Chapter 1

SIMPLE ARITHMETIC COMPUTATIONS

The following formulas will be very useful.

$(a+b)^2$	=	$a^2 + 2ab + b^2$
$(a-b)^2$	=	$a^2 - 2ab + b^2$
$(a+b)^3$	=	$a^3 + 3a^2b + 3ab^2 + b^3$
$(a-b)^3$	=	$a^3 - 3a^2b + 3ab^2 - b^3$
a^2-b^2	=	$(a+b)(a-b)$
$a^3 + b^3$	=	$(a+b)(a^2-ab+b^2)$
$a^3 - b^3$	=	$(a-b)(a^2+ab+b^2)$
$a^4 - b^4$	=	$(a-b)(a+b)(a^2+b^2)$
$(a+b)^2 + (a-b)^2$	=	$2(a^2 + b^2)$
$(a+b)^2 - (a-b)^2$	=	$4ab$.

H.C.F.

Factors which divide two or more numbers without a remainder are called their **Common Factor**. Out of the common factors, the greatest is called the **Highest Common Factor** (H.C.F.) of those numbers.

(e.g) The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30

Their common factors are 1, 2, 3, 6

The greatest of them is 6

Hence H.C.F. of 24 and 30 is 6.

L.C.M.

The lowest common multiple of two or more expressions or numbers is called the **Lowest** or **Least Common Multiple** (L.C.M.) of the two or more expressions.

(e.g) L.C.M. of 6, 8 and 10.

$6 = 2 \times 3$; $8 = 2 \times 2 \times 2$; $10 = 2 \times 5$

L.C.M. $= 2^3 \times 3 \times 5 = 120$.

Fractions and to simplify the expressions

In questions on fraction, signs +, –, ×, ÷ (of signifies multiplication) and brackets are often involved. In simplifying these questions, the following order must be followed.

1. **Begin with brackets, solve the part found within brackets. Also evaluate the brackets in the order ( ), { }, [ ].**
2. **Then, come to division.**
3. **Then, do multiplication.**
4. **Finally add and subtract.**

The rule is **BODMAS**

'**B**' stands for brackets; '**O**' for of; '**D**' for division; '**M**' for multiplication;

'**A**' for addition and '**S**' for subtraction.

(e.g) $(90 \times 35 + 90 \times 15) - (90 \times 10)$

$= 90(35+15) - (90 \times 10)$

$= 90 \times 50 - 90 \times 10$

$= 90 \times 50 - 900 = 4500 - 900 = 3600$

Decimal fraction

A fraction involving decimal point is called **Decimal Fraction**. For converting decimal fraction into vulgar fraction, write down the given number in the numerator omitting the decimal point and for the denominator write 1 followed by as many zeros as there are figures on the right of the decimal point.

(e.g)

$$46.78 = \frac{4678}{100}\ ;\ 188.0834 = \frac{1880834}{10000}$$

Square Root

To find the square root, divide the number into periods of two digits each, by placing dot over every second figure beginning with that in the units place and proceeding towards the left. Find the greatest number whose square is contained in the first period. This will be the first figure of the required square root, subtract its square from the first period, and to the remainder bring down the next period.

(e.g) Find the square root of 980100

		99
9	98,01,00	
	81	
189	1701	
	1701	
	0	

∴ Square root = 990.

EXAMPLE PROBLEMS

1. Find the value of 12 × 12 + 2 × 12 × 15 + 15 × 15.

12 × 12 + 2 × 12 × 15 + 15 × 15
= $(12 + 15)^2 = (27)^2$
= 729 [$\because (a+b)^2 = a^2 + 2ab + b^2$]

2. Find the value of $(99)^2$

$(99)^2 = (100 - 1)^2$
= $(100)^2 - 2 \times 100 \times 1 + (1)^2$
= 10000 – 200 + 1 = 9801

3. Find the value of $(999)^2$

$(999)^2 = (1000 - 1)^2$
= $(1000)^2 - 2 \times 100 \times 1 + (1)^2$
= 10,00,000 – 2000 + 1
= 9,98,001

4. Find the value of $(65^2 - 15^2)$

$(65^2 - 15^2) = (65 - 15)(65 + 15)$

= (50) (80) = 4000

5. Find the value of $(12.122)^2 - (12.022)^2$

$(12.122)^2 - (12.022)^2$

= (12.122 – 12.022) (12.122 + 12.022)

= (0.1) (24.144)

= 2.4144 [$\because a^2 - b^2 = (a+b)(a-b)$]

6. Find the value of $\frac{(25 \times 25) - (15 \times 15)}{(25 + 15)}$

$\left[\frac{a^2 - b^2}{a+b} = \frac{(a+b)(a-b)}{a+b} = a - b\right]$

Here, a = 25 ; b = 15

$\therefore \frac{(25 \times 25) - (15 \times 15)}{(25+15)} = \frac{(25+15) \times (25-15)}{(25+15)}$

= 25 – 15 = 10

7. Find the value of $\frac{15^3 - 4^3}{15^2 + 15 \times 4 + 4^2}$

$\left[\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b\right]$ Here a = 15 ; b = 4

$\therefore \frac{15^3 - 4^3}{15^2 + 15 \times 4 + 4^2} = 15 - 4 = 11$

8. Find the value of $\frac{1.7 \times 1.7 \times 1.7 - 1.5 \times 1.5 \times 1.5}{1.7 \times 1.7 + 1.7 \times 1.5 + 1.5 \times 1.5}$

The required value

$= \frac{(1.7)^3 - (1.5)^3}{(1.7)^2 + (1.7)(1.5) + (1.5)^2}$

= 1.7 – 1.5 = 0.20

9. Simplify

$$\frac{(.57)^3 - (0.1)^3}{(.57)^2 + (0.57)(0.1) + (0.1)^2} + \frac{0.67}{1 + \frac{24}{43}}$$

The first term can be simplified by using

$\frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$

Here a = 0.57 and b = 0.1

$\therefore$ The required value = $0.57 - 0.1 + \frac{0.67}{1 + \frac{24}{43}}$

$= 0.47 + \frac{0.67}{\frac{67}{43}} = 0.47 + 0.67 \times \frac{43}{67}$

$= 0.47 + 67 \times \frac{43}{6700} \left(\therefore \frac{0.67}{67} = \frac{67}{6700}\right)$

= 0.47 + 0.43 = 0.90

10. Find the value of $15^4 - 5^4$

$(a^4 - b^4) = (a+b)\,(a-b)\,(a^2+b^2)$

Here a = 15 ; b = 5

$15^4 - 5^4 = (15 + 5)\,(15 - 5)\,(15^2 + 5^2)$

$= 20 \times 10 \times 250 = 50{,}000$

11. Find the value of 6 + [5 – {4 + (3 – (2 + 1)}]

{The rule to be followed for this type of problems is BODMAS and brackets in the order (), {}, []}

$\therefore$ 6 + [5 – {4 + (3 – 2 + 1)}]

= 6 + [5 – {4 + (3 – 3)}]

= 6 + [5 – {4 + 0}]

= 6 + [5 – 4] = 7

12. Determine the value of
1 + [2 – {3 + (4 – 5 + 6)}]

The given expression

= 1 + [2 – {3 + (4 – 5 + 6)}]

= 1 + [2 – { 3 + (4 + 1)}]

= 1 + [2 – {3 + 5}]

= 1 + [2 – 8]

= 1 + [– 6] = 1 – 6 = –5

13. Simplify

$$\frac{2\frac{4}{9} \div 3\frac{2}{3} \text{ of } \frac{2}{5} \times \frac{3}{5} + 1\frac{1}{9}}{1\frac{1}{9} \times \frac{3}{4} \text{ of } 1\frac{2}{5} \div \frac{21}{38} - \frac{1}{3}} - \frac{5\frac{1}{2} - \frac{3}{4}}{2\frac{1}{5} \times 1\frac{9}{11}}$$

$$= \frac{\frac{22}{9} \div \frac{11}{3} \text{ of } \frac{2}{5} \times \frac{3}{5} + \frac{10}{9}}{\frac{10}{9} \times \frac{3}{4} \text{ of } \frac{7}{5} \div \frac{21}{38} - \frac{1}{3}} - \frac{\frac{11}{2} - \frac{3}{4}}{\frac{11}{5} \times \frac{20}{11}}$$

$$= \frac{\frac{22}{9} \div \frac{22}{15} \times \frac{3}{5} + \frac{10}{9}}{\frac{10}{9} \times \frac{21}{20} \div \frac{21}{38} - \frac{1}{3}} - \frac{22-3}{4}$$

$$= \frac{\left(\frac{22}{9} \times \frac{15}{22}\right) \times \frac{3}{5} + \frac{10}{9}}{\frac{10}{9} \times \left(\frac{21}{20} \times \frac{38}{21}\right) - \frac{1}{3}} - \frac{19}{4} \times \frac{1}{4}$$

$$= \frac{\frac{5}{3} \times \frac{3}{5} + \frac{10}{9}}{\frac{10}{9} \times \frac{19}{10} - \frac{1}{3}} - \frac{19}{16} = \frac{1 + \frac{10}{9}}{\frac{19}{9} - \frac{1}{3}} - \frac{19}{16}$$

$$= \frac{\frac{19}{9}}{\frac{16}{9}} - \frac{19}{16} = \left(\frac{19}{9} \times \frac{19}{16}\right) - \frac{19}{16}$$

$$= \frac{19}{16} - \frac{19}{16} = 0$$

14. Simplify

$$7\frac{1}{2} - \frac{1}{9}\left[3\frac{3}{4} \div \left\{\frac{5}{6} \text{ of } \frac{2}{3}\left(\frac{1}{3} - \left(\frac{1}{4} - \frac{1}{6}\right)\right)\right\}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{5}{6} \text{of} \frac{2}{3}\left(\frac{1}{3} - \left(\frac{3-2}{12}\right)\right)\right\}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{5}{6} \text{of} \frac{2}{3}\left(\frac{1}{3} - \frac{1}{12}\right)\right\}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{5}{6} \text{of} \frac{2}{3}\left(\frac{3}{12}\right)\right\}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{10}{18}\left(\frac{3}{12}\right)\right\}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \frac{5}{36}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \times \frac{36}{5}\right]$$

$$= 7\frac{1}{2} - \frac{1}{9} \times [3 \times 9] = 7\frac{1}{2} - \frac{1}{9} \times 27$$

$$= 7\frac{1}{2} - 3 = 4\frac{1}{2}$$

15. Simplify

$$\frac{0.1\times0.1\times0.1+0.01+0.01\times0.01}{0.2\times0.2\times0.2+0.08+0.04\times0.02}$$

$$=\frac{\frac{1}{10}\times\frac{1}{10}\times\frac{1}{10}+\frac{1}{100}+\frac{1}{100}\times\frac{1}{100}}{\frac{2}{10}\times\frac{2}{10}\times\frac{2}{10}+\frac{8}{100}+\frac{4}{100}\times\frac{2}{100}}$$

$$=\frac{\frac{1}{1000}+\frac{1}{100}+\frac{1}{10000}}{\frac{8}{1000}+\frac{8}{100}+\frac{8}{10000}}$$

$$=\frac{\left(\frac{1}{1000}+\frac{1}{100}+\frac{1}{10000}\right)}{8\left(\frac{1}{1000}+\frac{1}{100}+\frac{1}{10000}\right)}=\frac{1}{8}$$

16. Simplify 15 of

$$\frac{\frac{2}{3}\text{ of }\frac{4}{9}-3\times\frac{3}{5}\text{ of }\frac{4}{9}+3\times\frac{9}{25}\text{ of }\frac{2}{3}-0.6\text{ of }0.36}{\frac{4}{9}-2\times0.6\text{ of }\frac{2}{3}+0.36}$$

$= 15$ of

$$\frac{\frac{2}{3}\times\frac{4}{9}-3\times\frac{3}{5}\times\frac{4}{9}+3\times\frac{9}{25}\times\frac{2}{3}-\frac{6}{10}\times\frac{36}{100}}{\frac{4}{9}-2\times\frac{6}{10}\times\frac{2}{3}+\frac{36}{100}}$$

$$= 15 \text{ of } \frac{\frac{8}{27}-\frac{4}{5}+\frac{18}{25}-\frac{216}{1000}}{\frac{4}{9}-\frac{4}{5}+\frac{36}{100}}$$

$$= 15 \text{ of } \frac{\frac{8000-21600+19440-5832}{27000}}{\frac{400-720+324}{900}}$$

$$= 15 \text{ of } \frac{\frac{8}{27000}}{\frac{4}{900}} = 15 \text{ of } \frac{8}{27000}\times\frac{900}{4}$$

$$= 15\times\frac{1}{15} = 1$$

17. Simplify : $\frac{500\times0.00125}{6.25}$

$$=\frac{500\times\frac{125}{100000}}{\frac{625}{100}}=500\times\frac{125}{100000}\times\frac{100}{625}$$

$$=\frac{1}{10}=0.1$$

18. Find the H.C.F. of 630, 1050, 1260

$630 = 2.3.3.5.7$

$1050 = 2.3.5.5.7$

$1260 = 2.2.3.3.5.7$

$\therefore$ H.C.F. is $2.3.5.7 = 210$

19. Find the H.C.F. of 72 and 96

$72 = 2\times2\times2\times3\times3$

$96 = 2\times2\times2\times2\times2\times3$

$\therefore$ H.C.F. of 72 and 96

$= 2\times2\times2\times3 = 24$

20. Find the L.C.M. of 70, 80, 90

$70 = 2\times5\times7$

$80 = 2^4\times5$

$90 = 2\times3^2\times5$

$\therefore$ L.C.M. $= 2^4.3^2.5.7 = 5040$

21. What is the highest number of four digits which will leave a remainder of 1 when divided by any of the numbers 6,9,12,15 or 18 ?

L.C.M. of 6, 9, 12, 15, 18 = 180

Greatest no. of 4 digits = 9999

Greatest no. of 4 digits divisible by 180 is

$= 9999 - 99 = 9900$

$$\begin{array}{r|l} & 55 \\ \hline 180 & 9999 \\ & 900 \\ \hline & 999 \\ & 900 \\ \hline & 99 \end{array}$$

$\therefore$ Required Number $= 9900 + 1 = 9901$