TRB – Mathematics (PG)

Solved Original Question Paper - 2018-2019

Marks : 150
Duration : 3 hours

Held on : 27.09.2019

1. **Let G be the set of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a,b,c,d are integers modulo 2 such that ad − bc≠0. Under matrix multiplication, G is a group of order ....**
   A) 8 B) 6
   C) 4 D) 2 **Ans : (B)**

2. **Let G be defined as all formal symbols $\{x^i y^j$, i, j = 0, 1, 2, ..., n − 1 where**
   i. $x^i y^j = x^{i'} y^{j'}$ if and only if i=i', j=j'
   ii. $x^2 = y^n = e$, n>2
   iii. $xy = y^{-1}x\}$.
   Then G is :
   A) an abelian group of order n
   B) an abelian group of order 2n
   C) a non-abelian group of order n
   D) a non-abelian group of order 2n
   Ans : (D)

3. **Let J(√2) be the ring of real numbers of the form m+n √2, where m,n are integers, with respect to usual addition and multiplication of real numbers. Define φ : J(√2)→ J(√2) by φ (m+n√2)=m–n√2. Then the kernel of φ is equal to:**
   A) J(√2) B) 0
   C) J(–√2) D) √2 **Ans : (B)**

4. **Consider the following set of vectors over the real numbers R:**
   a) (1,1,0),(1,1,1) and (0,1,–1)
   b) (1,1,0,0), (0,1,–1,0) and (0,0,0,3)
   c) (1,–1,0), (1,3,–1) and (5,3,–2)
   d) (2,–3) and (6,–9)
   Which of the following statements hold true?
   A) a) and b) are linearly dependent over R, c) and d) are linearly independent over R
   B) a) and d) are linearly dependent over R, b) and c) are linearly independent over R
   C) a) and c) are linearly independent over R, b) and d) are linearly dependent over R
   D) a) and b) are linearly independent over R, c) and d) are linearly dependent over R
   Ans : (D)

5. **Let F_0 be the field of rational numbers. Let $\varpi = e^{2\pi i/5}, K = F_0(\varpi)$. Then G(K,$F_0$) is a:**
   A) cyclic group of order 1
   B) cyclic group of order 2
   C) cyclic group of order 5
   D) cyclic group of order 4 **Ans : (D)**

6. **If U is any vector space over F of dimension m, then the dimension of Hom (U,U) and Hom (U,F) are respectively:**
   A) m,m B) m,m^2
   C) m^2,m D) m^2, m^2 **Ans : (C)**

7. **If $E \subset [a,b], E' = [a,b] - E$, then $\overline{m}E + \underline{m}E'$ is:**
   A) b+a B) b − a
   C) $\frac{b+a}{2}$ D) $\frac{b-a}{2}$
   Ans : (B)

8. **Let f be a bounded function on the closed and bounded interval [a,b]. Let σ be any subdivision of [a,b]. The upper sum for f and the upper integral for f over [a,b] are:**
   A) $U[f;\sigma] = \sum_{K=1}^{n} M[f;I_K]|I_K|$
   $\int_a^{-b} f(x)dx = \text{g.I.b } U[f;\sigma]$
   $U[f;\sigma] = \sum_{K=1}^{n} M[f;I_K]|I_K|$
   $\int_a^{-b} f(x)dx = \text{g.I.b } U[f;\sigma]$
   B) $U[f;\sigma] = \sum_{K=1}^{n} m[f;I_K]|I_K|$
   $\int_a^{-b} f(x)dx = \text{I.U.b } U[f;\sigma]$
   $U[f;\sigma] = \sum_{K=1}^{n} m[f;I_K]|I_K|$
   $\int_a^{-b} f(x)dx = \text{I.U.b } U[f;\sigma]$

C) $U[f;\sigma] = \sum_{K=1}^{n} m[f;I_K]|I_K|$

$\int_a^{-b} f(x)dx = \text{g.I.b } U[f;\sigma]$

$U[f;\sigma] = \sum_{K=1}^{n} m[f;I_K]|I_K|$

$\int_a^{-b} f(x)dx = \text{g.I.b } U[f;\sigma]$

D) $U[f;\sigma] = \sum_{K=1}^{n} M[f;I_K]|I_K|$

$\int_a^{-b} f(x)dx = \text{I.U.b } U[f;\sigma]$

$U[f;\sigma] = \sum_{K=1}^{n} M[f;I_K]|I_K|$

$\int_a^{-b} f(x)dx = \text{I.U.b } U[f;\sigma]$

Ans : (A)

9. **Let f(x)=sin x, g(x)=cos x , $-\frac{\pi}{2} \le x < 0$ The value of 'C' which satisfies $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$ is:**

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$

C) $-\frac{\pi}{2}$ D) $-\frac{\pi}{4}$

Ans : (D)

10. **Which of the following sequence of function is uniformly convergent?**

A) $f_n(x) = nx(1-x)^n$, on [0,1]

B) $f_n(x) = nxe^{-nx^2}, x \ge 0$ on [0,k], k>0

C) $f_n(x) = \frac{1}{1+x^n}$, on $[0,\infty)$

D) $f_n(x) = \frac{1}{x+n}$, on [0,K], $k > 0$

Ans : (D)

11. **Let [a,b] be a closed and bounded interval in R'. Let G_1 and G_2 are open subsets of [a,b]. Then $|G_1|+|G_2|$ is:**

A) $|G_1 \cup G_2|$

B) $|G_1 \cap G_2|$

C) $|G_1 \cup G_2| + |G_1 \cap G_2|$

D) $|G_1 \cup G_2| - |G_1 \cap G_2|$

Ans : (C)

12. **Which of the following statements is not true?**

A) Continuous image of a connected set is connected

B) A metric space is connected if and only if every two valued function on it is constant

C) Every open interval is connected

D) Metric space S=R-{0} is connected

Ans : (D)

13. **Let $E \subseteq M$ Where M is measurable, m(M)<∞ E is measurable if and only if:**

A) m(M)=m*(E) – m*(M–E)

B) m(M)=m*(E) + m*(M–E)

C) m(M)=m(E) + m(M–E)

D) m(M)=m(E) – m(M–E)

Ans : (B)

14. **The value of $\int_0^{\infty} \frac{\cos ax}{b^2+x^2} dx, b>0$ is:**

A) $\frac{\pi}{2b} e^{-|a|b}$ B) $\frac{\pi}{b} e^{-|a|b}$

C) $\frac{\pi}{2b} e^{-ab}$ D) $\frac{\pi}{2b} e^{ab}$

Ans : (A)

15. **Let $\{\phi_1, \phi_2, ...\}$ be an orthonormal on I and $f \in L^2(I)$. Further let $s_n(x) = \sum_{K=0}^{n} c_K \varphi_K(x)$ and $t_n(x) = \sum_{K=0}^{n} b_K \varphi_K(x)$, where $C_K = (f, \varphi_K)$ for k=0.1,2...... and b_0, b_1, b_2,...... are arbitrary complex numbers. Then for each n:**

A) $||f-s_n|| = ||f-t_n||$ always

B) $||f-s_n|| \le ||f-t_n||$

C) $||f-s_n|| \ge ||f-t_n||$

D) $||f-s_n||$ and $||f-t_n||$ are not comparable

Ans : (B)

16. Fourier series expansion of x in $(-\pi, \pi)$ is :

A) $x = 2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos nx}{n}$

B) $x = 2\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sin nx}{n}$

C) $x = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos nx}{n^2}$

D) $x = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{\sin nx}{n^2}$

Ans : (B)

17. The Fourier series expansion of $\pi x - x^2$ in $(0, \pi)$ is :

A) $\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{(2n-1)^3}$

B) $\frac{8}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^3}$

C) $2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$

D) $2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos nx}{n}$

Ans : (A)

18. Let $\{f_0, f_1, \ldots\ldots\}$ be linearly independent system on [a,b]. Define a system $\{g_0, g_1, \ldots\ldots\}$ recursely as $g_0 = f_0$, $g_{r+1} = f_{r+1} - \sum_{K=1}^{r} a_K g_K$, where $a_K = \frac{(f_{r+1}, g_K)}{(g_K, g_K)}$ if $||g_k|| \neq 0$ and $a_k = 0$ if $||g_k|| = 0$. Let $(f,g) = \int_{-1}^{+1} f(t)g(t)dt$.

Applying the Gram-Schmidt process to the system of polynomials $\{1, t, t^2, \ldots\}$ on the interval $[-1,1]$ the the value of $g_4(t)$ is:

A) $t^4 - \frac{6}{7}t^3 + \frac{3}{35}$ B) $t^4 - \frac{6}{7}t^2 + \frac{3}{35}$

C) $t^4 - \frac{6}{7}t^2 + \frac{2}{35}$ D) $t^4 + \frac{6}{7}t^2 + \frac{3}{35}$

Ans : (B)

19. Which of the following statements is not correct?

A) Fourier series can be integrated term by term

B) If f is of bounded variation on the compact interval $[x-\delta, x+\delta]$ for some $\delta < \pi$, then the limt $S(x)$ exists and the Fourier series generates by f converges to $S(x)$

C) Convolution of two functions can be interpreted as a special kind of integral transform in which the Kernel $K(x,y)$ depends only on the difference $x-y$

D) Fourier series cannot be integrated term by term

Ans : (D)

20. Which of the following statements are correct?

a) if $\{\varphi_n\}$ is an orthonormal set on [a,b], then $\int_a^b |\varphi_n(x)|^2 dx = 1$

b) $\frac{e^{inx}}{\sqrt{2\pi}}, n = 0, 1, 2, \ldots\ldots$ is an orthonormal set on $[-\pi, \pi]$

c) $\left\{\frac{1}{\sqrt{2\pi}}, \frac{\cos x}{\sqrt{\pi}}, \frac{\sin x}{\sqrt{\pi}}, \frac{\cos 2x}{\sqrt{\pi}}, \frac{\sin 2x}{\sqrt{\pi}}, \ldots\ldots\right\}$ is an orthonormal set on $[-\pi, \pi]$

Select the correct answer :

A) a) and b) only true

B) b) and c) only true

C) a) and c) only true

D) All a),b),c) are true

Ans : (D)

21. Choose the statement which is not correct

a) Fourier transform converts multiplication by a character into translation

b) Fourier transform converts translation into multiplication by a character

c) Fourier transform converts convolution into pointwise translation.

Select the correct answer :

A) All a),b), and c) B) c) only

C) b) and c) only D) a) and b) only

Ans : (B)

22. If $\hat{f}$ denotes the Fourier transform of f, then which of the following is not correct?

A) If $g(x) = \overline{f(-x)}$, , then $\hat{g}(t) = \overline{\hat{f}(t)}$

B) If $g(x) = f\left(\frac{x}{\lambda}\right)$ and $\lambda > 0$, then $\hat{g}(t) = \hat{f}(\lambda t)$

C) If $g(x) = -ixf(x)$ and $g \in L'$, then $\hat{g}(t) = \hat{f}'(t)$

D) If $g(x) = g(x-\alpha)$ then $\hat{g}(t) = \hat{f}(t)e^{-i\alpha t}$

Ans : (B)

23. If f and g are continuous functions on the compact interval [a,b] and if $\{\phi_0, \phi_1, \ldots\}$ is an orthonormal system on [a,b], then which of the following statements are equivalent?

a) $(f, \phi_n)=(g, \phi_n)$ for all n implies $f=g$

b) $(f, \phi_n)=(g, \phi_n)$ for all n implies $f\neq g$

c) $(f, \phi_n)=0$ for all n implies $f=0$

d) $(f, \phi_n)=0$ for all n implies $\phi_n=0$

Select the correct answer :

A) a) and c) B) b) and d)

C) b) and c) D) c) and d)

Ans : (A)

24. The length of the curve given as the intersection of the surfaces $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $x = a\cosh\left(\frac{z}{a}\right)$ from the point (a,0,0) to the point (x, y, z) is :

A) $\frac{(a^2+b^2)y}{b}$ B) $\frac{(a^2+b^2)^{1/2}y}{b}$

C) $\frac{(a^2+b^2)^{1/2}y^2}{b}$ D) $\frac{(a^2+b^2)y^2}{b}$

Ans : (B)

25. The tortion of the cubic curve $\bar{r} = (U, U^2, U^3)$ is:

A) $\frac{3}{(3U^4+3U^2+1)}$ B) $\frac{3}{(9U^4+9U^2+1)}$

C) $\frac{1}{(9U^4+9U^2+1)}$ D) $\frac{9}{(9U^4+9U^2+1)}$

Ans : (B)

26. The metric of the curves of the family $\frac{v^3}{u^2}$ = constant is (here U>0 and V>0):

A) $(1+u^2)du^2-2uvdudv+(1+v^2)dv^2$

B) $(1+u^2)du^2+2uvdudv+(1+v^2)dv^2$

C) $v^2du^2-2uvdudv+2u^2dv^2$

D) $v^2du^2+2uvdudv+2u^2dv^2$ **Ans : (C)**

27. The curvature of a curve at a point P is equal to the normal curvature at P in the direction of that curve if the angle between principal normal $\bar{n}$ and surface normal $\bar{N}$ at P is :

A) 0 B) $\frac{\pi}{2}$

C) $\frac{\pi}{3}$ D) $\frac{\pi}{4}$ **Ans : (A)**

28. Which of the following statements is true?

A) The two families of lines of curvature are not orthogonal

B) The direction of a line of curvature at any point is along a direction perpendicular to principal direction at that point

C) The curvature of a line of curvature at any point is not necessarily equal to a principal curvature at that point

D) The combined equation of two families of lines of curvature is $(Em-FL)du^2 - (EN-GL)du+(FN-GM)=0$

Ans : (C)

29. Which of the following statements are true?

i. A necessary and sufficient condition for a curve to be a straight line is that the curvature is zero at all points at the curve

ii. A necessary and sufficient condition for a curve to be a plane curve is that torsion is zero at all points at the curve

iii. A necessary and sufficient condition for a curve to be a plane curve is that $[\bar{r}', \bar{r}'', \bar{r}'''] = 0$

Select the correct answer :

A) All i), ii) and iii)

B) none of i), ii) and iii)

C) i), ii) only

D) ii), and iii) only **Ans : (A)**

30. The torsion of the spherical indicatrix of the binormal is:

A) $\dfrac{K'\tau - K\tau'}{\tau(K^2+\tau^2)}$ B) $\dfrac{K'\tau + K\tau'}{\tau(K^2+\tau^2)}$

C) $\dfrac{K'\tau - K\tau'}{K(K^2+\tau^2)}$ D) $\dfrac{K'\tau + K\tau'}{K(K^2+\tau^2)}$

Ans : (A)

31. If ABC is the geodesic triangle formed by geodesic arcs AB, BC, CA, then the total curvature of the geodesic triangle ABC is:

A) $A+B+C+\pi$ B) $A+B+C-\pi$

C) $A+B+C+2\pi$ D) $A+B+C-2\pi$

Ans : (B)

32. The envelope of the family of planes $3a^2x-3ay+z=a^3$ is :

A) $(xy+z)^2 = 4(y^2-zx)(x^2-y)$

B) $(xy-z)^2 = 4(y^2-zx)(x^2-y)$

C) $(xy+z) = 4(y^2-zx)(x^2-y)$

D) $(xy-z) = 4(y^2-zx)(x^2-y)$

Ans : (C)

33. The number of basic feasible solution to the following system equations

$2x_1+6x_2+2x_3+x_4=3$

$6x_1+4x_2+4x_3+6x_4=2$ is equal to:

A) 0 B) 1

C) 2 D) 3 **Ans : (D)**

34. Consider the following statements:

i. Revised simplex method is more economical as it stores only relevant information needed currently.

ii. In solving linear programming problem by revised simplex method we have (m+1) simultaneous equations in (n+1) variables.

Then :

A) (i) is correct and (ii) is not correct

B) Both (i) and (ii) are not correct

C) Both (i) and (ii) are correct

D) (i) is not correct and (ii) is correct

Ans : (C)

35. In a network analysis if for any activity i–j, the earliest start time of the event j is 13 hours, the latest completion of the event i is 6 hours and the duration of the activity i–j is 3 hours then the independent float of the activity i–j is equal to :

A) 0 hours B) 7 hours

C) 10 hours D) 4 hours **Ans : (D)**

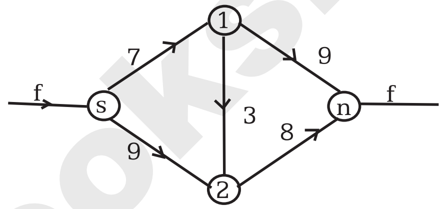
36. In M|M|1 model the probability density function of waiting time + service time distribution is (Here λ is arrival rate and μ is service rate)

A) $(\mu-\lambda)e^{-(\mu-\lambda)t}$ B) $(\lambda-\mu)e^{-(\mu-\lambda)t}$

C) $\dfrac{\lambda}{\mu}(\mu-\lambda)e^{-(\mu-\lambda)t}$ D) $\lambda\left(1-\dfrac{\lambda}{\mu}\right)$

Ans : (A)

37. The maximal flow from the source s to the sink n in the following flow network is



A) 10 units B) 12 units

C) 15 units D) 18 units

Ans : (C)

38. Which of the following two games is not fair?

I. $\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix} \end{array}$

II. $\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 0 & 2 \\ -1 & 4 \end{pmatrix} \end{array}$

Select the correct answer :

A) (I) only B) (II) only

C) both (I) and (II) D) neither (I) nor (II)

Ans : (D)

39. The value of the following 2 person zero–sum game is

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{pmatrix} 9&3&1&8&0 \\ 6&5&4&6&7 \\ 2&4&3&3&8 \\ 5&6&2&2&1 \end{pmatrix} \end{array}$$

A) not determinable

B) 2

C) 4

D) 3 **Ans : (C)**

40. The minimum value of $z=U_1^2+U_2^2+U_3^2$ subject to the constraints $U_1+U_2+U_3=10$, U_1,U_2,U_3 are non - negative integers is:

A) $3\left(\frac{10}{3}\right)^2$ B) $\frac{100}{3}$

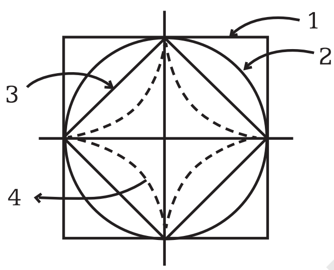
C) 27 D) 34 **Ans : (D)**

41. The failure time of a component of a machine is exponentially distributed with mean life equal to 350 hours. The design is modified so that the mean life is increased to 400 hours. Then the increase in reliability if in both cases the component is tested for 800 hours is :

A) $e^{-0.5}-e^{-\frac{16}{7}}$ B) $e^{-\frac{16}{7}}-e^{-0.5}$

C) $e^{-0.5}+e^{-\frac{16}{7}}$ D) $e^{-\frac{39}{14}}$ **Ans : (A)**

42. In the following diagram, the arrow (1) arrow (2) arrow (3) and arrow (4) represent:



A) $\|x\|_\infty=1,\ \|x\|_1=1,\ \|x\|_2=1,\ \|x\|_{1/2}=1$

B) $\|x\|_1=1,\ \|x\|_2=1,\ \|x\|_\infty=1,\ \|x\|_{1/2}=1$

C) $\|x\|_\infty=1,\ \|x\|_2=1,\ \|x\|_1=1,\ \|x\|_{1/2}=1$

D) $\|x\|_{1/2}=1,\ \|x\|_1=1,\ \|x\|_2=1,\ \|x\|_\infty=1$

Ans : (C)

43. If T is an arbitrary operator on H, and if α and β are scalar, $\alpha T+\beta T^*$ is normal if:

A) $\|\alpha\| = \|\beta\|$ B) $\alpha = 2, \beta = 1$

C) $\alpha = \frac{1}{2}, \beta = \frac{1}{4}$ D) $\alpha = 1, \beta = 2$

Ans : (A)

44. If $S_n = x_1 + x_2 + \ldots\ldots + x_n$ where $\|x_n\| < \frac{1}{2^{n-1}}$ the value of $\|S_n\|$ is :

A) $<\frac{1}{2}$ B) ≤ 1

C) $=2$ D) <2 **Ans : (D)**

45. Which of the following statement is true about conjugate operator T* and adjoint operator T*?

A) Conjugate operator T* operates on functionals and Adjoint operator T* operates on functionals

B) Conjugate operator T* operates on vectors and Adjoint operator T* operates on vectors

C) Conjugate operator T* operates on functionals and Adjoint operator T* operates on vectors

D) Conjugate operator T* operates on vectors and Adjoint operator T* operates on functionals

Ans : (C)

46. Cauchy Schwartz inequality? $|(x,y)| \leq \|x\|\,\|y\|$ reduces to an equality when:

A) $\|x+y\| < \|x\| + \|y\|$

B) $\|x+y\| > \|x\| + \|y\|$

C) $\|x+y\| = \|x\| + \|y\|$

D) $\|x+y\|^2 = 2[\|x\|^2 + \|y\|^2]$ **Ans : (C)**

47. In a Banach Algebra A, for every element x such that $\|x-1\| < 1$ is regular the value of x^{-1} is given by:

A) $x^{-1} = 1 - \sum_{n=1}^{\infty}(1-x)^n$

B) $x^{-1} = 1 + \sum_{n=1}^{\infty}(1-x)^n$

C) $x^{-1} = 1 + \sum_{n=1}^{\infty}(1+x)^n$

D) $x^{-1} = \sum_{n=1}^{\infty}(x-1)^n$ **Ans : (B)**

48. Operators T_1, T_2, T_3 from $R^2 \to R^2$ defined as

$(x_1, x_2) \to (x_1, 0)$

$(x_1, x_2) \to (0, x_2)$

$(x_1, x_2) \to (x_2, x_1)$

Range and Null space of T_1, T_2, T_3 are respectively:

A) x_1 axis, x_2 axis, R^2 — Range x_1 axis, x_2 axis, R^2 — Null space

B) x_1 axis, x_2 axis, (0,0) — Range x_2 axis, x_1 axis, (0,0) — Null space