1$]

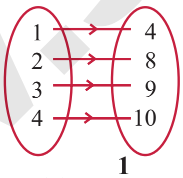
Hint:
$$n(A) = m$$
$$n(B) = n$$
$$n(A \times B) = m \times n = mn$$
$$\text{No. of relations} = 2^{n(A\times B)} = 2^{mn}$$
$$\text{Non-empty relations} = 2^{mn}-1$$

8. If $\{(a, 8),(6, b)\}$ represents an identity function, then the value of a and b are respectively [PTA - 1]
(A) $(8,6)$ (B) $(8,8)$ (C) $(6,8)$ (D) $(6,6)$
[Ans. (A) $(8,6)$]

Hint: $\{(a, 8), (6, b)\} \Rightarrow a = 8 \Rightarrow b = 6$

9. Let A = {1, 2, 3, 4} and B = {4, 8, 9, 10}. A function f :A→ B given by f = {(1, 4), (2, 8),(3, 9),(4, 10)} is a [PTA - 4]
(A) Many-one function
(B) Identity function
(C) One-to-one function
(D) Into function
[Ans. (C) One-to one function]

Hint: A = {1, 2, 3, 4), B = {4, 8, 9, 10}



10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, Then fog is [Hy - 2019]
(A) $\frac{3}{2x^2}$ (B) $\frac{2}{3x^2}$ (C) $\frac{2}{9x^2}$ (D) $\frac{1}{6x^2}$
[Ans. (C) $\frac{2}{9x^2}$]

Hint:
$$f(x) = 2x^2 \Rightarrow g(x) = \frac{1}{3x}$$
$$fog = f(g(x)) = f\left(\frac{1}{3x}\right) = 2\left(\frac{1}{3x}\right)^2$$
$$= 2 \times \frac{1}{9x^2} = \frac{2}{9x^2}$$

11. If f : A → B is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to [PTA - 2]
(A) 7 (B) 49 (C) 1 (D) 14
[Ans. (A) 7]

Hint: In a bijective function, $n(A) = n(B) \Rightarrow n(A) = 7$

12. Let f and g be two functions given by
f = {(0, 1), (2, 0), (3, −4), (4, 2), (5, 7)}
g = {(0, 2),(1, 0), (2, 4), (−4, 2), (7,0)} then the range of fog is
(A) {0,2,3,4,5} (B) {−4,1,0,2,7}
(C) {1,2,3,4,5} (D) {0,1,2}
[Ans. (D) {0, 1, 2}]

Hint:
$$gof = g(f(x))$$
$$fog = f(g(x))$$
$$= \{(0, 2),(1, 0),(2, 4),(-4, 2),(7, 0)\}$$
$$\text{Range of } fog = \{0, 1, 2\}$$

13. Let $f(x) = \sqrt{1+x^2}$ then
(A) $f(xy) = f(x).f(y)$ (B) $f(xy) \geq f(x).f(y)$
(C) $f(xy) \leq f(x).f(y)$ (D) None of these
[Ans. (C) $f(xy) \leq f(x).f(y)$]

Hint: $\sqrt{1+x^2y^2} \leq \sqrt{(1+x^2)}\sqrt{(1+y^2)}$
$\Rightarrow f(xy) \leq f(x) . f(y)$

14. If g = {(1, 1),(2, 3),(3, 5),(4, 7)} is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are [PTA - 6]
(A) $(-1, 2)$ (B) $(2, -1)$
(C) $(-1, -2)$ (D) $(1,2)$ **[Ans.(B) $(2, -1)$]**

Hint:
$$g(x) = \alpha x + \beta$$
$$\alpha = 2$$
$$\beta = -1$$
$$g(x) = 2x - 1$$
$$g(1) = 2(1) - 1 = 1$$
$$g(2) = 2(2) - 1 = 3$$
$$g(3) = 2(3) - 1 = 5$$
$$g(4) = 2(4) - 1 = 7$$

15. $f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is [PTA - 5; Qy - 2019]
(A) linear (B) cubic
(C) reciprocal (D) quadratic
[Ans. (D) quadratic]

Hint:
$$f(x) = (x + 1)^3 - (x - 1)^3$$
$$= x^3 + 3x^2 + 3x + 1 - [x^3 - 3x^2 + 3x - 1]$$
$$= \cancel{x^3} + 3x^2 + \cancel{3x} + 1 - \cancel{x^3} + 3x^2 - \cancel{3x} + 1 = 6x^2 + 2$$
It is a quadratic function.

Unit Exercise - 1

1. If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2,5)$ are equal, then find x and y.

Sol.
$$\begin{aligned}(x^2 - 3x, y^2 + 4y) &= (-2, 5)\\ x^2 - 3x &= -2\\ x^2 - 3x + 2 &= 0\\ (x-2)(x-1) &= 0\\ x &= 2, 1\\ y^2 + 4y &= 5\\ y^2 + 4y - 5 &= 0\\ (y+5)(y-1) &= 0\\ y &= -5, 1\end{aligned}$$

2: -2, -1

-5: $+5$, -1

2. The cartesian product A × A has 9 elements among which (–1, 0) and (0,1) are found. Find the set A and the remaining elements of A × A.

Sol.
$$\begin{aligned}A &= \{-1, 0, 1\}, B = \{1, 0, -1\}\\ A \times B &= \{(-1, 1), (-1, 0), (-1, -1), (0, 1), (0, 0), (0, -1), (1, 1), (1, 0), (1, -1)\}\end{aligned}$$

3. Given that $f(x) = \begin{cases}\sqrt{x-1} & x \geq 1\\ 4 & x < 1\end{cases}$. Find (i) $f(0)$ (ii) $f(3)$ (iii) $f(a+1)$ in terms of a.(Given that $a \geq 0$)

Sol. (i) $f(0) = 4$

(ii) $f(3) = \sqrt{3-1} = \sqrt{2}$

(iii) $f(a+1) = \sqrt{a+1-1} = \sqrt{a}$

4. Let A={9,10,11,12,13,14,15,16,17} and let f : A→ N be defined by $f(n)$= the highest prime factor of $n \in$ A. Write f as a set of ordered pairs and find the range of f.

Sol. A = {9, 10, 11, 12, 13, 14, 15, 16, 17}

$f : A \to \mathbb{N}$

$f(n)$ = the highest prime factor of $n \in$ A

f = {(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)}

Range = {3, 5, 11, 13, 7, 2, 17} = {2, 3, 5, 7, 11, 13, 17}

5. Find the domain of the function $f(x) = \sqrt{1+\sqrt{1-\sqrt{1-x^2}}}$.

Sol.
$$f(x) = \sqrt{1+\sqrt{1-\sqrt{1-x^2}}}$$

Domain of $f(x)$ = {–1, 0, 1}

(x^2 = 1, –1, 0, because $\sqrt{1-x^2}$ should be +ve, or 0)

6. If $f(x) = x^2$, $g(x) = 3x$ and $h(x) = x - 2$. Prove that $(f o g) o h = f o (g o h)$.

Sol.
$$\begin{aligned}f(x) &= x^2\\ g(x) &= 3x\\ h(x) &= x-2\\ (fog)oh &= x-2\\ \text{LHS} &= fo(goh)\\ fog &= f(g(x)) = f(3x) = (3x)^2 = 9x^2\\ (fog)oh &= (fog)\,h(x) = (fog)(x-2)\\ &= 9(x-2)^2 = 9(x^2-4x+4)\\ &= 9x^2 - 36x + 36 \quad ...(1)\\ \text{RHS} &= fo(goh)\\ (goh) &= g(h(x)) = g(x-2)\\ &= 3(x-2) = 3x-6\\ fo(goh) &= f(3x-6) = (3x-6)^2\\ &= 9x^2 - 36x + 36 \quad ...(2)\\ (1) &= (2)\\ \text{LHS} &= \text{RHS}\end{aligned}$$

$(fog)oh = fo(goh)$ is proved.

7. Let A = {1, 2} and B = {1, 2, 3, 4} , C ={5, 6} and D = {5, 6, 7, 8} . Verify whether A×C is a subset of B×D?

Sol.
$$\begin{aligned}A &= \{1, 2\}, B = \{1, 2, 3, 4\}\\ C &= \{5, 6\}, D = \{5, 6, 7, 8\}\\ A \times C &= \{\underline{(1, 5)}, \underline{(1, 6)}, \underline{(2, 5)}, \underline{(2, 6)}\}\\ B \times D &= \{\underline{(1, 5)}, \underline{(1, 6)}, (1, 7), (1, 8), \underline{(2, 5)}, \underline{(2, 6)}, (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}\end{aligned}$$

$(A \times C) \subset (B \times D)$ It is proved.

8. If $f(x) = \dfrac{x-1}{x+1}$, $x \neq 1$ show that $f(f(x)) = -\dfrac{1}{x}$, provided $x \neq 0$.

Sol.
$$f(x) = \frac{x-1}{x+1}, x \neq 1$$

$$f(f(x)) = f\left(\frac{x-1}{x+1}\right) = \frac{\left(\frac{x-1}{x+1}\right)-1}{\left(\frac{x-1}{x+1}\right)+1}$$

$$= \frac{\frac{\cancel{x}-1-x-1}{\cancel{(x+1)}}}{\frac{\cancel{x}-1+x+1}{\cancel{(x+1)}}} = \frac{-2}{2x} = \frac{-1}{x}$$

Hence it is proved.

9. The function f and g are defined by $f(x) = 6x + 8$; $g(x) = \frac{x-2}{3}$.

(i) Calculate the value of $gg\left(\frac{1}{2}\right)$

(ii) Write an expression for $gf(x)$ in its simplest form.

Sol.
$$f(x) = 6x + 8$$
$$g(x) = \frac{x-2}{3}$$

(i) $gg(x) = g(g(x))$

$$= g\left(\frac{x-2}{3}\right) = \frac{\frac{x-2}{3} - 2}{3}$$

$$= \frac{x-2-6}{3} \times \frac{1}{3} = \frac{x-8}{9}$$

$$gog\left(\frac{1}{2}\right) = \frac{\frac{1}{2} - 8}{9} = \frac{1-16}{2} \times \frac{1}{9} = \frac{-15}{18} = \frac{-5}{6}$$

(ii) $gof(x) = g(f(x)) = g(6x + 8)$

$$= \frac{6x+8-2}{3} = \frac{6x+6}{3}$$

$$= \frac{\cancel{3}(2x+2)}{\cancel{3}} = 2x + 2 = 2(x + 1)$$

10. Write the domain of the following real functions

(i) $f(x) = \frac{2x+1}{x-9}$ [PTA - 6]

(ii) $p(x) = \frac{-5}{4x^2+1}$

(iii) $g(x) = \sqrt{x-2}$ [PTA - 6]

(iv) $h(x) = x + 6$

Sol. (i) $f(x) = \frac{2x+1}{x-9}$

The denominator should not be zero as the function is a real function.

$\therefore$ The domain = R – {9}

(ii) $p(x) = \frac{-5}{4x^2+1}$

The domain is R.

(iii) $g(x) = \sqrt{x-2}$

The domain = [2, ∝]

(iv) $h(x) = x + 6$

The domain is R.

PTA EXAM QUESTION & ANSWERS

1 MARK

1. If $n(A) = p$, $n(B) = q$ then the total number of relations that exist between A and B is [PTA -1]

(A) 2^p (B) 2^q (C) 2^{p+q} (D) 2^{pq}

[Ans. (D) 2^{pq}]

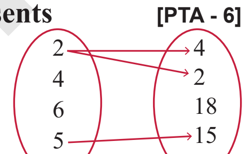
2. Given $f(x)$ $(-1)^x$ is a function from $\mathbb{N}$ to $\mathbb{Z}$. Then the range of f is [PTA - 3]

(A) {1} (B) $\mathbb{N}$ (C) {1, –1} (D) $\mathbb{Z}$

[Ans. (C){1, –1}]

3. The given diagram represents [PTA - 6]

(A) an onto function
(B) constant function
(C) an one-one function
(D) not a function



[Ans. (D) not a function]

Hint: 4 has no image

2 MARKS

1. A relation 'f' is defined by $f(x) = x^2 - 2$ where, $x \in \{-2, -1, 0, 3\}$ (i) List the elements of f (ii) Is f a function? [PTA - 1; Qy - 2019]

Sol.
$f(x) = x^2 - 2$ where $x \in \{-2, -1, 0, 3\}$

(i) $f(-2) = (-2)^2 - 2 = 2$;
$f(-1) = (-1)2 - 2 = -1$
$f(0) = 0^2 - 2 = -2$
$f(3) = 3^2 - 2 = 9 - 2 = 7$
$\therefore f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$

(ii) We note that each element in the domain of f has a unique image.

Therefore f is a function.

2. A relation R is given by the set $\{(x, y)/y = x^2 + 3, x \in \{0,1,2,3,4,5\}\}$ Determine its domain and range. [PTA - 2]

Sol.
Domain = {0, 1, 2, 3, 4, 5}
$x = 0, y = 0^2 + 3 = 3$
$x = 1, y = 1^2 + 3 = 4$
$x = 2, y = 2^2 + 3 = 7$
$x = 3, y = 3^2 + 3 = 12$
$x = 4, y = 4^2 + 3 = 19$
$x = 5, y = 5^2 + 3 = 28$
Range = {3, 4, 7, 12, 19, 28}

3. Find k, if $f(k) = 2k - 1$ and $fof(k) = 5$. [PTA - 4]

Sol.
$f(k) = 2k - 1$

Consider $fo\,f(k) = f(f(k)) = f(2k - 1)$

$[\because f(x) = 2k - 1]$

$$= 2(2k-1)-1$$
[In $f(k) = 2k - 1$, replace k by $2k - 1$]
$$= 4k-2-1 = 4k-3$$
$$\Rightarrow \quad 4k-3 = 5 \Rightarrow 4k = 5+3 = 8$$
$$\Rightarrow \quad k = \frac{8}{4} = 2$$
$$\therefore k = 2$$

4. Let A = {1, 2, 3, ..., 100} and R be the relation defined as "is cube of" on A. Find the domain and range of R. **[PTA - 4]**

Sol. R = {(1,1) (2,8), (3,27), (4, 64)}
Domain = {1, 2, 3, 4}
Range = {1, 8, 27, 64}

5. Let A = {1, 2, 3, 4} and B = ℕ. Let f : A → B be defined by $f(x) = x^2$ (i) the range of f (ii) identify the type of function. **[PTA - 5]**

Sol. $f(1) = 1; f(2) = 4; f(4) = 9; f(4) = 16$
(i) Range = {1, 4, 9, 16}
(ii) One - one and into function

6. Let f be a function from ℝ to ℝ defined by $f(x) = 3x - 5$ Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f. **[PTA - 6]**

Sol. $f(x) = 3x - 5$ can be written as
$f = \{(x, 3x-5) | x \in \mathbb{R}\}$
$(a, 4)$ mean the image of a is 4.
That is, $f(a) = 4$
$3a - 5 = 4 \Rightarrow a = 3$
$(1, b)$ means the image of 1 is b. That is,
That is, $f(1) = b \Rightarrow b = -2$
$3(1) - 5 = b \Rightarrow b = -2$

7. R = {(x, – 2), (–5, y)} represents the identity function, find the values x and y. **[PTA - 6]**

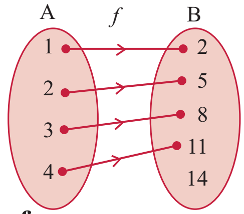
Sol. $x = -2$
$y = -5$

5 MARKS

1. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let f : A → B be a function given by $f(x) = 3x - 1$ Represent this function. **[PTA - 3]**
(i) by arrow diagram **[Sep.-2020]**
(ii) in a table form
(iii) as a set of ordered pairs
(iv) in a graphical form

Sol. Let A = {1, 2, 3, 4} ; B = {2, 5, 8, 11, 14};
$f(x) = 3x - 1$
$f(1) = 3(1) - 1 = 3 - 1 = 2; f(2) = 3(2) - 1 = 6 - 1 = 5$
$f(3) = 3(3) - 1 = 9 - 1 = 8; f(4) = 4(3) - 1 = 12 - 1 = 11$

(i) Arrow diagram
Let us represent the function f :A → B by an arrow diagram

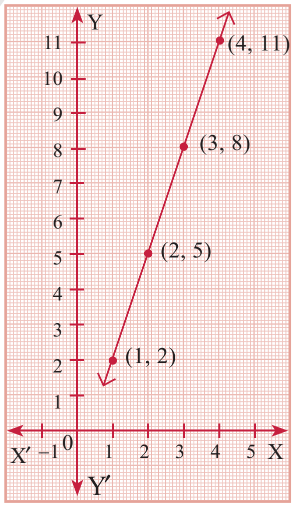


(ii) Table form
The given function f can be represented in a tabular form as given below

x	1	2	3	4
$f(x)$	2	5	8	11

(iii) Set of ordered pairs
The function f can be represented as a set of ordered pairs as
f = (1, 2),(2, 5),(3, 8),(4, 11)

(iv) Graphical form



In the adjacent xy -plane the points (1,2), (2,5), (3,8), (4,11) are plotted

2. Let A = {$x \in \mathbb{W}$/ 0 < x < 5}, B = {$x \in \mathbb{W}$/ 0 ≤ x ≤ 2}, C = {$x \in \mathbb{W}$/ x < 3} then verify that A × (B ∩ C) = (A × B) ∩ (A× C) **[PTA - 3]**

Sol. A = {1, 2, 3, 4}
B = {0, 1, 2}
C = {0, 1, 2}
B ∩ C = {0, 1, 2} ∩ {0, 1, 2} = {0, 1, 2}
A × (B ∩ C) = {1, 2, 3, 4} × {0, 1, 2}
= {(1,0), (1,1), (1, 2), (2,0), (2,1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4,2)} ... (1)
A × B = {1, 2, 3, 4} × {0, 1, 2}
= {(1,0), (1,1), (1,2), (2,0),(2,1),(2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2)

$$\begin{aligned} A\times C &= \{1,2,3,4\}\times\{0,1,2\}\\ &= \{(1,0),(1,1),(1,2),(2,0),(2,1),(2,2),\\ &\quad (3,0),(3,1),(3,2),(4,0),(4,1),(4,2)\}\\ (A\times C)\cap(A\times C) &= \{1,2,3,4\}\times\{0,1,2\}\\ &= \{(1,0),(1,1),(1,2),(2,0),(2,1),(2,2),\\ &\quad (3,0),(3,1),(3,2),(4,0),(4,1),(4,2)\}\quad \ldots(2)\end{aligned}$$

$(1) = (2)$ Hence it is proved.

3. $f(x) = 2x + 3$, $g(x) = 1 - 2x$ and $h(x) = 3x$, prove that $fo(g\ o\ h) = (f\ o\ g)\ oh$. [PTA - 5]

Sol.
$$\begin{aligned} f(x) &= 2x+3,\ g(x) = 1-2x,\\ h(x) &= 3x\\ \text{Now, } (f\ o\ g)(x) &= f(g(x)) = f(1-2x)\\ &= 2(1-2x)+3 = 5-4x\end{aligned}$$

Then,
$$\begin{aligned} (f\ o\ g)oh(x) &= (f\ o\ g)h(x)) = (f\ o\ g)(3x)\\ &= 5-4(3x) = 5-12x \quad \ldots(1)\\ (g\ o\ h)(x) &= g(h(x)) = g(3x) = 1-2(3x)\\ &= 1-6x\end{aligned}$$

So,
$$\begin{aligned} f\ o\ (g\ o\ h)(x) &= f(1-6x)\\ &= 2(1-6x)+3\\ &= 5-12x \quad \ldots(2)\end{aligned}$$

From (1) and (2), we get
$$(f\ o\ g)\ oh = f\ o\ (g\ o\ h)$$

GOVT. EXAM QUESTION & ANSWERS

1 MARK

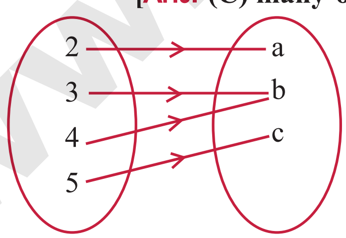
Multiple choice questions.

1. $f = \{(2, a), (3, b), (4, b), (5, c)\}$ is a ________. [Govt. MQP - 2019]

(A) identity function (B) one-one function
(C) many-one function (D) constant function

[Ans. (C) many-one function]

Hint:



2. Let $f(x) = x^2 - x$, then $f(x-1) - f(x+1)$ is : [Sep.-2020]

(A) $4x$ (B) $2-2x$ (C) $2-4x$ (D) $4x-2$

[Ans. (C) $2-4x$]

Hint:
$$\begin{aligned} f(x-1) &= (x-1)^2-(x-1)\\ &= x^2-2x+1-(x-1)\\ &= x^2-2x+1-x+1\\ &= x^2-3x+2\\ f(x+1) &= (x+1)^2-(x+1)\\ &= x^2+2x+\cancel{1}-x-\cancel{1}\\ &= x^2+x\\ \therefore f(x-1)-f(x+1) & \\ &= (x^2-3x+2)-(x^2+x)\\ &= \cancel{x^2}-3x+2-\cancel{x^2}-x\\ &= -4x^2+2\end{aligned}$$

3. If $n(A)= p$ and $n(B)= q$ then $n(A\times B)=$ ________ [Qy - 2019]

(A) $p+q$ (B) $p-q$ (C) $p\times q$ (D) $\frac{p}{q}$

[Ans. (C) $p\times q$]

Hint: $n(A\times B) = n(A)\times n(B) = p\times q$

2 MARKS

1. Define a function. [Govt. MQP - 2019]

Sol. A relation f between two non-empty sets X and Y is called a function from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x, y) \in f$.

That is, $f = \{(x, y) \mid \text{for all } x \in X, y \in Y\}$

2. Let f be a function $f : \mathbb{N} \to \mathbb{N}$ be defined by $f(x) = 3x + 2, x \in \mathbb{N}$. [Govt. MQP - 2019]

(i) Find the images of 1, 2, 3
(ii) Find the pre-images of 29, 53
(iii) Identify the type of function

Sol. $f : \mathbb{N} \to \mathbb{N}$ is defined by $f(x) = 3x + 2$,

(i)
$$\begin{aligned} f(1) &= 3(1)+2 = 3+2 = 5\\ f(2) &= 3(2)+2 = 6+2 = 8\\ f(3) &= 3(3)+2 = 9+2 = 11\end{aligned}$$

The images of 1, 2, 3 are 5, 8, 11 respectively.

(ii) If x is the pre-image of 29, then $f(x) = 29$.
$$\begin{aligned} \Rightarrow\quad 3x+2 &= 29\\ 3x &= 27\\ \Rightarrow\quad x &= 9.\end{aligned}$$

Similarly, if x is the pre-image of 53, then $f(x) = 53. \Rightarrow 3x+2 = 53$
$$\begin{aligned} 3x &= 51\\ \Rightarrow\quad x &= 17.\end{aligned}$$

$\therefore$ the pre-images of 29 and 53 are 9 and 17 respectively.