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NOTE FROM PUBLISHER

It gives me great pride and pleasure in bringing to you **Sura's Mathematics Guide** for **9th Standard**. It is prepared as per the Latest New Textbook.

This guide encompasses all the requirements of the students to comprehend the text and the evaluation of the textbook.

- Additional questions have been provided exhaustively for clear understanding of the units under study.
- Chapter-wise Unit Test are given.

In order to learn effectively, I advise students to learn the subject section-wise and practice the exercises given. It will be a teaching companion to teachers and a learning companion to students.

Though these salient features are available in this Guide, I cannot negate the indispensable role of the teachers in assisting the student to understand the subject thoroughly.

I sincerely believe this guide satisfies the needs of the students and bolsters the teaching methodologies of the teachers.

I pray the almighty to bless the students for consummate success in their examinations.

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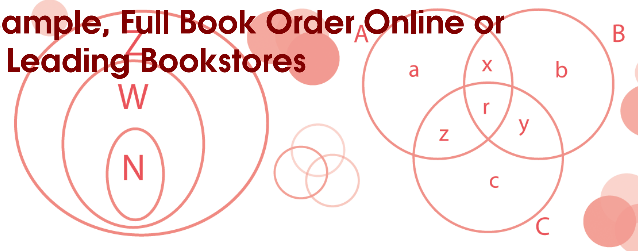
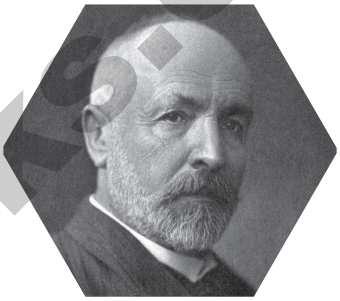


1

SET LANGUAGE

1.1 Introduction



In our daily life, we often deal with collection of objects like books, stamps, coins, etc. Set language is a mathematical way of representing a collection of objects.

1.2 Set :

(i) A set is a well - defined collection of objects.

(ii) The objects of a set are called its members or elements.

For example,

1. The collection of all books in a District Central Library.
2. The collection of all colours in a rainbow.

1.3 Representation of a Set :

The collection of odd numbers can be described in many ways:

(1) "The set of odd numbers" is a fine description.

(2) It can be written as $\{1, 3, 5, \ldots\}$.

(3) Also, it can be said as the collection of all numbers x where x is an odd number.

1.3.1 Descriptive Form :

In descriptive form, a set is described in words.

For Example,

(i) The set of all vowels in English alphabets.

(ii) The set of all whole numbers.

1.3.2 Set Builder Form or Rule Form :

In set builder form, all the elements are described by a rule.

For example,

(i) $A = \{x : x \text{ is a vowel in English alphabets}\}$

(ii) $B = \{x \mid x \text{ is a whole number}\}$

1.3.3 Roster Form or Tabular Form

A set can be described by listing all the elements of the set.

For example,

(i) $A = \{a, e, i, o, u\}$

(ii) $B = \{0,1,2,3,\ldots\}$

Exercise 1.1

1. Which of the following are sets?

(i) The Collection of prime numbers upto 100.

(ii) The Collection of rich people in India.

(iii) The Collection of all rivers in India.

(iv) The Collection of good Hockey players.

Sol. (i) A = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97}

As the collection of prime numbers upto 100 is known and can be counted (well defined). Hence this is a set.

(ii) The collection of rich people in India. Rich people has no definition.
Hence, it is not a set.

(iii) A = {Cauvery, Sindhu, Ganga, ...}
Hence, it is a set.

(iv) The collection of good hockey players is not a well - defied collection because the criteria for determining a hockey player's talent may vary from person to person.

Hence, this collection is not a set.

2. List the set of letters of the following words in Roster form.

(i) INDIA **(ii) PARALLELOGRAM**

(iii) MISSISSIPPI **(iv) CZECHOSLOVAKIA**

Sol. (i) A = {I, N, D, A}

(ii) B = {P, A, R, L, E, O, G, M}

(iii) C = {M, I, S, P}

(iv) D = {C, Z, E, H, O, S, L, V, A, K, I}.

3. Consider the following sets A = {0, 3, 5, 8} B = {2, 4, 6, 10} C = {12, 14, 18, 20}

(a) State whether True or false.

(i) $18 \in C$ (ii) $6 \notin A$ (iii) $14 \notin C$ (iv) $10 \in B$ (v) $5 \in B$ (vi) $0 \in B$

(b) Fill in the blanks?

(i) $3 \in$ ____ (ii) $14 \in$ _____ (iii) 18 ____ B (iv) 4 _____ B

Sol. (a) (i) True (ii) True (iii) False (iv) True (v) False (vi) False.

(b) (i) A (ii) C (iii) $\notin$ (iv) $\in$

4. Represent the following sets in Roster form.

(i) A = The set of all even natural numbers less than 20. *[QY-2019]*

(ii) $B = \{y : y = \frac{1}{2n}, n \in \mathbb{N}, n \leq 5\}$

(iii) $C = \{x : x$ is perfect cube, $27 < x < 216\}$

(iv) $D = \{x : x \in \mathbb{Z}, -5 < x \leq 2\}$

Sol. (i) A = {2, 4, 6, 8, 10, 12, 14, 16, 18}

(ii) N = { 1, 2, 3, 4, 5}

$$\text{if, } n = 1, y = \frac{1}{2n} = \frac{1}{2\times1} = \frac{1}{2}$$

$$n = 2, y = \frac{1}{2\times2} = \frac{1}{4}$$

$$n = 3, y = \frac{1}{2\times3} = \frac{1}{6}$$

$$n = 4, y = \frac{1}{2\times4} = \frac{1}{8}$$

$$n = 5, y = \frac{1}{2\times5} = \frac{1}{10}$$

$$\therefore \text{B} = \left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\right\}$$

(iii) C = {64, 125}

(iv) D = {–4, –3, –2, –1, 0, 1, 2}

5. Represent the following sets in set builder form.

(i) B = The set of all Cricket players in India who scored double centuries in One Day Internationals.

(ii) $\mathbf{C} = \left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\ldots\right\}$.

(iii) D = The set of all tamil months in a year.

(iv) E = The set of an odd Whole numbers less than 9.

Sol. (i) B = {x : x is an Indian player who scored double centuries in One Day Internationals}

(ii) $\text{C} = \{x : x = \frac{n}{n+1}, n \in \mathbb{N}\}$

(iii) D = {x : x is a tamil month in a year}

(iv) E = {x : x is an odd number, $x \in \mathbb{W}$, $x < 9$, where W is the set of whole numbers}.

6. Represent the following sets in descriptive form.

(i) P = { January, June, July}

(ii) Q = {7,11,13,17,19,23,29}

(iii) R = {x : $x \in \mathbb{N}$, $x < 5$}

(iv) S = {x : x is an consonant in English alphabets}

Sol. (i) P is the set of English Months begining with J.

(ii) Q is the set of all prime numbers between 5 and 31.

(iii) R is the set of all natural numbers less than 5.

(iv) S is the set of all English consonants.

1.4. Types of sets

1.4.1 Empty Set or Null Set :

A set consisting of no element is called the empty set or null set or void set.

For example,

A=$\{x : x$ is an odd integer and divisible by $2\}$

$\therefore$ A=$\{ \}$ or $\varnothing$

1.4.2 Singleton Set :

A set which has only one element is called a singleton set.

For example,

$A = \{x : 3 < x < 5, x \in \mathbb{N}\}$ where $A = \{4\}$

1.4.3 Finite Set :

A set with finite number of elements is called a finite set.

For example,

1. The set of family members.
2. The set of indoor/outdoor games you play.

1.4.4 Infinite Set :

A set which is not finite is called an infinite set.

For example,

(i) $\{5,10,15,...\}$ (ii) The set of all points on a line.

1.4.5 Equivalent Sets :

Two finite sets A and B are said to be equivalent if they contain the same number of elements. It is written as $A \approx B$.

If A and B are equivalent sets, then $n(A) = n(B)$.

1.4.6 Equal Sets :

Two sets are said to be equal if they contain exactly the same elements, otherwise they are said to be unequal.

In other words, two sets A and B are said to be equal, if

(i) every element of A is also an element of B

(ii) every element of B is also an element of A.

1.4.7 Subset :

Let A and B be two sets. If every element of A is also an element of B, then A is called a subset of B. We write $A \subseteq B$.

1.4.8 Proper Subset :

Let A and B be two sets. If A is a subset of B and A≠B, then A is called a proper subset of B and we write $A \subset B$.

For example,

If A=$\{1,2,5\}$ and B=$\{1,2,3,4,5\}$ then A is a proper subset of B i.e. $A \subset B$.

1.4.9 Power set :

The set of all subsets of A is said to be the power set of the set A and is denoted as P(A)

For example,

Let $A = \{-3, 4\}$

The subsets of A are , $\varnothing$, $\{-3\}$, $\{4\}$, $\{-3, 4\}$

Then the power set of A is $P(A) = \{\varnothing, \{-3\}, \{4\}, \{-3, 4\}\}$

Exercise 1.2

1. Find the cardinal number of the following sets.

(i) $M = \{p, q, r, s, t, u\}$

(ii) $P = \{x : x = 3n + 2, n \in \mathbb{W} \text{ and } x < 15\}$

(iii) $Q = \{y : y = \frac{4}{3n}, n \in \mathbb{N} \text{ and } 2 < n \leq 5\}$

(iv) $R = \{x : x \text{ is an integers}, x \in \mathbb{Z} \text{ and } -5 \leq x < 5\}$

(v) **S = The set of all leap years between 1882 and 1906.**

Sol. (i) $n(M) = 6$

(ii) $W = \{0, 1, 2, 3, \dots\dots\dots\}$

if $n = 0$, $x = 3(0) + 2 = 2$

if $n = 1$, $x = 3(1) + 2 = 5$

if $n = 2$, $x = 3(2) + 2 = 8$

if $n = 3$, $x = 3(3) + 2 = 11$

if $n = 4$, $x = 3(4) + 2 = 14$

$\therefore P = \{2, 5, 8, 11, 14\}$

$n(P) = 5$

(iii) $N = \{1, 2, 3, 4, \dots\dots\}$

$n \in \{3, 4, 5\}$

if $n = 3$, $y = \frac{4}{3(3)} = \frac{4}{9}$

if $n = 4$, $y = \frac{4}{3(4)} = \frac{4}{12}$

if $n = 5$, $y = \frac{4}{3(5)} = \frac{4}{15}$

$Q = \left\{\frac{4}{9}, \frac{4}{12}, \frac{4}{15}\right\}$

$n(Q) = 3$

(iv) $x \in z$

$R = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$n(R) = 10.$

(v) $S = \{1884, 1888, 1892, 1896, 1904\}$

$n(S) = 5.$

2. Identify the following sets as finite or infinite.

(i) **X = The set of all districts in Tamilnadu.**

(ii) **Y = The set of all straight lines passing through a point.**

(iii) $A = \{x : x \in \mathbb{Z} \text{ and } x < 5\}$

(iv) $B = \{x : x^2 - 5x + 6 = 0, x \in \mathbb{N}\}$

Sol. (i) Finite set

(ii) Infinite set

(iii) A = { ...., –2, –1, 0, 1, 2, 3,4} ∴ Infinite set

(iv) $x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

B = {3, 2} ∴ Finite set.

3. Which of the following sets are equivalent or unequal or equal sets?

(i) A = The set of vowels in the English alphabets.
B = The set of all letters in the word "VOWEL"

(ii) C = {2, 3, 4, 5}
D = $\{x : x \in \mathbb{W}, 1 < x < 5\}$

(iii) X = $\{x : x$ is a letter in the word "LIFE"$\}$
Y = {F, I, L, E}

(iv) G = $\{x : x$ is a prime number and $3 < x < 23\}$
H = $\{x : x$ is a divisor of $18\}$

Sol. (i) A = {*a*, *e*, *i*, *o*, *u*}

B = {V, O,W, E, L}

The sets A and B contain the same number of elements. ∴ Equivalent sets

(ii) C = {2, 3, 4, 5}

D = {2, 3, 4} ∴ Unequal sets

(iii) X = {L, I, F, E}

Y = {F, I, L, E}

The sets X and Y contain exactly the same elements. ∴ Equal sets.

(iv) G = {5, 7, 11, 13, 17, 19}

H = {1, 2, 3, 6, 9, 18} ∴ Equivalent sets.

4. Identify the following sets as null set or singleton set.

(i) A = $\{x : x \in \mathbb{N}, 1 < x < 2\}$

(ii) B = The set of all even natural numbers which are not divisible by 2.

(iii) C = {0}.

(iv) D = The set of all triangles having four sides.

Sol. (i) A = { } ∵ There is no element in between 1 and 2 in Natural numbers.∴ Null set

(ii) B = { }∵ All even natural numbers are divisible by 2. ∴ B is Null set

(iii) C = {0} ∴ Singleton set

(iv) D = { } ∵ No triangle has four sides. ∴ D is a Null set.

5. State which pairs of sets are disjoint or overlapping?

(i) A=$\{f,i,a,s\}$ and B=$\{a,n,f,h,s\}$

(ii) C =$\{x : x$ is a prime number, $x > 2\}$ and D =$\{x : x$ is an even prime number$\}$

(iii) E=$\{x : x$ is a factor of 24$\}$ and F=$\{x : x$ is a multiple of 3, $x < 30\}$

Sol. (i) A = $\{f, i, a, s\}$

B = $\{a, n, f, h, s\}$

$A \cap B = \{f, i, a, s\} \cap \{a, n, f, h, s\} = \{f, a, s\}$

Since $A \cap B \neq \phi$, A and B are overlapping sets.

(ii) $C = \{3, 5, 7, 11, \text{.........}\}$

$D = \{2\}$

$C \cap D = \{3, 5, 7, 11, \text{........}\} \cap \{2\} = \{\ \}$

Since $C \cap D = \varnothing$, C and D are disjoint sets.

(iii) $E = \{1, 2, 3, 4, 6, 8, 12, 24\}$

$F = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

$E \cap F = \{1, 2, 3, 4, 6, 8, 12, 24\} \cap \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

$= \{3, 6, 12, 24\}$

Since $E \cap F \neq \phi$, E and F are overlapping sets.

6. If S = {square, rectangle, circle, rhombus, triangle}, list the elements of the following subset of S.

(i) The set of shapes which have 4 equal sides.

(ii) The set of shapes which have radius.

(iii) The set of shapes in which the sum of all interior angles is 180°

(iv) The set of shapes which have 5 sides.

Sol. (i) {Square, Rhombus} (ii) {Circle}

(iii) {Triangle} (iv) Null set.

7. If A=$\{a,\{a,b\}\}$, write all the subsets of A.

Sol. $A = \{a, \{a, b\}\}$ subsets of A are $\{\ \}$ $\{a\}$, $\{a, b\}$, $\{a, \{a, b\}\}$.

8. Write down the power set of the following sets.

(i) A=$\{a,b\}$ (ii) B={1,2,3} (iii) D=$\{p,q,r,s\}$ (iv) E = $\varnothing$

Sol. (i) The subsets of A are $\varnothing$, $\{a\}$, $\{b\}$, $\{a, b\}$

The power set of A

$P(A) = \{\varnothing, \{a\}, \{b\}, \{a, b\}\}$

(ii) The subsets of B are φ, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}

The power set of B

$P(B) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$

(iii) The subset of D are $\varnothing$, $\{p\}$, $\{q\}$, $\{r\}$, $\{s\}$, $\{p, q\}$, $\{p, r\}$, $\{p, s\}$, $\{q, r\}$, $\{q, s\}$, $\{r, s\}$, $\{p, q, r\}$, $\{q, r, s\}$, $\{p, r, s\}$, $\{p, q, s\}$, $\{p, q, r, s\}\}$

The power set of D

$P(D) = \{\varnothing, \{p\}, \{q\}, \{r\}, \{s\}, \{p, q\}, \{p, r\}, \{p, s\}, \{q, r\}, \{q, s\}, \{r, s\}, \{p, q, r\}, \{q, r, s\}, \{p, r, s\}, \{p, q, s\}, \{p, q, r, s\}\}$

(iv) The power set of E

$P(E) = \{\{\ \}\}$.

9. Find the number of subsets and the number of proper subsets of the following sets.

(i) W={red,blue,yellow} (ii) X =$\{x^2 : x \in \mathbb{N}, x^2 \leq 100\}$.

Sol. (i) Given W = {red, blue, yellow}

Then $n(W) = 3$

The number of subsets $= n[P(W)] = 2^3 = 8$

The number of proper subsets $= n[P(W)] - 1 = 2^3 - 1 = 8 - 1 = 7$

(ii)
$$\begin{aligned} \text{Given X} &= \{1, 2, 3, \ldots\ldots\} \\ X^2 &= \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\} \\ n(X) &= 10 \\ \text{The Number of subsets} &= n[P(X)] = 2^{10} = 1024 \\ \text{The Number of proper subsets} &= n[P(X)]-1 = 2^{10} - 1 = 1024 - 1 = 1023. \end{aligned}$$

10. (i) If $n(A) = 4$, find $n[P(A)]$. ***[QY-2019]*** **(ii) If $n(A) = 0$, find $n[P(A)]$.**

(iii) If $n[P(A)] = 256$, find $n(A)$.

Sol. (i)
$$\begin{aligned} n(A) &= 4 \\ n[P(A)] &= 2^n = 2^4 = 16 \end{aligned}$$

(ii)
$$\begin{aligned} n(A) &= 0 \\ n[P(A)] &= 2^0 = 1 \end{aligned}$$

(iii)
$$n[P(A)] = 256$$

$$\begin{array}{r|l} 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$\begin{aligned} n[P(A)] &= 2^8 \\ \therefore n(A) &= 8. \end{aligned}$$

1.5 Set operations :

1.5.1 Complement of a Set

The Complement of a set A is the set of all elements of U (the universal set) that are not in A.

It is denoted by A' or A^C. In symbols $A' = \{x : x \in U, x \notin A\}$

1.5.2 Union of Two Sets

The union of two sets A and B is the set of all elements which are either in A or in B or in both. It is denoted by $A \cup B$ and read as A union B.

In symbol, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

1.5.3 Intersection of Two Sets

The intersection of two sets A and B is the set of all elements common to both A and B. It is denoted by $A \cap B$ and read as A intersection B.

In symbol , $A \cap B = \{x : x \in A \text{ and } x \in B\}$

1.5.4 Difference of Two Sets

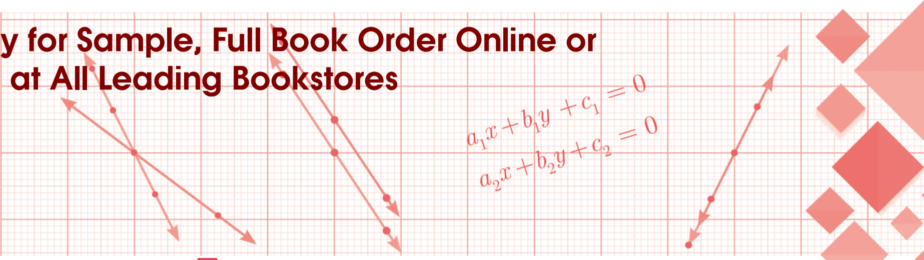
Let A and B be two sets, the difference of sets A and B is the set of all elements which are in A, but not in B. It is denoted by A–B or A\B and read as A difference B.

In symbol, $A - B = \{x : x \in A \text{ and } x \notin B\}$; $B - A = \{y : y \in B \text{ and } y \notin A\}$.

1.5.5 Symmetric Difference of Sets

The symmetric difference of two sets A and B is the set $(A - B) \cup (B - A)$. It is denoted by $A \,\Delta\, B$.

$A \,\Delta\, B = \{x : x \in A - B \text{ or } x \in B - A\}$



3

ALGEBRA

3.1 Introduction

Algebra, a branch of mathematics consists of proving the most obvious thing in the least obvious way. Algebra is a generalized form of Arithmetic. In Arithmetic we use numbers which have one single definite value while in algebra, we use letters or alphabets which may have any value, we assign to them. These letters are called literal numbers or variables.

This technique called algebra enable many difficult problems in the real world to be analysed and solved using the same methods like we do for any other branch of mathematics.

Constants : Any real number is a constant.

Variables : Letters used for representing unknown real numbers are called variables.

Co-efficients : The constant part of a term that is multiplied by the variable part of the term is called co-efficient.

Standard form of a polynomial: A polynomial f(x) in the increasing or decreasing order of the power of x. This way of writing a polynomial is called the standard form of a polynomial.

Degree of the polynomial : In case of a polynomial having more than one variable, the sum of the powers of the variables in each term is considered and the highest sum so obtained is called the degree of the polynomial.

A very special polynomial : It is zero polynomial having all its Co-efficients to be zero.

3.2 Types of polynomials :

(i) Polynomial on the basis of number of terms
  (1) Monomial – A polynomial having only one term.
  (2) Binomial – A polynomial having two terms.
  (3) Trinomial – A polynomial having three terms.

(ii) Polynomial based on degree.
  (1) Constant – A polynomial of degree zero is called constant polynomial.
  (2) Linear – A polynomial of degree one is called linear polynomial.
  (3) Quadratic – A polynomial of degree two is called quadratic polynomial.
  (4) Cubic – A polynomial of degree three is called cubic polynomial.

3.2.1 Arithmetic of Polynomials :

We can add polynomials, subtract one from another, multiply polynomials, divide one by another.

3.3.1 Addition of Polynomials :

Only like terms can be added. But addition of unlike terms gives a new polynomial.

3.3.2 Subtraction of Polynomials :

Only like terms can be subtracted. But subtraction of unlike terms gives a new polynomial.

3.3.3 Multiplication of Polynomials :

Any two polynomials can be multiplied and the product will also be a polynomial.

Exercise 3.1

1. Which of the following expressions are polynomials. If not give reason:

(i) $\frac{1}{x^2}+3x-4$ (ii) $x^2(x-1)$ (iii) $\frac{1}{x}(x+5)$ (iv) $\frac{1}{x^{-2}}+\frac{1}{x^{-1}}+7$

(v) $\sqrt{5}x^2+\sqrt{3}x+\sqrt{2}$ (vi) $m^2-\sqrt[3]{m}+7m-10$

Sol.

	Given Polynomial	Polynomial / Not	Reason
(i)	$\frac{1}{x^2}+3x-4$ $=x^{-2}+3x-4$	Not a polynomial	Negative integral power
(ii)	$x^2(x-1)=x^3-x^2$	Polynomial	Non-negative integral power
(iii)	$\frac{1}{x}(x+5)$ $=x^{-1}(x^1+5)=x^{-1+1}+5x^{-1}$ $=x^0+5x^{-1}=1+5x^{-1}$	Not a polynomial	One of the power of x is negative
(iv)	$\frac{1}{x^{-2}}+\frac{1}{x^{-1}}+7=x^2+x^1+7$	Polynomial	Non-negative integral power
(v)	$\sqrt{5}x^2+\sqrt{3}x+\sqrt{2}$	Polynomial	Non-negative integral power
(vi)	$m^2-\sqrt[3]{m}+7m-10$ $m^2-m^{\frac{1}{3}}+7m-10$	Not a polynomial	One of the power of m is a fraction $\frac{1}{3}$

2. **Write the coefficient of x^2 and x in each of the following polynomials.**

(i) $4+\frac{2}{5}x^2-3x$ *[QY-2019]* (ii) $6-2x^2+3x^3-\sqrt{7}x$ (iii) πx^2-x+2

(iv) $\sqrt{3}x^2+\sqrt{2}x+0.5$ (v) $x^2-\frac{7}{2}x+8$

Sol.

	Polynomial	Co-efficient of x^2	Co-efficient of x
(i)	$4+\frac{2}{5}x^2-3x$	$\frac{2}{5}$	-3
(ii)	$6-2x^2+3x^3-\sqrt{7}x$	-2	$-\sqrt{7}$
(iii)	πx^2-x+2	π	-1
(iv)	$\sqrt{3}x^2+\sqrt{2}x+0.5$	$\sqrt{3}$	$\sqrt{2}$
(v)	$x^2-\frac{7}{2}x+8$	1	$-\frac{7}{2}$

3. **Find the degree of the following polynomials.**

(i) $1-\sqrt{2}y^2+y^7$ (ii) $\frac{x^3-x^4+6x^6}{x^2}$ (iii) $x^3(x^2+x)$

(iv) $3x^4+9x^2+27x^6$ (v) $2\sqrt{5}p^4-\frac{8p^3}{\sqrt{3}}+\frac{2p^2}{7}$

Sol.

	Polynomial	Degree
(i)	$1-\sqrt{2}y^2+y^7$	7
(ii)	$\frac{x^3-x^4+6x^6}{x^2}$ $=\frac{x^{\cancel{3}\,1}}{\cancel{x^2}}-\frac{x^{\cancel{4}\,2}}{\cancel{x^2}}+\frac{6x^{\cancel{6}\,4}}{\cancel{x^2}}$ $=x-x^2+6x^4$	4
(iii)	$x^3(x^2+x)=x^5+x^4$	5
(iv)	$3x^4+9x^2+27x^6$	6
(v)	$2\sqrt{5}p^4-\frac{8p^3}{\sqrt{3}}+\frac{2p^2}{7}$	4

4. **Rewrite the following polynomial in standard form.**

(i) $x - 9 + \sqrt{7}x^3 + 6x^2$ (ii) $\sqrt{2}x^2 - \frac{7}{2}x^4 + x - 5x^3$ (iii) $7x^3 - \frac{6}{5}x^2 + 4x - 1$

(iv) $y^2 + \sqrt{5}y^3 - 11 - \frac{7}{3}y + 9y^4$

Sol.

	Polynomial	Standard form
(i)	$x - 9 + \sqrt{7}x^3 + 6x^2$	$\sqrt{7}x^3 + 6x^2 + x - 9$ (or) $-9 + x + 6x^2 + \sqrt{7}x^3$
(ii)	$\sqrt{2}x^2 - \frac{7}{2}x^4 + x - 5x^3$	$\frac{-7}{2}x^4 - 5x^3 + \sqrt{2}x^2 + x$ (or) $x + \sqrt{2}x^2 - 5x^3 - \frac{7}{2}x^4$
(iii)	$7x^3 - \frac{6}{5}x^2 + 4x - 1$	$7x^3 - \frac{6}{5}x^2 + 4x - 1$ (or) $-1 + 4x - \frac{6}{5}x^2 + 7x^3$
(iv)	$y^2 + \sqrt{5}y^3 - 11 - \frac{7}{3}y + 9y^4$	$9y^4 + \sqrt{5}y^3 + y^2 - \frac{7}{3}y - 11$ (or) $-11 - \frac{7}{3}y + y^2 + \sqrt{5}y^3 + 9y^4$

5. **Add the following polynomials and find the degree of the resultant polynomial.**

(i) $p(x) = 6x^2 - 7x + 2$ $q(x) = 6x^3 - 7x + 15$

(ii) $h(x) = 7x^3 - 6x + 1$ $f(x) = 7x^2 + 17x - 9$

(iii) $f(x) = 16x^4 - 5x^2 + 9$ $g(x) = -6x^3 + 7x - 15$

Sol.

	Polynomials	Addition	Degree of the resultant polynomial
(i)	$p(x) = 6x^2 - 7x + 2$ $q(x) = 6x^3 - 7x + 15$	$6x^2 - 7x + 2$ $6x^3 + 0x^2 - 7x + 15$ $6x^3 + 6x^2 - 14x + 17$	3
(ii)	$h(x) = 7x^3 - 6x + 1$ $f(x) = 7x^2 + 17x - 9$	$7x^3 + 0x^2 - 6x + 1$ $7x^2 + 17x - 9$ $7x^3 + 7x^2 + 11x - 8$	3
(iii)	$f(x) = 16x^4 - 5x^2 + 9$ $g(x) = -6x^3 + 7x - 15$	$16x^4 + 0x^3 - 5x^2 + 0x + 9$ $-6x^3 + 0x^2 + 7x - 15$ $16x^4 - 6x^3 - 5x^2 + 7x - 6$	4

6. **Subtract the second polynomial from the first polynomial and find the degree of the resultant polynomial.**

(i) $p(x) = 7x^2 + 6x - 1$ $\quad q(x) = 6x - 9$ ***[HY-2019]***

(ii) $f(y) = 6y^2 - 7y + 2$ $\quad g(y) = 7y + y^3$

(iii) $h(z) = z^5 - 6z^4 + z$ $\quad f(z) = 6z^2 + 10z - 7$

Sol.

	Polynomial	**Subtraction**	**Degree of the resultant polynomial**
(i)	$p(x) = 7x^2 + 6x - 1$ $q(x) = 6x - 9$	$p(x) - q(x) \Rightarrow 7x^2 + 6x - 1$ $6x - 9$ $(-) \quad (+)$ $7x^2 + 0x + 8$	2
(ii)	$f(y) = 6y^2 - 7y + 2$ $g(y) = 7y + y^3$	$f(y) - g(y) \Rightarrow 6y^2 - 7y + 2$ $y^3 + 0y^2 + 7y + 0$ $(-) \quad (-) \quad (-) \quad (-)$ $-y^3 + 6y^2 - 14y + 2$	3
(iii)	$h(z) = z^5 - 6z^4 + z$ $f(z) = 6z^2 + 10z - 7$	$h(z) - f(z) \Rightarrow z^5 - 6z^4 + 0z^3 + 0z^2 + z + 0$ $+ 6z^2 + 10z - 7$ $(-) \quad (-) \quad (+)$ $z^5 - 6z^4 + 0z^3 - 6z^2 - 9z + 7$	5

7. **What should be added to $2x^3 + 6x^2 - 5x + 8$ to get $3x^3 - 2x^2 + 6x + 15$?**

Sol. $(2x^3 + 6x^2 - 5x + 8) + Q(x) = 3x^3 - 2x^2 + 6x + 15$

$\therefore Q(x) = (3x^3 - 2x^2 + 6x + 15) - (2x^3 + 6x^2 - 5x + 8)$

$$\begin{array}{r} 3x^3 - 2x^2 + 6x + 15 \\ 2x^3 + 6x^2 - 5x + 8 \\ (-) \quad (-) \quad (+) \quad (-) \\ \hline x^3 - 8x^2 + 11x + 7 \\ \hline \end{array}$$

The required polynomial is $x^3 - 8x^2 + 11x + 7$

8. **What must be subtracted from $2x^4 + 4x^2 - 3x + 7$ to get $3x^3 - x^2 + 2x + 1$?**

Sol.

$$\begin{aligned} (2x^4 + 4x^2 - 3x + 7) - Q(x) &= 3x^3 - x^2 + 2x + 1 \\ Q(x) &= (2x^4 + 4x^2 - 3x + 7) - (3x^3 - x^2 + 2x + 1) \\ \text{The required polynomial} &= 2x^4 + 4x^2 - 3x + 7 - 3x^3 + x^2 - 2x - 1 \\ &= 2x^4 - 3x^3 + 5x^2 - 5x + 6 \\ \therefore \qquad Q(x) &= 2x^4 - 3x^3 + 5x^2 - 5x + 6 \end{aligned}$$

9. **Multiply the following polynomials and find the degree of the resultant polynomial:**

(i) $p(x) = x^2 - 9$ $\quad q(x) = 6x^2 + 7x - 2$

(ii) $f(x) = 7x + 2$ $\quad g(x) = 15x - 9$

(iii) $h(x) = 6x^2 - 7x + 1$ $\quad f(x) = 5x - 7$

Sol. **(i)** $p(x) = x^2 - 9$ $\quad q(x) = 6x^2 + 7x - 2$

$p(x) \times q(x) = (x^2 - 9)\,(6x^2 + 7x - 2)$

$= (x^2 - 9)\,(6x^2 + 7x - 2)$

$= 6x^4 + 7x^3 - 2x^2 - 54x^2 - 63x + 18$

$= 6x^4 + 7x^3 - 56x^2 - 63x + 18$

The required polynomial is $6x^4 + 7x^3 - 56x^2 - 63x + 18$, degree 4.

(ii) $f(x) = 7x + 2$ $\quad g(x) = 15x - 9$

$f(x) \times g(x) = (7x + 2)\,(15x - 9)$

$= (7x + 2)\,(15x - 9)$

$= 105x^2 - 63x + 30x - 18$

$= 105x^2 - 33x - 18$

The required polynomial is $105x^2 - 33x - 18$, degree 2.

(iii) $h(x) = 6x^2 - 7x + 1$ $\quad f(x) = 5x - 7$

$h(x) \times f(x) = (6x^2 - 7x + 1)\,(5x - 7)$

$= (6x^2 - 7x + 1)\,(5x - 7)$

$= 30x^3 - 42x^2 - 35x^2 + 49x + 5x - 7$

$= 30x^3 - 77x^2 + 54x - 7$

The polynomial is $30x^3 - 77x^2 + 54x - 7$, degree 3.

10. **The cost of a chocolate is Rs. $(x + y)$ and Amir bought $(x + y)$ chocolates. Find the total amount paid by him in terms of x and y. If $x = 10$, $y = 5$ find the amount paid by him.**

Sol. Amount paid = Number of chocolates × Cost of a chocolate

$= (x + y)\,(x + y) = (x + y)^2 = x^2 + 2xy + y^2$

$x = 10, y = 5$ (Given)

The total amount paid by him $= 10^2 + 2 \times 10 \times 5 + 5^2 = 100 + 100 + 25 =$ ₹ 225

∴ Required amount = ₹ 225

11. The length of a rectangle is (3x+2) units and it's breadth is (3x–2) units. Find its area in terms of x. What will be the area if x = 20 units.

Sol.

$$\begin{aligned}\text{Area of a rectangle} &= \text{length} \times \text{breadth}\\ &= (3x+2)\times(3x-2) = (3x)^2 - 2^2 = [9x^2 - 4] \text{ Sq. units}\\ \text{If } x = 20, \text{ then Area} &= 9 \times (20)^2 - 4 = (9 \times 400) - 4\\ &= 3600 - 4 = 3596 \text{ Sq. units}\end{aligned}$$

12. $p(x)$ is a polynomial of degree 1 and $q(x)$ is a polynomial of degree 2. What kind of the polynomial $p(x) \times q(x)$ is ?

Sol. $p(x)$ is a polynomial of degree 1.

$q(x)$ is a polynomial of degree 2.

Then the $p(x) \times q(x)$ will be the polynomial of degree (1 + 2) = 3 (or) Cubic polynomial

3.2.2 Value and Zeros of a Polynomial

The number of zeros depend on the line intersecting "x" axis.

Zeros of a polynomial $\leq$ the degree of the polynomial

Value of a polynomial $p(x)$ at $x = a$ is $p(a)$ obtained on replacing x by a ($a \in R$)

Zeros of Polynomial :

When the value of $p(x)$ at $x = 1$ is zero, we can say that 1 is one of the zeros of $p(x)$.

Example $q(x) = x^2 - 3x + 2$, where $q(1) = 1^2 - 3(1) + 2 = -2 + 2 = 0$

$\therefore$ 1 is a zeros of a polynomial $q(x)$.

Roots of a polynomial equation :

If 'a' is zero of polynomial $p(x)$, if $p(a) = 0$ then 'a' is the root of polynomial equation $p(x) = 0$.

(i) A zero of a polynomial can be any real number not necessarily zero.
(ii) A non-zero constant polynomial has no zero.
(iii) By convention, every real number is zero of the zero polynomial.

Exercise 3.2

1. Find the value of the polynomial $f(y) = 6y - 3y^2 + 3$ at (i) $y = 1$ (ii) $y = -1$ (iii) $y = 0$

Sol. (i)

$$\begin{aligned}\text{At } y &= 1,\\ f(1) &= 6(1) - 3(1)^2 + 3 = 6 - \cancel{3} + \cancel{3} = 6\end{aligned}$$

(ii)

$$\begin{aligned}\text{At } y &= -1,\\ f(-1) &= 6(-1) - 3(-1)^2 + 3 = -6 - 3(1) + 3 = -6 - \cancel{3} + \cancel{3} = -6\end{aligned}$$

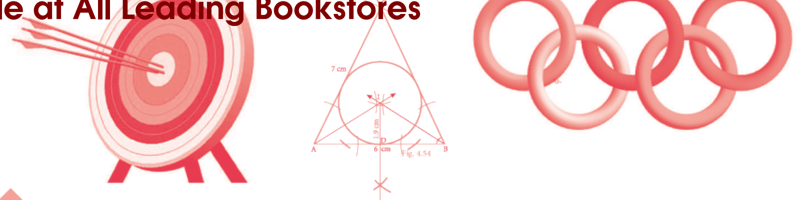
(iii)

$$\begin{aligned}\text{At } y &= 0,\\ f(0) &= 6(0) - 3(0)^2 + 3 = 0 - 0 + 3 = 3\end{aligned}$$

2. If $p(x) = x^2 - 2\sqrt{2}\,x + 1$, find $p(2\sqrt{2})$

Sol.

$$\begin{aligned}p(2\sqrt{2}) &= (2\sqrt{2})^2 - 2\sqrt{2}\,(2\sqrt{2}) + 1\\ &= (4 \times 2) - (4 \times 2) + 1\\ &= \cancel{8} - \cancel{8} + 1 = 1\end{aligned}$$



4

GEOMETRY

4.1 Introduction

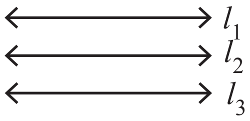
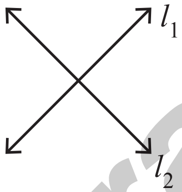
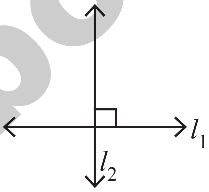
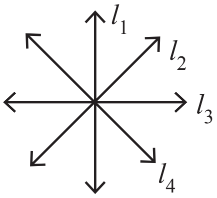
We describe different shapes by their properties.

Parallel lines : Two or more lines lying in the same plane that never meet.

Intersecting lines : Two lines which meet at a common point.

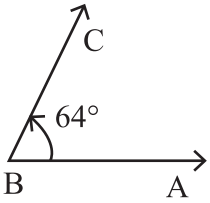
Perpendicular lines : Two lines which intersect each other at right angle.

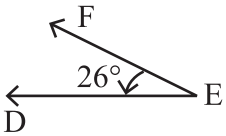
Concurrent lines : Three or more lines passing through the same point.

Parallel Lines	Intersecting Lines	Perpendicular Lines	Concurrent Lines
			
$l_1 \parallel l_2 \parallel l_3$		$l_1 \perp l_2$	

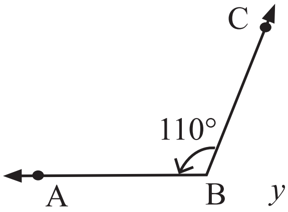
Types of angles :

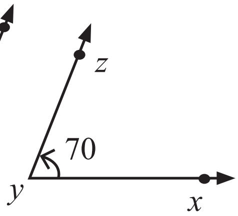
Acute angles



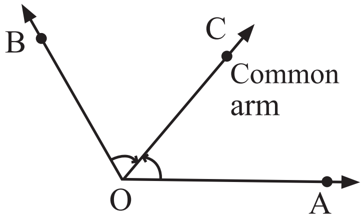


Obtuse angles

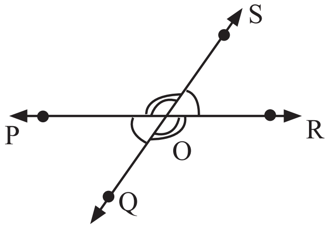




Adjacent Angles

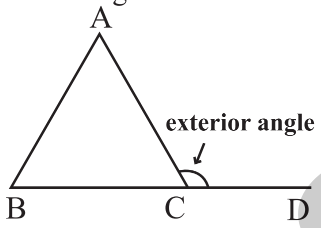


Opposite Angles

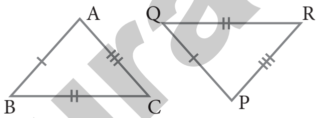
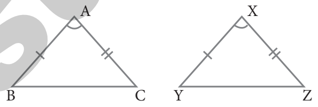
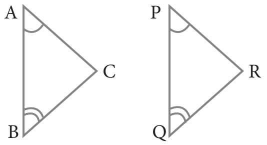
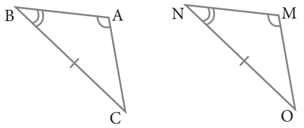
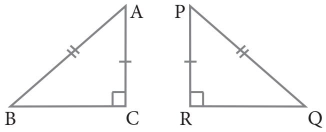


Transversal : A line which intersects two or more lines at a distinct points is called a transversal of lines.

Exterior angle property : If a side of a triangle is stretched, the exterior angle so formed is equal to the sum of the two remote interior angles.



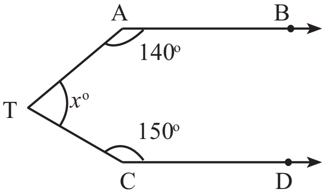
Congruent triangles : Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of another triangle.

Rule	Diagrams	Reason
SSS		AB = PQ BC = QR AC = PR ΔABC≅ ΔPQR
SAS		AB = XY ∠BAC = ∠YXZ AC = XZ ΔABC≅ ΔXYZ
ASA		∠A = ∠P AB = PQ ∠B = ∠Q ΔABC≅ ΔPQR
AAS		∠A = ∠M ∠B = ∠N BC = NO ΔABC≅ ΔMNO
RHS		∠ACB = ∠PRQ = 90°(R) AB = PQ hypotenuse (H) AC = PR (S) ΔABC ≅ ΔPQR

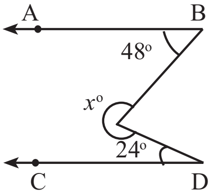
Exercise 4.1

1. In the figure, AB is parallel to CD, find x

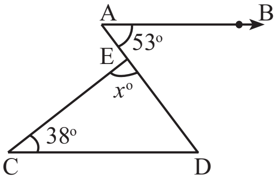
(i)



(ii)



(iii)



[HY-2019]

Sol. (i) From the figure

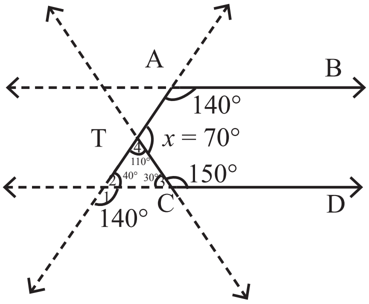
$\angle 1 = 140°$ ($\because$ corresponding angles are equal)

$\angle 2 = 40°$ ($\because \angle 1 + \angle 2 = 180° \Rightarrow \angle 2 = 180° – \angle 1 = 180° – 140° = 40°$)

$\angle 3 = 30°$ ($\because \angle 3 + 150° = 180°$)

$\angle 4 = 110°$ ($\because \angle 2 + \angle 3 + \angle 4 = 180°$)

$\therefore \angle x = 70°$ ($\because \angle 4 + \angle x = 180°$)



(ii) From the figure

$\angle 1 = 48°$

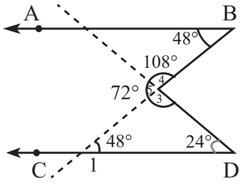
$\angle 3 = 108°$ ($\angle 1 + 24° + \angle 3 = 180°$)

$\angle 4 = 108°$ (If two lines are intersect, then the vertically the opposite angles are equal)

$\angle 5 = 72°$ ($\because \angle 3 + \angle 5 = 180°$)

$\therefore \angle 3 + \angle 4 + \angle 5 = 108° + 108° + 72°$

$x = 288°$



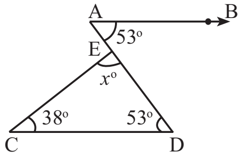
(iii) From the figure

$\angle D = 53°$ ($\because \angle B$ and $\angle D$ are alternate interior angles)

Sum of the three angles of a triangle is 180°

$\angle x° = 180° – (38° + 53°)$

$= 180° – 91° = 89°$



2. The angles of a triangle are in the ratio 1: 2 : 3, find the measure of each angle of the triangle.

Sol. Let the angles be x, $2x$ and $3x$ respectively.

Sum of the three angles of a triangle $= 180°$

$\therefore \quad x + 2x + 3x = 180°$

$$6x = 180^{\circ} \Rightarrow x = \frac{180^{\circ}}{6}$$

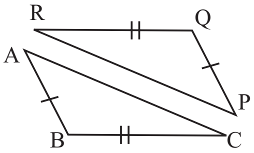
$$\therefore \quad x = 30^{\circ}$$

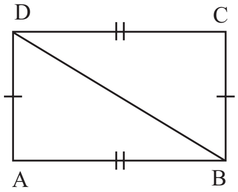
$$2x = 2 \times 30 = 60^{\circ}$$

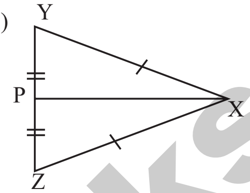
$$3x = 3 \times 30 = 90^{\circ}$$

The 3 angles of the triangle are 30°, 60°, 90°.

3. Consider the given pairs of triangles and say whether each pair is that of congruent triangles. If the triangles are congruent, say 'how'; if they are not congruent say 'why' and also say if a small modification would make them congruent:

(i)


(ii)


(iii)


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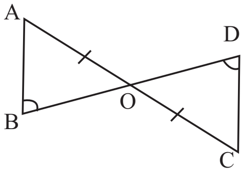

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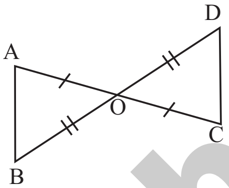

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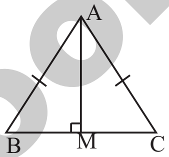


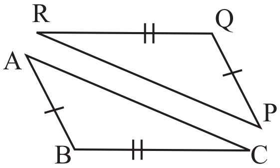
Sol. (i) Consider ΔPQR and ΔABC

Given, RQ = BC

PQ = AB

ΔABC is not congruent to ΔPQR

If PR = AC, then ΔABC ≅ ΔPQR

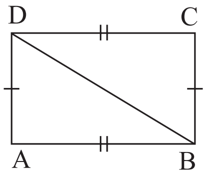
(i)


(ii) Consider ΔABD and ΔBCD for the triangles to be congruent.

Given, AB = DC

AD = BC and AB is common side.

∴ By SSS rule ΔABD ≅ ΔBCD.

(ii)


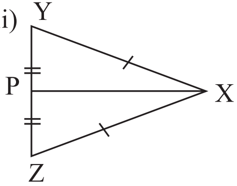
(iii) Consider ΔPXY and ΔPXZ,

Given, XY = XZ

PY = PZ

and PX is common

∴ By SSS rule ΔPXY ≅ ΔPXZ.

(iii)


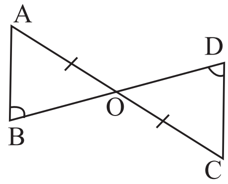
(iv) Consider ΔOAB and ΔODC,

Given, OA = OC

∠ABO = ∠ODC

and ∠AOB = ∠DOC [vertically opposite angles]

∴ By AAS rule, ΔOAB ≅ ΔODC.

(iv)


(v) Consider ΔAOB and ΔDOC,

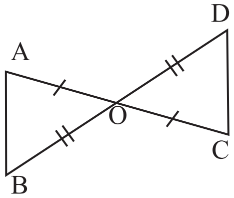
Given, AO = OC

OB = OD

and ∠AOB = ∠DOC [vertically opposite angles]

∴ By SAS rule, ΔAOB ≅ ΔDOC.

(v)



(vi) Consider ΔAMB and ΔAMC,

Given, AB = AC

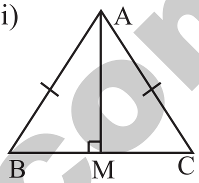
∠AMB = ∠AMC = 90°

∴ AM is common.

∴ By RHS rule

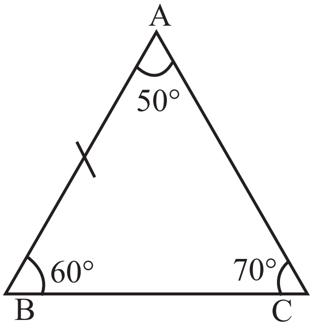
ΔAMB ≅ ΔAMC.

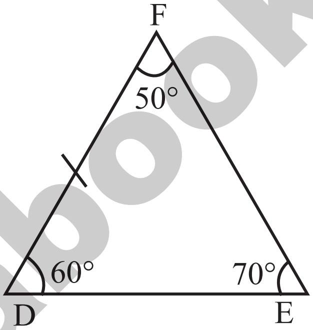
(vi)



4. **ΔABC and ΔDEF are two triangles in which AB=DF, ∠ACB=70°, ∠ABC=60°; ∠DEF=70° and ∠EDF=60°. Prove that the triangles are congruent.**

Sol.





In ΔABC, ∠ACB = 70°, ∠ABC = 60°

∴ ∠BAC = 180° – (70° + 60°)

= 180° – 130° = 50°

In ΔDEF, ∠DEF = 70°, ∠EDF = 60°

∴ ∠DFE = 180° – (70° + 60°)

= 180° – 130° = 50°

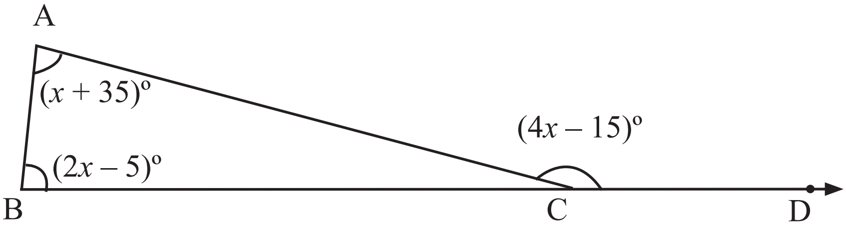
∠A = ∠F

AB = DF

and ∠B = ∠D

∴ By ASA rule ΔABC ≡ ΔFDE

5. **Find all the three angles of the ΔABC** ***[QY-2019]***



Sol. Exterior angle = Sum of the two opposite interior angles.

$$4x - 15 = 2x - 5 + x + 35$$
$$4x = 3x + 30 + 15$$
$$4x - 3x = 45^{\circ}$$
$$x = 45^{\circ}$$
$$\therefore \angle A = x + 35$$
$$= 45^{\circ} + 35^{\circ} = 80^{\circ}$$
$$\angle B = 2x - 5$$
$$= 2(45^{\circ}) - 5 = 90^{\circ} - 5^{\circ} = 85^{\circ}$$
$$\angle C = 4x - 15 = 4(45) - 15^{\circ}$$
$$= 180^{\circ} - 15^{\circ} = 165^{\circ}$$

4.3 Quadrilateral : Polygons with four sides are called quadrilateral.

Thinking corner :

1. If there is a polygon of n sides ($n \geq 3$), then the sum of all interior angles is $(n-2) \times 180^{\circ}$
2. For the regular polygon. Each interior angle is $\frac{(n-2)}{n} \times 180^{\circ}$.

   Each exterior angle is $\frac{360^{\circ}}{n}$

   The sum of all the exterior angles formed by producing the sides of a convex polygon in the order is 360° .

   If a polygon has 'n' sides , then the number of diagonals of the polygon is $\frac{n(n-3)}{2}$

4.3.1 Special names for some quadrilaterals :

1. A parallelogram is a quadrilateral in which opposite sides are parallel and equal.
2. A rhombus is a quadrilateral in which opposite sides are parallel and all sides are equal.
3. A trapezium is a quadrilateral in which one pair of opposite sides are parallel.

4.3.2 More special Names :

We thus have

A rhombus is an equilateral parallelogram.
A rectangle is an equiangular parallelogram.
A square is an equilateral and equiangular parallelogram.

4.4 Properties of a parallelogram :

(i) Opposite sides are parallel and congruent.
(ii) Opposite angles are congruent.
(iii) Diagonals bisect each other
(iv) Sum of any two adjacent angles is 180°
(v) Each diagonal divides the parallelogram into two triangles of equal area
(vi) Straight lines joining the midpoints of the adjacent sides of any quadrilateral form a parallelogram.
(vii) Area of Parallelogram = base × height.

6

TRIGONOMETRY

6.1 Introduction

i) $\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

ii) $\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

iii) $\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

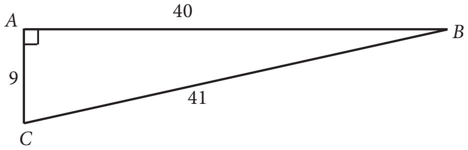
iv) $\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{opposite side}}$

v) $\text{secant}\,\theta = \dfrac{\text{hypotenuse}}{\text{adjacent side}}$

vi) $\text{cotangent}\,\theta = \dfrac{\text{adjacent side}}{\text{opposite side}}$

Exercise 6.1

1. **From the given figure, find all the trigonometric ratios of angle B.**



Sol. $\sin B = \dfrac{9}{41}$

$\cos B = \dfrac{40}{41}$

$\tan B = \dfrac{9}{40}$

$\text{cosec}\, B = \dfrac{1}{\sin B} = \dfrac{41}{9}$

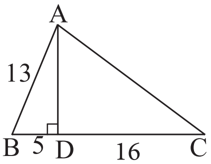
$\sec B = \dfrac{1}{\cos B} = \dfrac{41}{40}$

$\cot B = \dfrac{1}{\tan B} = \dfrac{40}{9}$

2. From the given figure, find the values of

(i) sin B (ii) sec B (iii) cot B

(iv) cos C (v) tan C (vi) cosec C



Sol. From the figure

(i) $\sin B = \frac{12}{13}$

(ii) $\sec B = \frac{1}{\cos B}$

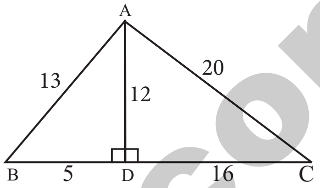
$= \frac{1}{\frac{5}{13}} = \frac{13}{5}$

(iii) $\cot B = \frac{1}{\tan B} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$

(iv) $\cos C = \frac{\overset{4}{\cancel{16}}}{\underset{5}{\cancel{20}}} = \frac{4}{5}$

(v) $\tan C = \frac{\overset{3}{\cancel{12}}}{\underset{4}{\cancel{16}}} = \frac{3}{4}$

(vi) $\text{cosec } C = \frac{1}{\sin C} = \frac{1}{\frac{12}{20}} = \frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{12}}} = \frac{5}{3}$



By the pythagoras theorem,

$$AD = \sqrt{13^2 - 5^2} = \sqrt{169 - 25}$$

$$= \sqrt{144} = 12$$

$$AC = \sqrt{12^2 + 16^2}$$

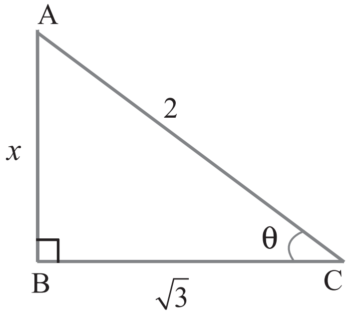
$$= \sqrt{144 + 256} = \sqrt{400} = 20$$

3. If 2 cos θ = $\sqrt{3}$, then find all the trigonometric ratios of angle θ.

Sol.

$$\text{If } 2\cos\theta = \sqrt{3}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$



By the Pythagoras theorem,

$$x = \sqrt{2^2 - \sqrt{3}^2} = \sqrt{4-3} = \sqrt{1} = 1$$

$$\therefore \sin\theta = \frac{1}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

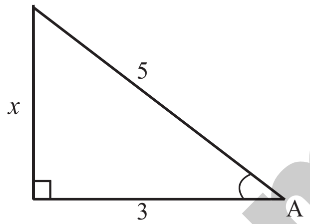
$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\text{cosec }\theta = 2$$

$$\sec\theta = \frac{2}{\sqrt{3}}$$

$$\cot\theta = \sqrt{3}$$

4. **If $\cos A = \frac{3}{5}$, then find the value of $\frac{\sin A - \cos A}{2\tan A}$**



Sol. By the Pythagoras theorem,

$$x = \sqrt{5^2 - 3^2}$$

$$= \sqrt{25 - 9}$$

$$= \sqrt{16} = 4$$

$$\sin A = \frac{4}{5}$$

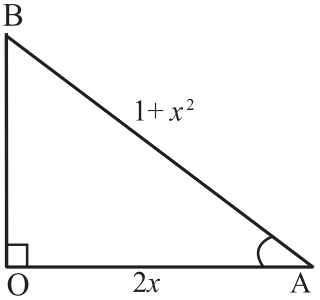
$$\cos A = \frac{3}{5}$$

$$\tan A = \frac{4}{3}$$

$$\therefore \frac{\sin A - \cos A}{2\tan A} = \frac{\frac{4}{5} - \frac{3}{5}}{2 \times \frac{4}{3}} = \frac{\frac{1}{5}}{2 \times \frac{4}{3}} = \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$$

5. **If $\cos A = \frac{2x}{1+x^2}$, then find the values of sin A and tan A in terms of x.**

Sol. By the pythagoras theorem,



$$(AB)^2 = (OA)^2 + (OB)^2$$

$$(1 + x^2)^2 = (2x)^2 + OB^2$$

$$(OB)^2 = (1 + x^2)^2 - (2x)^2 = 1 + x^4 + 2x^2 - 4x^2$$

$$= 1 + x^4 - 2x^2$$

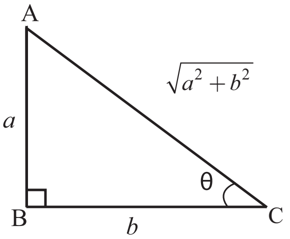
$$(OB)^2 = (1 - x^2)^2$$

$$OB = (1 - x^2)$$

$$\therefore \sin A = \frac{1-x^2}{1+x^2}$$

$$\therefore \tan A = \frac{1-x^2}{2x}$$

6. **If $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$, then show that $b\sin\theta = a\cos\theta$.**



Sol.

$$\sin\theta = \frac{a}{\sqrt{a^2+b^2}},$$

$$\cos\theta = \frac{b}{\sqrt{a^2+b^2}}$$

$$b\sin\theta = b \times \frac{a}{\sqrt{a^2+b^2}} = \frac{ab}{\sqrt{a^2+b^2}} \quad ...(1)$$

$$a\cos\theta = a \times \frac{b}{\sqrt{a^2+b^2}} = \frac{ab}{\sqrt{a^2+b^2}} \quad ...(2)$$

(1) = (2) ⇒ LHS = R.H.S.

Hence proved.

By the Pythagoras theorem

$$(BC)^2 = (AC)^2 - (AB)^2$$

$$= (\sqrt{a^2+b^2})^2 - a^2$$

$$= a^2 + b^2 - a^2$$

$$= b^2$$

$$BC = b$$

7. If 3 cot A = 2, then find the value of $\dfrac{4\sin A-3\cos A}{2\sin A+3\cos A}$.

Sol.

$$3\cot A = 2$$

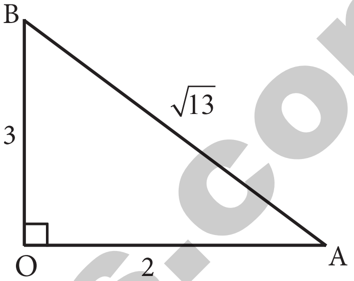
$$\cot A = \frac{2}{3}$$

$$\cot A = \frac{\text{Adjacent side}}{\text{Opposite side}}$$

$$\tan A = \frac{\text{Opp. side}}{\text{Adj. side}} = \frac{3}{2}$$

$$\sin A = \frac{3}{\sqrt{13}}$$

$$\cos A = \frac{2}{\sqrt{13}}$$



By Pythagoras theorem

In ΔOAB,

$$(AB)^2 = (OA)^2 + (OB)^2$$

$$= 2^2 + 3^2 = 4 + 9$$

$$= 13$$

$$AB = \sqrt{13}$$

$$\therefore \frac{4\sin A-3\cos A}{2\sin A+3\cos A} = \frac{\left(4\times\frac{3}{\sqrt{13}}\right)-\left(3\times\frac{2}{\sqrt{13}}\right)}{\left(2\times\frac{3}{\sqrt{13}}\right)+\left(3\times\frac{2}{\sqrt{13}}\right)}$$

$$= \frac{\frac{12}{\sqrt{13}}-\frac{6}{\sqrt{13}}}{\frac{6}{\sqrt{13}}+\frac{6}{\sqrt{13}}} = \frac{\frac{6}{\sqrt{13}}}{\frac{12}{\sqrt{13}}} = \frac{6}{\cancel{\sqrt{13}}}\times\frac{\cancel{\sqrt{13}}}{12} = \frac{\cancel{6}}{\underset{2}{\cancel{12}}} = \frac{1}{2}$$

8. If cos θ : sin θ = 1 : 2, then find the value of $\dfrac{8\cos\theta-2\sin\theta}{4\cos\theta+2\sin\theta}$.

Sol.

$$\cos\theta : \sin\theta = 1:2$$

$$\frac{\cos\theta}{\sin\theta} = \frac{1}{2}$$

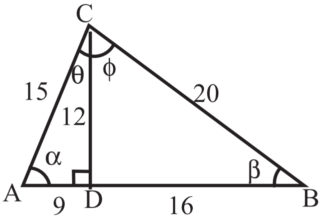
$$\therefore \cos\theta = \frac{1}{2}\sin\theta$$

$$\sin\theta = 2\cos\theta$$

$$\therefore \frac{8\cos\theta-2\sin\theta}{4\cos\theta+2\sin\theta} = \frac{\left(\overset{4}{\cancel{8}}\times\frac{1}{\cancel{2}}\sin\theta\right)-2\sin\theta}{\left(\overset{2}{\cancel{4}}\times\frac{1}{\cancel{2}}\sin\theta\right)+2\sin\theta} = \frac{4\sin\theta-2\sin\theta}{2\sin\theta+2\sin\theta} = {}_{2}\frac{\overset{1}{\cancel{2\sin\theta}}}{\cancel{4\sin\theta}} = \frac{1}{2}$$

$$\therefore \frac{8\cos\theta-2\sin\theta}{4\cos\theta+2\sin\theta} = \frac{1}{2}$$

9. From the given figure, prove that θ + ϕ = 90°. Also prove that there are two other right angled triangles. Find sin α, cos β and tan ϕ



Sol. In Δ ABC

$$(AC)^2 = 15^2 = 225 \quad \text{... (1)}$$

$$(BC)^2 = 20^2 = 400 \quad \text{... (2)}$$

$$(AB)^2 = (9+16)^2$$

$$= (25)^2 = 625 \quad \text{... (3)}$$

From (1), (2), (3)

$$\begin{aligned}(AB)^2 &= (AC)^2 + (BC)^2\\ 625 &= 225 + 400 = 625\\ \therefore \angle C &= \theta + \phi = 90^\circ\end{aligned}$$

($\therefore$ By Pythagoras theorem, in a right angled triangle square of hypotenuse is equal to sum of the squares of other two side)

And also in the figure, Δ ADC, Δ DBC are two other triangles.

As per the data given,

$$9^2 + 12^2 = 81 + 144 = 225 = 15^2$$

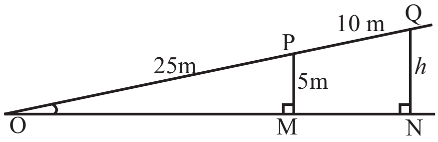
$\therefore \Delta$ ADC is a right angled triangle.

then $$12^2 + 16^2 = 144 + 256 = 400 = 20^2$$

$\therefore \Delta$ DBC is also a right angled triangle.

$$\sin\alpha = \frac{12}{15} = \frac{4}{5}, \cos\beta = \frac{16}{20} = \frac{4}{5}, \tan\phi = \frac{\overset{4}{\cancel{16}}}{\underset{3}{\cancel{12}}} = \frac{4}{3}$$

10. A boy standing at a point O finds his kite flying at a point P with distance OP = 25 m. It is at a height of 5 m from the ground. When the thread is extended by 10 m from P, it reaches a point Q. What will be the height QN of the kite from the ground? (use trigonometric ratios)



Sol. In the figure,

Δ OPM, Δ OQN are similar triangles. In similar triangles the sides are in the same proportional.

$$\begin{aligned}\therefore \frac{QN}{PM} &= \frac{QO}{PO}\\ \frac{h}{5} &= \frac{35}{25}\\ h &= \frac{5\times 35}{25}\\ h &= \frac{\cancel{5}\times \cancel{35}^{7}}{\cancel{25}\,\cancel{5}}\\ h &= 7 \text{ m.}\end{aligned}$$

Exercise 6.2

1. Verify the following equalities:

(i) $\sin^2 60^\circ + \cos^2 60^\circ = 1$

(ii) $1 + \tan^2 30^\circ = \sec^2 30^\circ$

(iii) $\cos 90^\circ = 1 - 2\sin^2 45^\circ = 2\cos^2 45^\circ - 1$

(iv) $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = \sin 90^\circ$

Sol. (i) $\sin^2 60° + \cos^2 60° = \left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

(ii) $1 + \tan^2 30° = \sec^2 30°$

LHS : $1 + \tan^2 30° = 1 + \left(\frac{1}{\sqrt{3}}\right)^2 = 1 + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}$ ... (1)

R.H.S : $\sec 30° = \frac{1}{\cos 30} = \frac{1}{\frac{\sqrt{3}}{2}} = 1 \times \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\sec^2 30 = \left(\frac{2}{\sqrt{3}}\right)^2$

$= \frac{4}{3}$ ... (2)

(1) = (2) ⇒ LHS = RHS

∴ Hence it is verified.

(iii) $\cos 90° = 1 - 2\sin^2 45° = 2\cos^2 45° - 1$

$\cos 90° = 0$ ... (1)

$1 - 2\sin^2 45° = 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 - \cancel{2} \times \frac{1}{\cancel{2}} = 1 - 1 = 0$ ... (2)

$2\cos^2 45° = 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 = 2 \times \frac{1}{2} = 1$

$\therefore\ 2\cos^2 45° - 1 = 1 - 1 = 0$ ... (3)

(1) = (2) = (3). Hence it is verified.

(iv) $\sin 30° \cos 60° + \cos 30° \sin 60° = \sin 90°$

$\sin 30° = \frac{1}{2},\ \sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2},\ \cos 60° = \frac{1}{2}$

$\sin 90° = 1$

$\therefore\ \sin 30° \cos 60° = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\cos 30° \sin 60° = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{4}$

$\therefore \sin 30° \cos 60° + \cos 30° \sin 60°$

$= \frac{1}{4} + \frac{3}{4} = \frac{\cancel{4}}{\cancel{4}} = 1 = \sin 90°.$

Hence it is verified.

	0º	30º	45º	60º	90º
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Study the above table thoroughly.

2. Find the value of the following:

(i) $\frac{\tan 45°}{\operatorname{cosec} 30°} + \frac{\sec 60°}{\cot 45°} - \frac{5\sin 90°}{2\cos 0°}$

(ii) (sin 90° + cos 60° + cos 45°) × (sin 30° + cos 0° – cos 45°)

(iii) $\sin^2 30° - 2\cos^3 60° + 3\tan^4 45°$

Sol. (i) $\frac{\tan 45°}{\operatorname{cosec} 30°} + \frac{\sec 60°}{\cot 45°} - \frac{5\sin 90°}{2\cos 0°}$

$$= \frac{1}{2} + \frac{2}{1} - \frac{5\times1}{2\times1}$$

$$= \frac{1}{2} + \frac{2}{1} - \frac{5}{2} = \frac{1+4-5}{2} = \frac{5-5}{2} = \frac{0}{2} = 0$$

(ii) (sin 90° + cos 60° + cos 45°) × (sin 30° + cos 0° – cos 45°)

$$= (1 + \frac{1}{2} + \frac{1}{\sqrt{2}}) \times (\frac{1}{2} + 1 - \frac{1}{\sqrt{2}})$$

$$= \left(1+\frac{1}{2}\right)^2 - \frac{1}{2} = \frac{9}{4} - \frac{1}{2} = \frac{7}{4}$$

(iii) $\sin^2 30° - 2\cos^3 60° + 3\tan^4 45°$

$$= \left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right)^3 + 3(1)^4$$

$$= \frac{1}{4} - \cancel{2}\left(\frac{1}{\cancel{8}_{4}}\right) + 3(1) = \frac{\cancel{1}}{4} - \frac{\cancel{1}}{4} + 3 = 3$$

3. Verify cos 3A = 4 cos³ A – 3 cos A, when A = 30°

Sol. L.H.S = cos 3A = cos 3 (30°)= cos 90°= 0 ... (1)

R.H.S = $4\cos^3 A - 3\cos A = 4\cos^3 30° - 3\cos 30°$

$$= 4\left(\frac{\sqrt{3}}{2}\right)^3 - \left(3\times\frac{\sqrt{3}}{2}\right) = \frac{\cancel{4}\times 3\sqrt{3}}{\cancel{8}_2} - \frac{3\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0 \quad \text{... (2)}$$

(1) = (2). Hence it is verified.

4. Find the value of 8 sin 2*x* cos 4*x* sin 6*x*, when *x* = 15°

Sol. 8 sin 2(15°). cos 4 (15°). sin 6(15°)

$$= 8\sin 30^\circ \cos 60^\circ \sin 90^\circ = 8 \times \frac{1}{2} \times \frac{1}{2} \times 1 = 2$$

Exercise 6.3

1. Find the value of the following:

(i) $\left(\dfrac{\cos 47^\circ}{\sin 43^\circ}\right)^2 + \left(\dfrac{\sin 72^\circ}{\cos 18^\circ}\right)^2 - 2\cos^2 45^\circ$

(ii) $\dfrac{\cos 70^\circ}{\sin 20^\circ} + \dfrac{\cos 59^\circ}{\sin 31^\circ} + \dfrac{\cos\theta}{\sin(90^\circ - \theta)} - 8\cos^2 60^\circ$

(iii) tan 15° tan 30° tan 45° tan 60° tan 75°

(iv) $\dfrac{\cot\theta}{\tan(90^\circ - \theta)} + \dfrac{\cos(90^\circ - \theta)\tan\theta\sec(90^\circ - \theta)}{\sin(90^\circ - \theta)\cot(90^\circ - \theta)\operatorname{cosec}(90^\circ - \theta)}$

Sol. (i) $\left(\dfrac{\cos 47^\circ}{\sin 43^\circ}\right)^2 + \left(\dfrac{\sin 72^\circ}{\cos 18^\circ}\right)^2 - 2\cos^2 45^\circ = \left(\dfrac{\cos(90^\circ - 43^\circ)}{\sin 43^\circ}\right)^2 + \left(\dfrac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ}\right)^2 - 2\cos^2 45^\circ$

$$= \left(\frac{\sin 43}{\sin 43}\right)^2 + \left(\frac{\cos 18}{\cos 18}\right)^2 - 2\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 1^2 + 1^2 - \cancel{2} \times \frac{1}{\cancel{2}} = 2 - 1 = 1$$

(ii) $\dfrac{\cos 70^\circ}{\sin 20^\circ} + \dfrac{\cos 59^\circ}{\sin 31^\circ} + \dfrac{\cos\theta}{\sin(90^\circ - \theta)} - 8\cos^2 60^\circ$

$$= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} + \frac{\cos\theta}{\cos\theta} - 8\left(\frac{1}{2}\right)^2$$

$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} + \frac{\cos\theta}{\cos\theta} - \overset{2}{\cancel{8}} \times \frac{1}{\cancel{4}} = 1 + 1 + 1 - 2 = 1$$

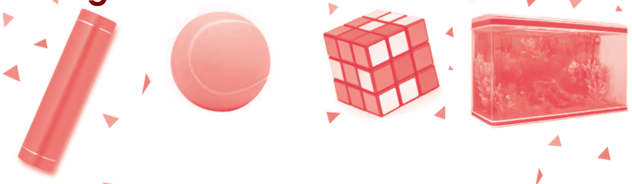
(iii) tan 15° tan 30° tan 45° tan 60° tan 75°

$$= \tan 15^\circ \times \frac{1}{\cancel{\sqrt{3}}} . 1 . \cancel{\sqrt{3}} . \tan 75^\circ$$

$$= \tan 15^\circ \times \tan(90 - 15^\circ) = \tan 15^\circ \times \cot 15^\circ = \cancel{\tan 15^\circ} \times \frac{1}{\cancel{\tan 15^\circ}} = 1$$

(iv) $\dfrac{\cot\theta}{\tan(90^\circ - \theta)} + \dfrac{\cos(90^\circ - \theta)\tan\theta\sec(90^\circ - \theta)}{\sin(90^\circ - \theta)\cot(90^\circ - \theta)\operatorname{cosec}(90^\circ - \theta)}$

$$= \frac{\cot\theta}{\cot\theta} + \frac{\sin\theta.\ \cancel{\tan\theta}\ .\operatorname{cosec}\theta}{\cos\theta.\ \cancel{\tan\theta}\ .\ \sec\theta} = 1 + \frac{\sin\theta}{\cos\theta}.\left(\frac{1}{\sin\theta}\middle/\frac{1}{\cos\theta}\right) = 1 + \frac{\cancel{\sin\theta}}{\cancel{\cos\theta}}.\frac{1}{\cancel{\sin\theta}} \times \frac{\cancel{\cos\theta}}{1} = 2$$



7

MENSURATION

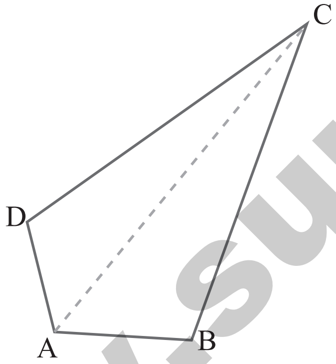
7.1 Introduction

- If a, b, c, are the sides of a triangle, then the area of a triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$ sq. units.

Where $s = \dfrac{a+b+c}{2}$, 's' is the semi - perimeter (that is half of the perimeter) of the triangle.

- Area of quadrilateral ABCD = Area of triangle ABC + Area of triangle ACD



Exercise 7.1

1. Using Heron's formula, find the area of a triangle whose sides are

(i) 10 cm, 24 cm, 26 cm (ii) 1.8 m, 8 m, 8.2 m

Sol. (i) sides : 10 cm, 24 cm, 26 cm

Using Heron's formula

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ sq. units

$$s = \frac{a+b+c}{2} = \left(\frac{10+24+26}{2}\right) \text{cm} = \frac{60}{2} = 30 \text{ cm}$$

$$\therefore \text{Area} = \sqrt{30(30-10)(30-24)(30-26)}$$

$$= \sqrt{30\times20\times6\times4} = \sqrt{600\times24} = \sqrt{14400} = 120 \text{ sq. cm}$$

(ii) Sides : 1.8 m, 8 m, 8.2 m

$$s = \frac{1.8+8+8.2}{2} = \frac{18}{2} = 9$$

$$\therefore \text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \text{ sq. units}$$

$$= \sqrt{9(9-1.8)(9-8)(9-8.2)} = \sqrt{9\times7.2\times1\times0.8}$$

$$= \sqrt{51.84} = 7.2 \text{ sq.m}$$

2. The sides of the triangular ground are 22 m, 120 m and 122 m. Find the area and cost of leveling the ground at the rate of ₹ 20 per m^2.

Sol. Sides : 22 m, 120 m, 122 m

Using Heron's formula

$$s = \frac{22+120+122}{2} = \frac{264}{2} = 132 \text{ m}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{132(132-22)(132-120)(132-122)}$$

$$= \sqrt{132\times110\times12\times10}$$

$$= \sqrt{1742400} = \sqrt{11\times11\times3\times3\times2\times\times2\times2\times2\times10\times10}$$

$$= 11 \times 3 \times 2 \times 2 \times 10 = 1320 \text{ m}^2$$

$$\begin{array}{r|l} 2 & 1742400 \\ \hline 2 & 871200 \\ \hline 2 & 435600 \\ \hline 2 & 217800 \\ \hline 9 & 108900 \\ \hline 11 & 12100 \\ \hline 11 & 1100 \\ \hline 10 & 100 \\ \hline & 10 \end{array}$$

Cost of levelling 1 m^2 = ₹ 20

∴Cost of levelling 1320 m^2 = 1320 × 20 = ₹ 26400

3. The perimeter of a triangular plot is 600 m. If the sides are in the ratio 5:12:13, then find the area of the plot.

Sol. We are given that the sides are in the ratio 5 : 12 : 13

Let the ratio be x

So, sides are $5x$, $12x$ and $13x$

Perimeter of a triangular plot = Sum of all sides = $5x + 12x + 13x = 30x$

The perimeter of a triangular plot is 600 m

$$\text{So, } 30x = 600 \text{ m}$$

$$x = \frac{600}{30} = 20 \text{ m}$$

$$\text{Sides} = 5x = 5(20) = 100 \text{ m}$$

$$12x = 12\,(20) = 240 \text{ m}$$

$$13x = 13(20) = 260 \text{ m}$$

$$\text{Area of triangular field} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{where } s = \frac{a+b+c}{2}$$

$$s = \frac{100+240+260}{2}$$

$$= \frac{600}{2} = 300 \text{ m}$$

$$\text{Area} = \sqrt{300(300-100)(300-240)(300–260)}$$

$$= \sqrt{300 \times 200 \times 60 \times 40}$$

$$= \sqrt{60000 \times 2400}$$

$$= \sqrt{144000000}$$

$$= 12000 \text{ sq.m}$$

4. **Find the area of an equilateral triangle whose perimeter is 180 cm.**

Sol. Perimeter of an equilateral triangle = 180 cm

$$\therefore \text{One side } (a) = \frac{180}{3} = 60 \text{ cm}$$

$$\text{Area of an equilateral triangle} = \frac{\sqrt{3}}{\cancel{4}} a^2 \text{ sq. units} = \frac{\sqrt{3}}{\cancel{4}} \times \overset{15}{\cancel{60}} \times 60$$

$$= 900\sqrt{3} \text{ m}^2 = 900 \times 1.732 = 1558.8 \text{ cm}^2$$

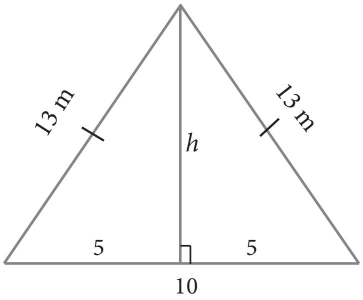
5. **An advertisement board is in the form of an isosceles triangle with perimeter 36m and each of the equal sides are 13 m. Find the cost painting it at ₹ 17.50 per square metre.**

Sol. Perimiter of an is isosceles triangle = 36 m

$$\therefore 13+13+x = 36$$

$$x = 36\text{-}13\text{-}13 = 10\text{m}$$

Let the height be 'h' and base be 'b'



$$h = \sqrt{13^2 - 5^2} = \sqrt{169 - 25}$$

$$= \sqrt{144} = 12 \text{ m}$$

$\therefore$ Area of the triangular board

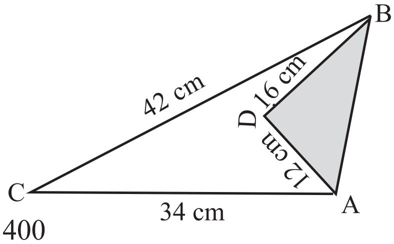
$$= \frac{1}{2} \times b\,h$$

$$= \frac{1}{2} \times 10 \times 12 = 60 \text{ m}^2$$

$$\text{Cost of painting } 1\text{m}^2 = ₹\ 17.50$$

$$\therefore \text{ Cost of painting } 60\text{m}^2 = 60 \times 17.50 = ₹\ 1050$$

6. **Find the area of the unshaded region.**



Sol. By the Pythagoras theorem

$$AB^2 = AD^2 + DB^2$$

$$= (12)^2 + (16)^2 = 144 + 256 = 400$$

$$AB = 20 \text{ cm.}$$

$\therefore$ Area of the Δ ABC $= \sqrt{s(s-a)(s-b)(s-c)}$ sq. units.

$$s = \frac{34+20+42}{2} = \frac{96}{2} = 48 \text{ cm}$$

$$= \sqrt{48(48-34)(48-20)(48-42)}$$

$$= \sqrt{48 \times 14 \times 28 \times 6} = \sqrt{112896}$$

$$= \sqrt{336 \times 336} = 336 \text{ sq. cm.}$$

$$\begin{array}{r|l} & 3\,3\,6 \\ \hline 3 & 11,28,96 \\ & 9 \\ \hline 63 \times 3 & 228 \\ & 189 \\ \hline 666 \times 6 & 3996 \\ & 3996 \\ \hline & 0 \end{array}$$

Area of the triangle ABD

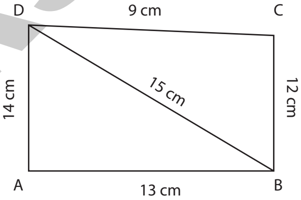
$$= \frac{1}{2} b\,h = \frac{1}{\cancel{2}} \times \overset{6}{\cancel{12}} \times 16 = 96 \text{ cm}^2$$

$\therefore$ Area of the unshaded region

$$= (\text{Area of } \Delta \text{ ABC}) - (\text{Area of } \Delta \text{ ABD})$$

$$= 336 - 96 = 240 \text{ cm}^2$$

7. Find the area of a quadrilateral ABCD whose sides are AB = 13 cm, BC = 12 cm, CD = 9 cm, AD = 14 cm and diagonal BD = 15 cm.



Sol. Area of the quadrilateral ABCD

$= (\text{Area of the } \Delta \text{ ABD}) + (\text{Area of the } \Delta \text{ BCD})$

Sides of the triangle ABD are 13 cm, 14 cm, 15 cm.

$$s = \frac{13+14+15}{2} \text{ cm} = \frac{42}{2} = 21 \text{ cm}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-13)(21-14)(21-15)}$$

$$= \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84 \text{ cm}^2$$

$$\begin{array}{r|l} & 8\,4 \\ \hline 8 & 7056 \\ & 64 \\ \hline 164 & 656 \\ & 656 \\ \hline & 0 \end{array}$$

Sides of the triangle BCD are 12 cm, 9 cm, 15 cm

$$\therefore s = \frac{12+9+15}{2} = \frac{36}{2} = 18 \text{ cm}$$

$$\text{Area} = \sqrt{18(18-12)(18-9)(18-15)}$$

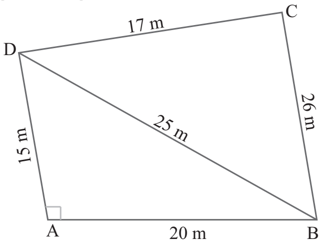
$$= \sqrt{18 \times 6 \times 9 \times 3} = \sqrt{2916} = 54 \text{ cm}^2$$

$$\begin{array}{r|l} & 5\,4 \\ \hline 5 & 29,16 \\ & 25 \\ \hline 104 & 416 \\ & 416 \\ \hline & 0 \end{array}$$

$\therefore$ Area of the quadrilateral

$$= 84 \text{ cm}^2 + 54 \text{ cm}^2 = 138 \text{ cm}^2$$

8. A park is in the shape of a quadrilateral. The sides of the park are 15 m, 20m, 26 m and 17 m and the angle between the first two sides is a right angle. Find the area of the park.



Sol. Area of the quadrilateral = Area of Δ ABD + Area of Δ BCD

Δ ABD is right angled triangle

$$\therefore \text{Area of } \Delta \text{ ABD} = \frac{1}{2}\, b\,h$$

$$= \frac{1}{\cancel{2}} \times \overset{10}{\cancel{20}} \times 15 \text{ m}^2 = 150 \text{ m}^2$$

$$\text{In } \Delta \text{ ABD, BD}^2 = \text{AD}^2 + \text{AB}^2$$

$$= (15)^2 + (20)^2 = 225 + 400 = 625 \text{ m}^2$$

$$BD = \sqrt{625} = 25 \text{ m}$$

$$\therefore \text{In } \Delta\, BCD,\ s = \frac{25+26+17}{2} = \frac{68}{2} = 34.$$

$$\text{Area of } \Delta\, BCD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{34(34-25)(34-26)(34-17)}$$

$$= \sqrt{34\times 9\times 8\times 17} = \sqrt{41616} = 204 \text{ m}^2$$

	2 0 4
2	4, 16, 16
	4
40	16
	0
404	1616
	1616
	0

$$\therefore \text{Area of the quadrilateral} = (150 + 204) \text{ m}^2 = 354 \text{ m}^2$$

9. A land is in the shape of rhombus. The perimeter of the land is 160 m and one of the diagonal is 48 m. Find the area of the land.

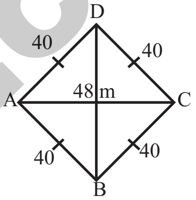
Sol. Perimeter of the rhombus land = 160 m

$$4a = 160 \text{ m}$$

$$a = 40 \text{ m}$$

One of the diagonal = 48 m

$\therefore$ Area of the land = 2 × Area of the Δ ABC.



$$s = \frac{40+40+48}{2} = \frac{128}{2} = 64 \text{ m}$$

$$\text{Area of } \Delta\, ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{64(64-40)(64-40)(64-48)}$$

$$= \sqrt{64\times 24\times 24\times 16} = \sqrt{589824}$$

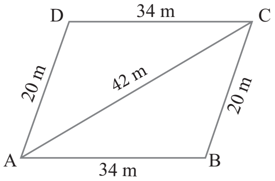
$$= 768 \text{ m}^2$$

	7 6 8
7	58, 98, 24
	49
146	998
	876
1528	12224
	12224
	0

$$\therefore \text{Area of the land} = 2 \times 768 \text{ m}^2 = 1536 \text{ m}^2$$

10. The adjacent sides of a parallelogram measures 34 m, 20 m and the measure of one of the diagonal is 42 m. Find the area of Parallelogram.

Sol. Area of the parallelogram = 2 × Area of the Δ ABC



$$s = \frac{34+20+42}{2} = \frac{96}{2} = 48 \text{ m}$$

$$\text{Area of } \Delta\, ABC = \sqrt{48(48-34)(48-20)(48-42)}$$

$$= \sqrt{48\times 14\times 28\times 6} = \sqrt{112896} = 336 \text{ m}^2$$

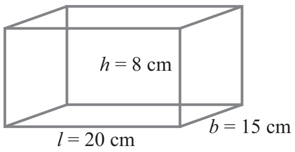
$$\therefore \text{Area of the parallelogram} = 2 \times 336 \text{ m}^2 = 672 \text{ m}^2$$

Exercise 7.2

1. Find the Total Surface Area and the Lateral Surface Area of a cuboid whose dimensions are :

(i) length = 20 cm, breadth = 15 cm and height = 8 cm

Sol. (i) (a)Total surface Area of a cuboid



$$= 2\,(lb + bh + hl) \text{ sq. units}$$

$$= 2\,(20 \times 15) + (15 \times 8) + (8 \times 20)$$

$$= 2\,(300 + 120 + 160) = 2 \times 580$$

$$= 1160 \text{ cm}^2$$

(b)Lateral surface area of a cuboid = $2h\,(l + b)$ sq. units

$$= (2 \times 8)\,(20 + 15) = 16 \times 35 = 560 \text{ cm}^2$$

2. The dimensions of a cuboidal box are 6 m × 400 cm × 1.5 m. Find the cost of painting its entire outer surface at the rate of ₹ 22 per m^2.

Sol. $l \times b \times h = 6 \text{ m} \times 400 \text{ cm} \times 1.5 \text{ m}$

$l = 6\text{m}, b = 4 \text{ m}, h = 1.5 \text{ m}$

$\therefore$ Total surface area of the cuboid = Outer surface area

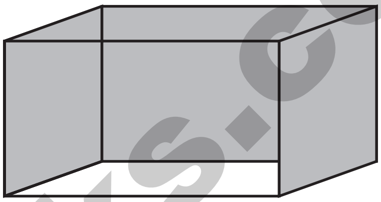
$= 2\,(lb + bh + hl) = 2\,((6 \times 4) + (4 \times 1.5) + (1.5 \times 6)) = 2\,(24 + 6 + 9) = 2\,(39) = 78 \text{ m}^2$

Cost of painting 1 m^2 = ₹ 22

Cost of painting 78 m^2 = 78 × 22 = ₹ 1716

3. The dimensions of a hall is 10 m × 9 m × 8 m. Find the cost of white washing the walls and ceiling at the rate of ₹ 8.50 per m^2.

Sol. Dimensions of a hall 10 m × 9 m × 8 m



$l = 10 \text{ m}$

$b = 9 \text{ m}$

$h = 8 \text{ m}$

White washing to be done for the area of the surface

$= \text{L.S.A} + lb$

$= 2\,(lh + bh) + lb$

$= 2\,(10 \times 8) + (9 \times 8) + (10 \times 9)$

$= 2\,(80 + 72) + 90 = (2 \times 152) + 90$

$= 304 + 90 = 394 \text{ m}^2$

Cost of white washing per m^2 = ₹ 8.50

Cost of white washing 394 m^2 = 394 × 8.50

Total cost = ₹ 3349

4. Find the TSA and LSA of the cube whose side is (i) 8 m (ii) 21 cm (iii) 7.5 cm

Sol. (i) side of a cube = 8 m

TSA of the cube = $6a^2 = 6 \times 64 = 384 \text{ m}^2$

LSA of the cube = $4a^2 = 4 \times 64 = 256 \text{ m}^2$

(ii) side a = 21 cm

TSA = $6a^2 = 6 \times 21 \times 21 = 2646 \text{ cm}^2$.

LSA = $4a^2 = 4 \times 21 \times 21 = 1764 \text{ cm}^2$.

(iii) side a = 7.5 cm

TSA = $6a^2 = 6 \times 7.5 \times 7.5 \text{ cm}^2 = 337.5 \text{ cm}^2$

LSA = $4a^2 = 4 \times 7.5 \times 7.5 \text{ cm}^2 = 225 \text{ cm}^2$

5. If the total surface area of a cube is 2400 cm^2 then, find its lateral surface area.

Sol.

$$6a^2 = 2400 \text{ cm}^2$$

$$\Rightarrow a^2 = \frac{2400}{6}$$

$$\therefore 4a^2 = \frac{4 \times \overset{400}{\cancel{2400}}}{\cancel{6}} = 1600 \text{ cm}^2.$$

6. A cubical container of side 6.5 m is to be painted on the entire outer surface. Find the area to be painted and the total cost of painting it at the rate of ₹ 24 per m^2.

Sol. $a = 6.5 \text{ m}$

$6a^2 = 6 \times 6.5 \times 6.5 = 253.5 \text{ m}^2$

9

PROBABILITY

9.1 Introduction

Outcome : While flipping a coin we get Head or Tail. Head and Tail are called outcomes. The result of the trial is called an outcome.

Sample space : In a single flip of a coin, the collection of sample points is given by

S = {H, T}.

Event : If a dice is rolled, it shows 4 which is called an outcome (since, it is a result of a single trial). In the same experiment the event of getting an even number is {2,4,6}. So any subset of a sample space is called an event.

Exercise 9.1

1. **You are walking along a street. If you just choose a stranger crossing you, what is the probability that his next birthday will fall on a Sunday?**

Sol. Days in a week (S) = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

n(S) = 7

∴ No. of days in week = 7

Event of selecting Sunday (A) = {Sunday}

n(A) = 1

∴ Probability of selecting Sunday $= \frac{n(A)}{n(S)} = \frac{1}{7}$

2. **What is the probability of drawing a King or a Queen or a Jack from a deck of cards?**

Sol. Number of cards n(S) = 52

No. of King cards n(A) = 4

No. of Queen cards n(B) = 4

No. of Jack cards n(C) = 4

Probability of drawing a King card $= \frac{n(A)}{n(S)} = \frac{4}{52}$

Probability of drawing a Queen card

$$= \frac{n(\text{B})}{n(\text{S})} = \frac{4}{52}$$

Probability of drawing a Jack card

$$= \frac{n(\text{C})}{n(\text{S})} = \frac{4}{52}$$

∴ The Probability of drawing a King or a Queen or a Jack from a deck of cards

$$= \text{P(A)} + \text{P(B)} + \text{P(C)} = \frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{4+4+4}{52} = \frac{12}{52} = \frac{3}{13}$$

3. What is the probability of throwing an even number with a single standard dice of six faces?

Sol. Faces of a dice (S) = {1, 2, 3, 4, 5, 6}

$n(\text{S}) = 6$

Event of throwing an even number

A = {2, 4, 6}, n(A) = 3

∴ Probability of throwing an even number

$$\text{P(A)} = \frac{n(\text{A})}{n(\text{S})} = \frac{3}{6} = \frac{1}{2}$$

4. There are 24 balls in a pot. If 3 of them are Red, 5 of them are Blue and the remaining are Green then, what is the probability of picking out (i) a Blue ball, (ii) a Red ball and (iii) a Green ball?

Sol. $n(\text{S}) = 24$

Red – $n(\text{R}) = 3$

Blue – $n(\text{B}) = 5$

Green – $n(\text{G}) = 16$

(i) Probability of picking a Blue ball $= \frac{n(\text{B})}{n(\text{S})} = \frac{5}{24}$

(ii) Probability of picking a Red ball $= \frac{n(\text{R})}{n(\text{S})} = \frac{3}{\underset{8}{\cancel{24}}} = \frac{1}{8}$

(iii) Probability of picking a Green ball $= \frac{n(\text{G})}{n(\text{S})} = \frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{24}}} = \frac{2}{3}$

5. When two coins are tossed, what is the probability that two heads are obtained?

Sol. Sample space when two coins are tossed (S) = {HH, TT, HT, TH}

$n(\text{S}) = 4$

Event of getting two heads (A) = {HH}

$n(\text{A}) = 1$

Probability of getting two heads P(A) $= \frac{n(\text{A})}{n(\text{S})} = \frac{1}{4}$

6. Two dice are rolled, find the probability that the sum is

(i) equal to 1 (ii) equal to 4 (iii) less than 13

Sol. When two dice are rolled Sample space

$$S = \left\{\begin{matrix}(1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6),\\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6),\\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6),\\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6),\\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6),\\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6)\end{matrix}\right\}$$

$$n(S) = 36$$

(i) Event of the sum is equal to 1 $= 0$

$$\therefore \text{Probability} = \frac{0}{n(S)} = 0$$

(ii) Event of the sum is equal to 4

$$B = \{(1,3), (2,2), (3,1)\}$$

$$n(B) = 3$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{\cancel{3}}{\underset{12}{\cancel{36}}} = \frac{1}{12}$$

(iii) Event of the sum is equal to less than 13

$$C = \left\{\begin{matrix}(1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6),\\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6),\\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6),\\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6),\\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6),\\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6)\end{matrix}\right\}$$

$$n(C) = 36$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{\cancel{36}}{\cancel{36}} = 1$$

7. A manufacturer tested 7000 LED lights at random and found that 25 of them were defective. If a LED light is selected at random, what is the probability that the selected LED light is a defective one.

Sol. $n(S) = 7000$ S – Total no. of lights.

$n(A) = 25$ A – Defective ones.

$$P(A) = \frac{n(A)}{n(S)} = \frac{\cancel{25}}{\underset{280}{\cancel{7000}}} = \frac{1}{280}$$

8. **In a football match, a goalkeeper of a team can stop the goal, 32 times out of 40 attempts tried by a team. Find the probability that the opponent team can convert the attempt into a goal.**

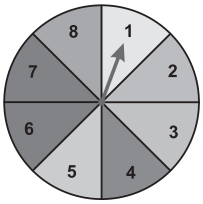
Sol. Total no. of attempts $n(\text{S}) = 40$

Total no. of attempts by A team $n(\text{A}) = 32$

Total no. of attempts by the opponent team B $= n(\text{B}) = 40 - 32 = 8$

$$\text{P(B)} = \frac{n(\text{B})}{n(\text{S})} = \frac{\cancel{8}}{\underset{5}{\cancel{40}}} = \frac{1}{5}$$

9. **What is the probability that the spinner will not land on a multiple of 3?**



Sol. Total no. of choices $n(\text{S}) = 8$

Total no. of multiples of 3A $= \{3, 6\}$

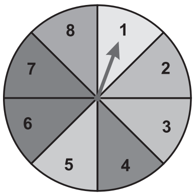
$n(\text{A}) = 2$

Event of non-multiples of 3B $= \{1, 2, 4, 5, 7, 8\}$

$n(\text{B}) = 6$

$$\therefore \text{P(B)} = \frac{n(\text{B})}{n(\text{S})} = \frac{\overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}}} = \frac{3}{4}$$

10. **Frame two problems in calculating probability, based on the spinner shown here.**



Sol. (i) What is the probability that the spinner will land on an even number?

(ii) What is the probability that the spinner will not land on a prime number.

Exercise 9.2

1. **A company manufactures 10000 Laptops in 6 months. Out of which 25 of them are found to be defective. When you choose one Laptop from the manufactured, what is the probability that selected Laptop is a good one.**

Sol.

$$\text{Total } n(S) = 10,000$$

$$\text{Defective } n(A) = 25$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{\overset{1}{\cancel{25}}}{\underset{400}{\cancel{10000}}} = \frac{1}{400}$$

$$\text{No. of good laptops} = 10000–25$$

$$n(B) = 9975$$

$$\text{Probability of a good one} = P(B) = \frac{n(B)}{n(S)} = \frac{\overset{399}{\overset{\cancel{1995}}{\cancel{9975}}}}{\underset{400}{\underset{\cancel{2000}}{\cancel{10000}}}}$$

$$= \frac{399}{400} = 0.9975$$

2. **In a survey of 400 youngsters aged 16-20 years, it was found that 191 have their voter ID card. If a youngster is selected at random, find the probability that the youngster does not have their voter ID card.**

Sol.

$$\text{No. of youngsters } n(S) = 400$$

$$\text{No. of youngsters having voter id } n(A) = 191$$

$$\text{No. of youngsters do not have their voter id } n(B) = 400 – 191 = 209$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{209}{400}$$

3. **The probability of guessing the correct answer to a certain question is $\frac{x}{3}$. If the probability of not guessing the correct answer is $\frac{x}{5}$, then find the value of x.**

Sol.

$$\frac{x}{3} + \frac{x}{5} = 1$$

$$\frac{5x+3x}{15} = 1$$

$$\frac{8x}{15} = 1$$

$$8x = 15$$

$$x = \frac{15}{8}$$

9th STD.

COMMON ANNUAL EXAMINATION - 2022

MATHEMATICS

Reg. No. ☐☐☐☐☐☐

(with Answers)

Time Allowed : 3.00 Hours] [Max. Marks : 100

Part - I

I. Choose the correct answer: 14 × 1 = 14

1. If $A = \{x, y, z\}$ then the number of non - empty subsets of A is

a) 8 b) 5 c) 6 d) 7

2. Which one of the following has a terminating decimal expansion?

a) $\frac{5}{64}$ b) $\frac{8}{9}$ c) $\frac{14}{15}$ d) $\frac{1}{12}$

3. An irrational number between 2 and 2.5 is

a) $\sqrt{11}$ b) $\sqrt{5}$ c) $\sqrt{2.5}$ d) $\sqrt{8}$

4. The root of the polynomial equation $2x+3=0$ is.

a) $\frac{1}{3}$ b) $\frac{-1}{3}$ c) $\frac{-3}{2}$ d) $\frac{-2}{3}$

5. Degree of the constant polynomial is........

a) 3 b) 2 c) 1 d) 0

6. $x^3 + y^3 + 3xy(x+y) =$ ________

a) $x^3 + y^3$ b) $(x + y)^3$
c) $x^3 - y^3$ d) $(x - y)^3$

7. PQ and RS are two equal chords of a circle with centre O such that $\angle POQ = 70^o$ then $\angle ORS =$ ....

a) 60^o b) 70^o c) 55^o d) 80^o

8. The point whose ordinate is 4 and which lies on the y – axis is

a) (4,0) b) (0,4) c) (1,4) d) (4,2)

9. The distance between two points (5, –1) and the origin is ....

a) $\sqrt{24}$ b) $\sqrt{37}$
c) $\sqrt{26}$ d) $\sqrt{17}$

10. If $2 \sin 2\theta = \sqrt{3}$, then the value of θ is

a) 90° b) 30°
c) 45° d) 60°

11. tan 90° = ...

a) $\frac{1}{\sqrt{3}}$ b) $\sqrt{3}$
c) 1 d) 0

12. If the lateral surface area of a cube is 600cm², then the total surface area is ....

a) 150 cm² b) 400 cm²
c) 900 cm² d) 1350 cm²

13. Probability of sure event is ....

a) 1 b) 0 c) – 1 d) 2

14. If A is any event in S and its complement is A′ then, P(A′) is equal to

a) 1 b) 0
c) 1 – A d) 1–P(A)

Part - II

Note : Answer any 10 questions : (Ques. No.28 is compulsory) 10 × 2 = 20

15. Write the set of letters of the following words in Roster form :

(i) INDIA

(ii) PARALLELOGRAM

16. If $n[P(A)] = 256$, find $n(A)$.

17. Verify that 1 = 0.9

18. Find the 5th root of 243

19. Expand $(2x + 3y + 4z)^2$

20. The base of a parallelogram is $(5x + 4)$. Find its height if the area is $25x^2 - 16$

21. Find the GCD of $2x^2 - 18$ and $x^2 - 2x - 3$

22. A Chord is 12 cm away from the centre of the circle of radius 15 cm. Find the Length of the Chord.

23. In Which quadrant does the following points lie?

(i) (3, –8) (ii) (–1, –3) (iii) (2, 5) (iv) (–7, 3)

24. The Centre of a circle is (–4,2). If one end of the diameter of the circle is (–3,7) then find the other end.

25. Find the TSA and LSA of the cube whose side is 8 m.

26. Find the volume of a cuboid whose dimensions are Length 12 cm, breadth 8 cm and height is 6 cm.

27. The Probability that it will rain tomorrow is $\frac{91}{100}$. What is the probability that it will not rain tomorrow?

28. Prove that : $\frac{\cos 30^\circ + \sin 60^\circ}{1 + \cos 60^\circ + \sin 30^\circ} = \frac{\sqrt{3}}{2}$

Part - III

Note : Answer any 10 questions. Question No. 42 is compulsory. 10 × 5 = 50

29. Find the number of subsets and the number of proper subsets of X ={ $x^2 : x \in \mathbb{N}, x^2 \leq 100$}

30. Express the rational number $\frac{1}{33}$ in recurring decimal from by using the recurring decimal expansion of $\frac{1}{11}$. Hence write $\mathbf{\frac{71}{33}}$ in recurring decimal form

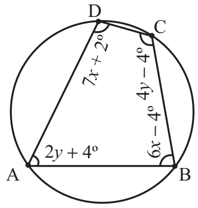
31. Represent 4.863 on the number line.

32. Find the value of m, if (x – 2) is a factor of the polynomial $2x^3 - 6x^2 + mx + 4$.

33. If $x^2 + \frac{1}{x^2} = 23$, then find the value of $x + \frac{1}{x}$ and $x^3 + \frac{1}{x^3}$.

34. Factorize : $x^3 - 10x^2 - x + 10$.

35. Find all the angles of the given cyclic quadrilateral ABCD in the figure.



36. Show that the points A(7,10), B(–2,5) C(3,–4) are the vertices of a right angled triangle.

37. If $2 \cos \theta = \sqrt{3}$, then find all the trigonometric ratios of angle θ.

38. Find the area of the Right angled triangle with hypotenuse 5 cm and one of the acute angle is 48º 30'[sin 48º = 0.7490, Cos 48º 30' = 0.6626]

39. Three identical cubes of side 4 cm are joinded end to end. Find the total surface area and lateral surface area of the new resulting cuboid.

40. A cubical tank can hold 64,000 litres of water. Find the length of its side in meters.

41. Two dice are rolled. Find the Probability that the sum is (i) equal to 1 (ii) equal to 4 (iii) Less than 13.

42. Find the Lengths of the medians of a ΔABC whose vertices are A(7, –3) B(5,3) and C(3, –1).

Part - IV

Note : Answer all the questions : 2 × 8 = 16

43. a) Draw an equilateral triangle of sides 6.5cm and Locate its Orthocentre.

(OR)

b) Draw the triangle ABC, where AB = 8cm, BC = 6cm and ∠B = 70° and Locate its circum centre and draw the circum circle.

44. a) Plot the following points A(2,2), B(–2,2), C(–2, –1) and D(2, –1) in the Cartesian plane also find the area of the shape.

(OR)

b) Plot the following points A(5,4), B(–1,4), C(–1,–2) and D(5, –2) in the Cartesian plane also find the area of the shape.

☆ ☆ ☆

Answers

Part - I

1. d) 7
2. a) $\frac{5}{64}$
3. b) $\sqrt{5}$
4. c) $\frac{-3}{2}$
5. d) 0
6. d) $(x + y)^3$
7. d) 55º
8. b) (0,4)
9. c) $\sqrt{26}$
10. b) 30°
11. *) Not defined
12. c) 900 cm^2
13. a) 1
14. d) 1 – P(A)
15. (i) A = {I, N, D, A}
(ii) B = {P, A, R, L, E, O, G, M}

Part - II

16. $n[P(A)] = 256$

$$\begin{array}{r|l} 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$n[P(A)] = 2^8$$
$$\therefore n(A) = 8.$$

17. Let $x = 0.\overline{9} = 0.99999 \ldots (1)$

(Multiply equation(1) by 10)

$10x = 9.99999 \ldots\ldots (2)$

Subtract (1) from (2)

$9x = 9$ or $x = 1$

Thus, $0.\overline{9} = 1$

18. $\sqrt[5]{243} = 243^{\frac{1}{5}} = \left(3^5\right)^{\frac{1}{5}} = 3^{\cancel{5} \times \frac{1}{\cancel{5}}} = 3$

19. We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

substituting, a $= 2x$, b $= 3y$ and c $= 4z$

$(2x + 3y + 4z)^2 = (2x)^2 + (3y)^2 + (4z)^2 + 2(2x)(3y) + 2(3y)(4z) + 2(4z)(2x)$

$= 4x^2 + 9y^2 + 16z^2 + 12xy + 24yz + 16xz$

20. Area of parallelogram $= b \times h$

$= 25x^2 - 16$

base, $b = 5x + 4$

$$\text{height, } h = \frac{25x^2 - 16}{\text{base}} = \frac{25x^2 - 16}{5x + 4}$$

$$\begin{array}{r|l} & 5x - 4 \\ \hline 5x + 4 & 25\cancel{x}^2 + 0x - 16 \\ & \underset{(-)}{2\cancel{5}x^2} + \underset{(-)}{20x} \\ \hline & -2\cancel{0}x - 1\cancel{6} \\ & \underset{(+)}{-\cancel{2}0x} \underset{(+)}{- \cancel{1}6} \\ \hline & 0 \end{array}$$

$\therefore$ Height $= 5x - 4$

21. $2x^2 - 18 = 2(x^2 - 9) = 2(x^2 - 3^2) = 2(x+3)(x-3)$

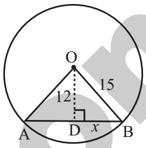
$x^2 - 2x - 3 = x^2 - 3x + x - 3$

$= x(x-3) + 1(x-3)$

$= (x-3)(x+1)$

Therefore, GCD $= (x-3)$

22.



$$BD = \sqrt{15^2 - 12^2}$$
$$= \sqrt{225 - 144}$$
$$= \sqrt{81} = 9 \text{ cm}$$

$\therefore$ Length of the chord AB = 9 + 9 =18 cm

23. i) The x - coordinate is positive and y - coordinate is negative. So, point (3, –8) lies in the IV quadrant.

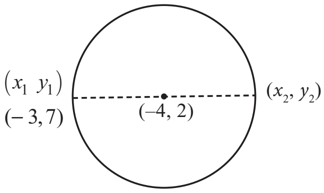
ii) The x - coordinate is negative and y - coordinate is negative. So, point (–1, –3) lies in the III quadrant.

iii) The x - coordinate is positive and y - coordinate is positive. So point (2,5) lies in the I quadrant.

iv) The x - coordinate is negative and y - coordinate is positive. So point (–7,3) lies in the II quadrant.

24. $$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$(-4, 2) = \left(\frac{-3 + x_2}{2}, \frac{7 + y_2}{2}\right)$$



$\frac{-3 + x_2}{2} = -4$	$\frac{7 + y_2}{2} = 2$
$-3 + x_2 = -8$	$7 + y_2 = 4$
$x_2 = -8 + 3$	$y_2 = 4 - 7 = -3$
$x_2 = -5$	

$\therefore$ The other end is (–5, –3)

25. Side of a cube = 8 m

TSA of the cube = $6a^2 = 6 \times 64 = 384$ m^2

LSA of the cube = $4a^2 = 6 \times 64 = 256$ m^2

26. l = 12 cm; b = 8 cm; h = 6 cm

Volume of the cuboid $= lbh$

$= 12 \times 8 \times 6$ cm^3

$= 576$ cm^3

27. Let E be the event that it will rain tomorrow. Then E′ is the event that it will not rain tomorrow since P(E) = 0.91, We have P(E′) = 1 – 0.91 = 0.09

Therefore, the probability that it will not rain tomorrow = 0.09.

28. $\cos 30^\circ = \frac{\sqrt{3}}{2}$; $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$; $\sin 30^\circ = \frac{1}{2}$

$$\text{LHS} = \frac{\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}}{1+\frac{1}{2}+\frac{1}{2}} \Rightarrow \frac{\frac{\sqrt{3}+\sqrt{3}}{2}}{\frac{2+1+1}{2}} = \frac{\sqrt{3}}{2} \Rightarrow \text{RHS}$$

Hence proved.

Part - III

29. Given X $= \{1, 2, 3, \ldots\ldots\}$

$X^2 = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

$n(X) = 10$

The Number of subsets

$= n[P(X)] = 2^{10} = 1024$

The Number of proper subsets

$= n[P(X)]–1 = 2^{10} – 1 = 1024 – 1 = 1023.$

30. The recurring decimal expansion of

$$\frac{1}{11} = 0.09090909\ldots. = 0.\overline{09}$$

$$\frac{1}{33} = \frac{1}{11}\times\frac{1}{3}$$

$$= 0.\overline{09}\times\frac{1}{3} = 0.\overline{03}$$

$$\therefore \frac{1}{33} = 0.03030303\ldots. = 0.\overline{03}$$

Also, $\frac{71}{33} = 2\frac{5}{33} = 2+\frac{5}{33}$

$$= 2+\left(5\times\frac{1}{33}\right)$$

$$= 2+(5\times 0.\overline{03})$$

$$= 2+(5\times 0.030303\cdots)$$

$$= 2+0.151515\cdots\cdots$$

$$= 2.151515\cdots\cdots = 2.\overline{15}$$

31. (i) Divide the distance between 4 and 5 into 10 equal intervals.

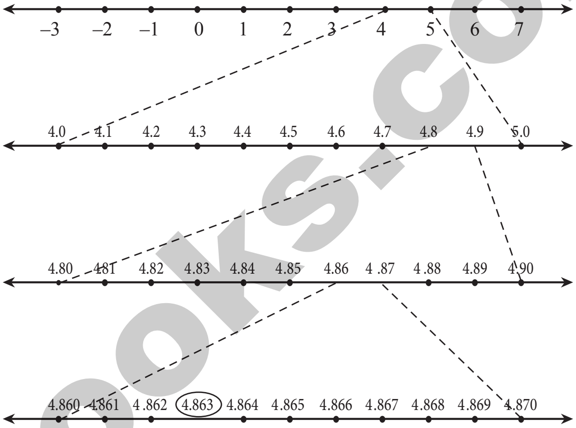
(ii) Mark the point 4.8 which is second from the left of 5 and eighth from the right of 4

(iii) 4.86 lies between 4.8 and 4.9. Divide the distance into 10 equal intervals.

(iv) Mark the point 4.86 which is fourth from the left of 4.9 and sixth from the right of 4.8

(v) 4.863 lies between 4.86 and 4.87. Divide the distance into 10 equal intervals.

(vi) Mark point 4.863 which is seventh from the left of 4.87 and third from the right of 4.86.



32. Let $p(x) = 2x^3 -6x^2 +mx + 4$

By factor theorem, $(x -2)$ is a factor of $p(x)$

if $p(2) = 0$

$2(2)^3 -6(2)^2 + m\ (2) + 4 = 0$

$2(8)-6(4)+2\ m + 4 = 0$

$-4 +2\ m = 0$

$m = 2$

33. $\left(x+\frac{1}{x}\right)^2 = x^2 + 2\cancel{x}\times\frac{1}{\cancel{x}}+\frac{1}{x^2}$

$$\left(x+\frac{1}{x}\right)^2 = x^2+2+\frac{1}{x^2} = \left(x^2+\frac{1}{x^2}\right)+2$$

$$\left(x+\frac{1}{x}\right)^2 = 23+2 = 25$$

$$\left(x+\frac{1}{x}\right) = \sqrt{25} = \pm 5$$

$$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\cancel{x}\times\frac{1}{\cancel{x}}\left[x+\frac{1}{x}\right]$$

$$= (5)^3 – 3(5) = 125 – 15$$

$$= 110 \text{ (when } x = 5)$$

When $x = –5$, $x^3+\frac{1}{x^3} = (–5)^3 – 3(–5)$

$$= –125 + 15 = –110$$

34. Let $p(x) = x^3 - 10x^2 - x + 10$

Sum of the co-efficients = 11 – 11 = 0

$\therefore (x - 1)$ is a factor

Sum of co-efficients of even powers of x with constant = –10 + 10 = 0

Sum of co-efficients of odd powers of x = 1 – 1 = 0

$\therefore (x + 1)$ is a factor

Synthetic division

+1	1	–10	–1	10
	0	1	–9	–10
–1	1	–9	–10	0
	0	–1	10	
	1	–10	0	

$\therefore x^3 + 10x^2 - x + 10 = (x - 1)(x + 1)(x - 10)$

35. In the cyclic quadrilateral $\angle A + \angle C = 180^o$

$$2y + \cancel{4} + 4y - \cancel{4} = 180^o$$

$$6y = 180^o$$

$$y = \frac{180^o}{6} = 30^o$$

$$\angle B + \angle D = 6x - 4 + 7x + 2$$

$$13x - 2 = 180^o$$

$$13x = 180 + 2 = 182^o$$

$$x = \frac{182}{13} = 14^o$$

$\therefore \angle A = 2(30) + 4^o = 64^o$

$\angle B = 6(14) - 4^o = 84 - 4^o = 80^o$

$\angle C = 4(30) - 4 = 120 - 4 = 116^o$

$\angle D = 7(14) + 2 = 98 + 2 = 100^o$

36. Here A = (7, 10), B = (-2, 5), C = (3,-4)

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-2-7)^2 + (5-10)^2}$$

$$= \sqrt{(-9)^2 + (-5)^2}$$

$$= \sqrt{(81+25)} = \sqrt{106}$$

$AB^2 = 106$ ... (1)

$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3-(-2))^2 + (-4-5)^2}$$

$$= \sqrt{(5)^2 + (-9)^2} = \sqrt{25+81} = \sqrt{106}$$

$BC^2 = 106$ ... (2)

$$AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3-7)^2 + (-4-10)^2}$$

$$= \sqrt{(-4)^2 + (-14)^2}$$

$$= \sqrt{16+196} = \sqrt{212}$$

$AC^2 = 212$ ... (3)

From (1), (2) & (3) we get,

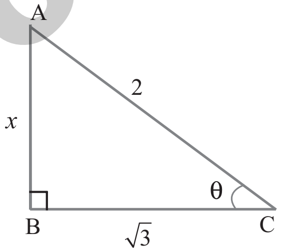
$AB^2 + BC^2 = 106 + 106 = 212 = AC^2$

Since $AB^2 + BC^2 = AC^2$

$\therefore$ ΔABC is a right angled triangle, right angled at B.

37. If $2\cos\theta = \sqrt{3}$; $\cos\theta = \frac{\sqrt{3}}{2}$

By the Pythagoras theorem,



$$x = \sqrt{2^2 - \sqrt{3}^2} = \sqrt{4-3} = \sqrt{1} = 1$$

$\therefore \sin\theta = \frac{1}{2}$; $\cos\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{1}{\sqrt{3}}$; $\text{cosec}\,\theta = 2$

$\sec\theta = \frac{2}{\sqrt{3}}$; $\cot\theta = \sqrt{3}$

38. From the figure,

$$\sin\theta = \frac{AB}{AC}$$

$$\sin 48^o 30' = \frac{AB}{5} ; 0.7490 = \frac{AB}{5}$$

$$5 \times 0.7490 = AB$$

$$AB = 3.7450 \text{ cm}$$

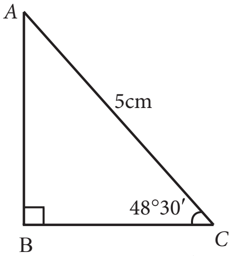
$$\cos\theta = \frac{BC}{AC}$$

$$\cos 48^o 30' = \frac{BC}{5}$$

$$0.6626 = \frac{BC}{5}$$

$$0.6626 \times 5 = BC$$

BC = 3.313 cm



Area of right triangle $= \frac{1}{2}\, bh$

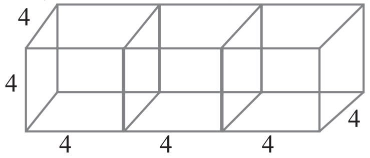
$= \frac{1}{2} \times \text{BC} \times \text{AB}$

$= \frac{1}{2} \times 3.3130 \times 3.7450$

$= 1.6565 \times 3.7450 = 6.2035925 \text{ cm}^2$

39. *a* = 4 *cm;*

TSA of the cuboid = 2 (lb + bh + hl)



$l = 12$ cm ; $b = 4$ cm; $h = 4$ cm

$\therefore$ TSA $= 2\,(12 \times 4) + (4 \times 4) + (4 \times 12)$

$= 2\,(48 + 16 + 48) = 2 \times 112$

$= 224 \text{ cm}^2$

LSA $= 2\,\text{h}\,(l + b) = (2 \times 4)\,(12 + 4)$

$= 8 \times 16 = 128 \text{ cm}^2$

40. Let 'a' be the side of cubical tank.

Here, volume of the tank = 64,000 litres

i.e., $a^3 = 64{,}000 = \frac{64000}{1000}$

[since,1000 litres=1m³]

$a^3 = 64 \text{ m}^3$

$a = \sqrt[3]{64} \Rightarrow a = 4$ m

Therefore, length of the side of the tank is 4 metres.

41. When two dice are rolled Sample space

$$S = \left\{\begin{array}{llllll}(1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6),\\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6),\\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6),\\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6),\\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6),\\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6)\end{array}\right\}$$

$n(\text{S}) = 36$

(i) Event of the sum is equal to 1 = 0

$$\therefore \text{Probability} = \frac{0}{n(\text{S})} = 0$$

(ii) Event of the sum is equal to 4

B = {(1,3), (2,2), (3,1)}

$n(\text{B}) = 3$

$$\text{P(B)} = \frac{n(\text{B})}{n(\text{S})} = \frac{\cancel{3}}{\underset{12}{\cancel{36}}} = \frac{1}{12}$$

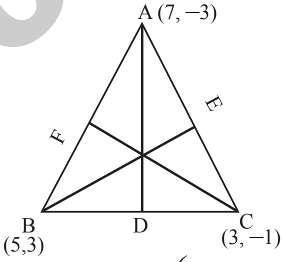
(iii) Event of the sum is equal to less than 13

$$\text{C} = \left\{\begin{array}{llllll}(1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6),\\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6),\\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6),\\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6),\\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6),\\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6)\end{array}\right\}$$

$n(\text{C}) = 36$

$$\text{P(C)} = \frac{n(\text{C})}{n(\text{S})} = \frac{\cancel{36}}{\cancel{36}} = 1$$

42. Let AD, BE and CF be the median of ΔABC.



Midpoint of BC $= \left(\frac{5+3}{2}, \frac{3-1}{2}\right)$

$= \left(\frac{8}{2}, \frac{2}{2}\right) = (4,\ 1)$

Mid point of AC $= \left(\frac{7+3}{2}, \frac{-3-1}{2}\right)$

$= \left(\frac{10}{2}, \frac{-4}{2}\right)$

$= (5, -2)$

Midpoint of AB $= \left(\frac{7+5}{2}, \frac{-3+3}{2}\right)$

$= \left(\frac{12}{2}, 0\right)$

$= (6,\ 0)$

$\therefore$ The coordinates of D,E and F are (4,1) (5, –2) and (6,0) respectively.

Length of Median AD

$= \sqrt{(4-7)^2 + (1+3)^2}$

$= \sqrt{(-3)^2+(4)^2} = \sqrt{9+16} = \sqrt{25}$ = 5 units

Length of median BE

$= \sqrt{(5-5)^2+(-2-3)^2}$

$= \sqrt{0+(-5)^2} = \sqrt{25}$ = 5 units

Length of Median CF

$= \sqrt{(6-3)^2+(0+1)^2}$

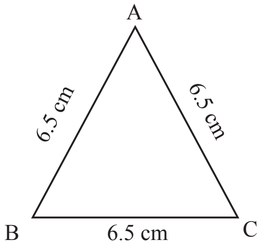
$= \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$ units

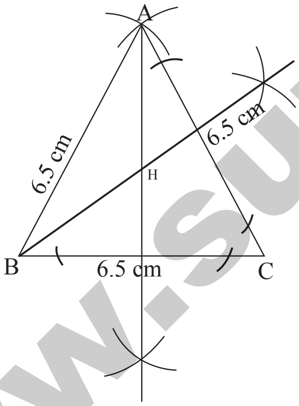
Part - IV

43.

a) Equilateral triangle at sides 6.5 cm.

Rough diagram





Construction :

(i) Draw the ΔABC with the given measurements.

(ii) Construct altitudes from any two vertices A and B, to their opposite sides BC and AC respectively.

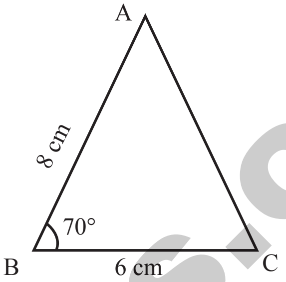
(iii) The point intersection of the altitude H is the orthocentre of the given ΔABC.

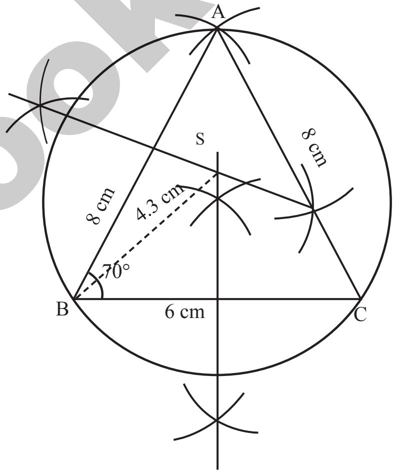
(OR)

b) ΔABC, where AB = 8 cm, BC = 6 cm,

B = 70°

Rough diagram





Construction :

(i) Draw the ΔABC with the given measurements.

(ii) Construct the perpendicular bisector at any two sides (AB and BC) and let them meet at S which is the circumcircle.

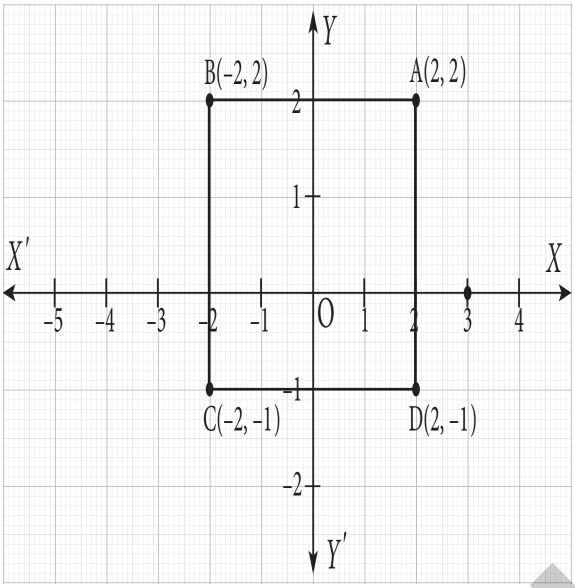
(iii) S as centre and SA = SB = SC as radius, draw the circumcircle to pass through A, B, and C.

Circum radius = 4.3 cm.

44.

a)

Point	A	B	C	D
Quadrant	I	II	III	IV



ABCD is a rectangle.

Distance between BC

$= \sqrt{(-2+2)^2+(-1-2)^2} = \sqrt{0+(-3)^2}$

$= \sqrt{0+9} = \sqrt{9}$ = 3 units

Distance between AB

$= \sqrt{(2+2)^2+(2-2)^2} = \sqrt{4+0} = \sqrt{4}$

= 2 units

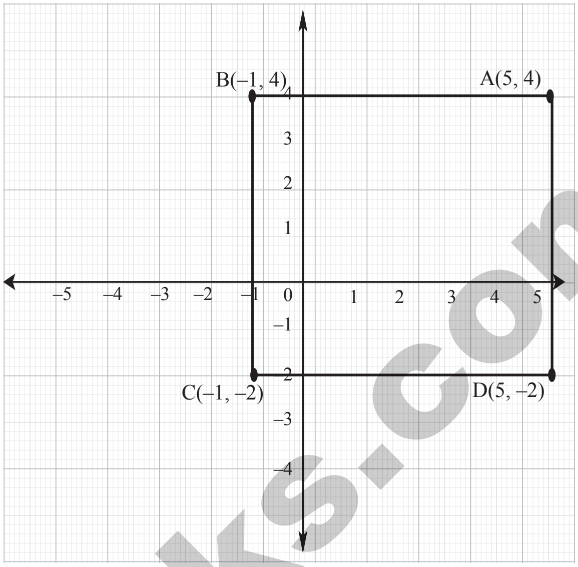
Here BC = AD = 3 units

AB = CD = 2 units

∴Area of the rectangle = 3 × 2 = 6 units

(OR)

b)



Distance between CD

$= \sqrt{(5+1)^2+(-2+2)^2}$

$= \sqrt{(6)^2+(0)^2} = \sqrt{36}$ = 6 units

Distance between AB

$= \sqrt{(5+1)^2+(4-4)^2}$

$= \sqrt{(6)^2+(0)^2} = \sqrt{36}$ = 6 units

Distance between BC

$= \sqrt{(-1+1)^2+(-2-4)^2}$

$= \sqrt{0+(-6)^2} = \sqrt{36}$ = 6 units

Distance between AD

$= \sqrt{(5-5)^2+(-2-4)^2}$

$= \sqrt{0+(-6)^2} = \sqrt{36}$ = 6 units

Since all the distance are equal to 6 units, the shape is square.

∴Area of square = a × a = 6 × 6 = 36 sq. units

☆ ☆ ☆