1 Number System

* **Digit :-** 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called digits.
* **Natural Numbers :** Numbers which are used for counting i.e. 1, 2, 3, 4.......are called natural numbers. Smallest natural number is 1.
* **Whole Number :** When zero is included in natural numbers then it is called a set of whole numbers. i.e. 0, 1, 2, 3, 4.......Zero is the smallest whole number.
* **Even and Odd Number :** Numbers which are divisible by 2 are called even and which are not divisible by 2 are called odd numbers.

In general even numbers can be represented by 2n, and odd numbers can be represented by $2n \pm 1$ where n is an integer.

SOME FACTS

* The sum and product of any number of even number is an even number.
* The difference of two even numbers is even number.
* The sum of odd numbers depends on the number of numbers. (1), If the number of numbers is odd, the sum is an odd number. (2), If the number of numbers are even there is an even number.
* If the product of certain numer is even then at least one of the number has to be even.

Integers : Whole numbers and negative natural numbers constitute the set of integers.

i.e. Z = ...–4, –3, –2, –1, 0, 1, 2, 3.........

* **Rational and Irrational Numbers :** Any number which can be expressed in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is called a rational number e.g $\frac{4}{5}, \frac{-1}{2}$, 0 etc. Every natural number, whole number and integer is also a rational number. And any number which cannot be expressed in the form of $\frac{p}{q}$ is called an irrational number e.g. $\sqrt{2}$, $\sqrt{5}$ etc.
* **Terminating Decimals :** The rational number which terminates after a finite number of step in process of division is called a terminating decimal e.g., 1.25, 3.14 etc.
* **Non-Terminating Decimals :** The number which doesn't terminates in the process of division is called non terminating decimals. It is of two types.

1. **Non-Terminating Repeating Decimal :** The number which doesn't terminate but repeats the same numbers in process of divisions e.g.

   $\frac{1}{3} = 0.3333............ = 0.\bar{3}$
2. **Non-Terminating Non-Repeated Decimals :** The number which neither terminates nor repeats in process of division e.g. 1.030030003..........., $\sqrt{3}$ etc.

* If x and y are any two rational numbers then $\frac{x+y}{2}$ is a rational numbers between them.
* If x and y are any two distinct irrational number then $\sqrt{xy}$ is an irrational number between them.

* **Real Numbers :** All these numbers which are described above are called real numbers i.e. when natural numbers, whole numbers, integers, rational numbers and irrational numbers are collected together they are called real numbers.
* **Prime Number :** A number greater than 1 having exactly two factors, 1 and itself is called a prime number e.g. 3, 3, 5, 7......
* **Composite Number :** A number which has factors besides other than 1 and itself is called composite number e.g. 6, 8, 12, 15......
* **Co-Primes or Relative Primes :** Two numbers are called co-primes or relative primes if they have only one common factor which is 1. *e.g.* (7, 9) is co-prime but 15 and 21 is not co-prime.
* **Twin-Primes :** Pair of prime numbers whose difference is called twin primes *e.g.* (3,5), (5, 7) etc.

SOME FACTS

* 1 is neither prime nor composite.
* 2 is the only even prime number.
* Any prime number greater than 3 can be expressed as 6k + 1 or 6k – 1 where k is a natural number.
* A pair of co-prime may have
  (i) both number as primes *e.g.* (3, 5)
  (ii) one prime and one composite *e.g.* (7, 6)
  (iii) both composites e.g. (8, 15)
* Co-prime need not necessarily be prime themselves.

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* π (the ratio of diameter and circumference of a circle) is irrational number. $\frac{22}{7}$, is rational, π is not exactly equal to $\frac{22}{7}$ as it is taken generally.
* Sum, difference & product of two rational numbers is again a rational numbers
* Sum, difference, product and quotient of two irrational numbers need not to be an irrational number.
* **Factorial of a Number :** For any natural number n factorial (symbolised as $n!$) is defined as product of natural numbers from 1 to n.

  So $n! = 1 \times 2 \times 3 \times 4 \ldots\ldots \times (n-2) \times (n-1) \times n$
* Sum of first n natural numbers

$$\sum n = 1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$$

* Sum of the squares of the n natural number

$$\sum n^2 = 1^2 + 2^2 + 3^3 + \ldots\ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

* Sum of the cubes of the n natural number

$$\sum n^3 = 1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \left[\frac{n(n+1)^2}{2}\right]$$

* **Indices :** Any number multiplied by itself m times is written as a^m where a is called base and m is called power or index or exponent.
* Any non zero real number when raised to the power zero is equal to 1 i.e. $a^0 = 1$
* When m is a positive integer,

  $a^m = a \times a \times a \times a \ldots\ldots. m$ times.

  For example if $m = -3$ then $a^m = a^{-3} = \frac{1}{a^3}$
* When m is a fraction (either positive or negative):

(i) Suppose m = 1/3, then $a^m = a^{1/3}$. This is defined as equal to $\sqrt[3]{a}$ (cube root of a).

(ii) Suppose $m = 3/5$, then $a^m = a^{3/5} = \sqrt[5]{a^3}$ (fifth root of a^3).

(iii) Suppose $m = -3/5$ then $a^m = a^{3/5} = 1/a^{3/5} = \frac{1}{\sqrt[5]{a^3}}$.

* If $a \neq 1, a \neq 0, x \neq -1$ and $a^x = a^y$ then $x = y$
* If $a \neq 0$, $b \neq 0$, and $ax = bx$ then

  (i) $a = b$ if x is odd

  (ii) $a = \pm b$ if x is even

* **LAWS OF EXPONENTS**

	Law	Examples
1.	$a^m \times a^n = a^{m+n}$	$2^2 \times 2^3 = 2^{2+3} = 2^5$
2.	$a^m + a^n = a^{m-n}$	$2^6 \div 2^2 = 2^{6-2} = 24$
3.	$a^{-m} = 1/a^m$	$2^{-4} = 1/2^4$
4.	$a^{p/q} = \sqrt[q]{a^p}$	$2^{4/3} = \sqrt[3]{2^4}$
5.	$(a^m)^n = a^{mn}$	$(2^3)^2 = 2^6$
6.	$(ab)^m = a^m \cdot b^m$	$(2\times3)^6 = 2^6 \times 3^6$
7.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{2}{3}\right)^6 = \frac{2^6}{3^6}$

* **Rationalising Factor :** If product of two surds is a rational number then they are rationalising factor of each other e.g.

  $\sqrt[3]{16} \times \sqrt[3]{4} = 4$ so RF of $\sqrt[3]{16}$ is $\sqrt[3]{4}$

  and RF of $\sqrt[3]{4}$ is $\sqrt[3]{16}$
* **Rationalisation of Surds :** Process of converting surd into rational number is called its rationalisation. It is carried our by multiplying surd with an appropriate rationalising factor.
* The conjugate surd of $\sqrt{a} + \sqrt{b}$ is $\pm\left(\sqrt{a} - \sqrt{b}\right)$
* To rationalise $\frac{1}{\sqrt{a} + \sqrt{b}}$, multiply it by $\frac{\left(\sqrt{a} - \sqrt{b}\right)}{\sqrt{a} + \sqrt{b}}$ or $\frac{\left(\sqrt{b} - \sqrt{a}\right)}{\left(\sqrt{b} - a\right)}$
* If a+ $\sqrt{b} = c + \sqrt{d}$, then $a = c$ and $b = d$
* To find $\sqrt{\left(a + \sqrt{b}\right)}$ write it in the form m+n+ $2\sqrt{mn}$, such that m+n = a and 4mn = b, then $\sqrt{\left(a + \sqrt{b}\right)} = \pm\left(\sqrt{m} + \sqrt{n}\right)$
* $\sqrt{a.\sqrt{a.\sqrt{a \ldots\ldots\ldots \infty}}} = a$
* $\sqrt{a.\sqrt{a.\sqrt{a.\sqrt{a \ldots\ldots\ldots n \text{ times}}}}} = a^{1-\frac{1}{2^n}}$
* If $\sqrt{a + \sqrt{a + \sqrt{a \ldots\ldots\ldots \infty}}} = p$, then $p(p-1) = a$
* The rationalising factor of $\sqrt{a} + \sqrt{b}$ is $\sqrt{a} - \sqrt{b}$
* The rationalising factor of $\sqrt{a} - \sqrt{b}$ is $\sqrt{a} + \sqrt{b}$
* The rationalising factor of $a + \sqrt{b}$ is $a - \sqrt{b}$

COMPARISON OF SURDS :

1. Find LCM of order of surds.
2. Divide this LCM by order surds indisivually and find their quotients.
3. Now replace order by LCM, and put this obtained quotient as power of radicand.
4. Now we canput it in required order.

e.g. Find which is greater $\sqrt[4]{2}$ or $\sqrt[5]{3}$

1. Orders of surds are 4 and 5
2. LCM of orders is 20
3. After dividing LCM obtained by orders we get

$$\frac{20}{4} = 5$$

$$\frac{20}{5} = 4$$

4. Now replacing order by LCM and putting quotient as power of radicand we get

$$\sqrt[20]{(2)^5} \text{ and } \sqrt[20]{3^4}$$

Now surds are

$$\sqrt[20]{32} \text{ and } \sqrt[20]{81}$$

Now compare 32 and 81 > 32 < 81

So $\sqrt[4]{2} < \sqrt[5]{3}$

HCF AND LCM OF NUMBERS

* **Factor :** If y is divisible by x leaving remainder zero then x is a factor of y.
* **Prime Factorization :** When factorization of a number expressed completely involving only prime factors.
* **Highest Common Factors :** The greatest number which divides two or more number is called their HCF or GCD (greatest common divisors).
* **Least Common Multiple :** The least number which is divisible by two or more given number is called their LCM.
* **LCM × HCF :** As a rule, for any two numbers *m* and *n,* the product of their LCM and HCF equals the product of the numbers themselves.

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

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If the HCF of m and n is h then the HCF of

(i) $m, m + n$ is also h

(ii) $m, m - n$ is also h

(iii) $m + n, m - n$ is also h

MULTIPLE CHOICE QUESTIONS

1. Which of the following number divides the product of 8 consecutive integers?

(A) 4! (B) 8!
(C) 6! (D) 7!

2. Number divisible by 11 is :

(A) 7284342 (B) 6543832
(C) 8432644 (D) 5599998

3. The square of a prime numbers between 10 and 50 which is same after interchanging its digits is :

(A) 1936 (B) 1089
(C) 484 (D) 121

4. Which of the following number of 4 digits divides a number which is obtained by repeating a number of two digits is :

(A) 7 (B) 19
(C) 83 (D) 101

5. If 93P 25Q is divisible by 88 then value of P and Q is :

(A) 2 and 8 (B) 8 and 2
(C) 8 and 6 (D) 6 and 8

6. If a < 0 and b < 0 then which of the following is always true :

(A) $a - b > 0$ (B) $a + b > 0$
(C) $a + b < 0$ (D) $a - b < 0$

7. The total number of factors of $(4)^{11} \times (14)^5 \times (11)^2$ is :

(A) 34
(B) 36
(C) 38
(D) 43

8. In a fraction sum of numerator and denominator is 8. If 2 is added to both the numerator and denominator then value of new fraction is $\frac{4}{35}$ more than original fraction. The fraction is :

(A) $\frac{1}{7}$ (B) $\frac{3}{5}$
(C) $\frac{5}{3}$ (D) $\frac{2}{5}$

9. If $39x3$ is divisible by 7 then x is :

(A) 1 (B) 3
(C) 7 (D) 9

10. Common factor in (13^7+11^7) and (13^5+11^5) is :

(A) 24
(B) 13^5+11^5
(C) 13^2+11^2
(D) None of these

11. $\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\ldots\left(1-\frac{1}{n}\right)$ is equal to

(A) $\frac{1}{n}$ (B) $\frac{2}{n}$

(C) $\frac{2(n-1)}{n}$ (D) $\frac{2}{n(n+1)}$

12. If addition and product of two number is 12 and 35 then addition of their reciprocals is :

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$

(C) $\frac{12}{35}$ (D) $\frac{35}{12}$

13. Digit at unit place in $(264)^{102}+(264)^{103}$ is :

(A) 0 (B) 4

(C) 6 (D) 8

14. Simplified value of $\frac{9|3-5|-5|4|\div 10}{-3(5)-2\times 4\div 2}$ is :

(A) $\frac{9}{10}$ (B) $-\frac{8}{17}$

(C) $-\frac{16}{19}$ (D) $\frac{4}{7}$

15. A two digit number is 54 more than sum of tis digit. The difference between its digit is :

(A) 2 (B) 6

(C) 1 (D) None of the above

16. What is the remainder when $2851 \times (2862)^2 \times (2873)^3$ is divided by 23?

(A) 18 (B) 17

(C) 10 (D) 5

17. First 20 natural numbers are written as N = 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 when this number is divided by 16 then remainder is :

(A) 0 (B) 84

(C) 7 (D) 9

18. 899

(A) is prime

(B) has four prime factor

(C) has more than four prime factors

(D) is a perfect cube

19. If in a number the difference between the totals of digits appearing at even places and odd places is 0 then number is divisible by

(A) 3 (B) 5

(C) 7 (D) 11

20. What will be difference between 0.44 & 0.00044?

(A) 0.0043956 (B) 0.043956

(C) 0.0043956 (D) 0.43956

21. The value of $\frac{1}{3+\frac{1}{2-\frac{1}{7/9}}}+\frac{17}{22}$ is

(A) $\frac{12}{22}$ (B) $\frac{22}{5}$

(C) $\frac{5}{22}$ (D) 1

22. The value of $2^3+4^3+6^3+\ldots\ldots+20^3$ is

(A) 6050 (B) 9075

(C) 12100 (D) 24200

23. If $2 = x+\frac{1}{1+\frac{1}{3+\frac{1}{4}}}$ then value of x is

(A) $\frac{18}{17}$ (B) $\frac{21}{17}$

(C) $\frac{13}{17}$ (D) $\frac{12}{17}$

24. If difference of squares of two consecutive even numbers is 84 then sum of the number is :

(A) 38 (B) 34

(C) 42 (D) 46

25. The digit at unit place in 81 × 82 × 83 × 84 × 85 × 86 × 87 × 88 × 89 is :

(A) 0 (B) 9

(C) 7 (D) 2

26. Sum of the reciprocals of 3 integers is again an integer. Such three integers are :

(A) 2, 3, 4 (B) 3, 4, 5

(C) 5, 6, 7 (D) 2, 3, 6

27. After rationalisation of $\frac{7}{3+\sqrt{n}}$ we get $3-\sqrt{n}$ then value of n is :

(A) 1 (B) 2

(C) 3 (D) 7

Direction (Q. 28–30) : What will be the number at the place of ? In the given series?

28. 1 4 27 256 3125 46656?

(A) 117649 (B) 279936

(C) 705894 (D) 823543

29. 15 12 17 10 ? 8 27 6

(A) 3 (B) 17

(C) 21 (D) 19

30. 12 14 17 13 8 14 21 13 4?

(A) 14 (B) 13

(C) 15 (D) 2