SURA'S

Mathematics

11th Standard

VOLUME - I & II

Based on the New Syllabus and New Textbook

Salient Features

- Prepared as per the updated New Textbook.
- Exhaustive Additional Questions & Answers in all chapters.
- Govt. Model Question Paper-2018 [Govt. MQP-2018], First Mid-Term Test (2018) [First Mid-2018], Quarterly Exam - 2018 [QY-2018], Half Yearly Exam - 2018 [HY-2018], March Question Paper - 2019[March - 2019], June Question Paper - 2019[June - 2019],Quarterly Exam - 2019 [QY-2019], Half Yearly Exam - 2019 [HY-2019] are incorporated at appropriate sections.
- Govt. Model Question Paper
- June - 2019 Question Paper
- Common Quarterly Examination 2019 Question Paper
- Common Half Yearly Examination 2019 Question Paper
- Public Examination March - 2020 Question Paper with answers

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Chennai

CONTENTS

Volume - I

Volume - II

Volume - I

MATHEMATICS

11th Standard

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01 Sets Relations and Functions

MUST KNOW DEFINITIONS

A set is a collection of well defined objects.

Type of sets

Empty set	:	A set containing no element.
Finite set	:	The number of elements in the set is finite.
Infinite set	:	The number of elements in the set is not finite.
Singleton set	:	A set containing only one element.
Equivalent set	:	Two sets having same number of elements.
Equal sets	:	Two sets exactly having the same elements.
Subset	:	A set X is a subset of Y if every element of X is also an element of Y. $(X \subseteq Y)$
Proper subset	:	X is a proper subset of Y if $X \subseteq Y$ and $X \neq Y$.
Power set	:	The set of all subsets of A is the power set of A.
Universal set	:	The set contains all the elements under consideration

Algebra of sets

Union	:	The union of two sets A and B is the set of elements which are either in A or in B $(A \cup B)$
Intersection	:	The intersection of two sets A and B is the set of all elements common to both A and B $(A \cap B)$.
Complement of a set	:	The set of all elements of U (Universal set) that are not elements of A. (A') Set different$(A \backslash B)$ or $(A - B)$ The difference of the two sets A and B is the set of all elements belonging to A but not to B
Disjoint sets	:	Two sets A and B are said to be disjoint if there is no element common to both A and B.
Open interval	:	The set $\{x: a < x < b\}$ is called an open interval and denoted by (a, b)
Closed interval	:	The set $\{x: a \leq x \leq b\}$ is called a closed interval and denoted by $[a, b]$
Neighbourhood of a point	:	Let a be any real number. Let $\in > 0$ be arbitrarily small real number. Then $(a - \in, a + \in)$ is called an "$\in$" neighbourhood of the point a and denoted by $N_{a,\in}$

$$= \frac{9\left(\frac{5x-160}{9}\right)+160}{5}$$

$$= \frac{5x-160+160}{5} = \frac{5x}{5} = x$$

and $fog(y) = f(g(y)) = f\left(\frac{9y+160}{5}\right)$

$$= \frac{5\left(\frac{9y+160}{5}\right)-160}{9} = \frac{9y+160-160}{9} = y$$

Thus $gof = I_x$ and $fog = I_y$.

This implies that f and g are bijections and inverses to each other.

$$f^{-1}(y) = \frac{9y+160}{5}$$

Replacing y by x, we get $f^{-1}(x) = \frac{9x+160}{5} = \frac{9x}{5} + 32$

20. A simple cipher takes a number and codes it, using the function $f(x) = 3x - 4$. Find the inverse of this function, determine whether the inverse is also a function and verify the symmetrical property about the line $y = x$ (by drawing the lines).

Solution : Given $f(x) = 3x - 4$

Let $y = 3x - 4 \Rightarrow y + 4 = 3x$

$$\Rightarrow x = \frac{y+4}{3}$$

Let $g(y) = \frac{y+4}{3}$.

Now $gof(x) = g(f(x)) = g(3x-4)$

$$= \frac{3x-4+4}{3} = \frac{3x}{3} = x$$

and $fog(y) = f(g(y)) = f\left(\frac{y+4}{3}\right)$

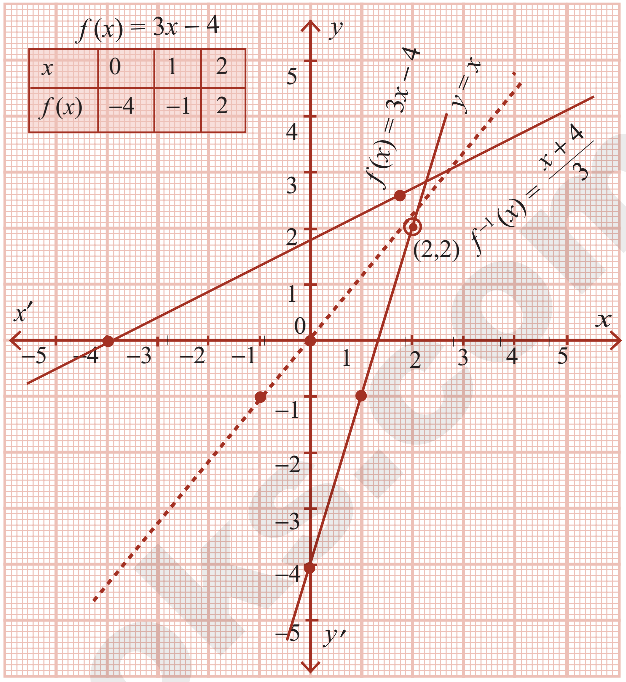
$$= 3\left(\frac{y+4}{3}\right) - 4 = y + 4 - 4 = y$$

Thus, $gof(x) = I_x$ and $fog(y) = I_y$.

This implies that f and g are bijections and inverses to each other.

Hence f is bijection and $f^{-1}(y) = \frac{y+4}{3}$

Replacing y by x, we get $f^{-1}(x) = \boxed{\frac{x+4}{3}}$.



Hence, the graph of $y = f^{-1}(x)$ is the reflection of the graph of f in $y = x$

EXERCISE 1.4

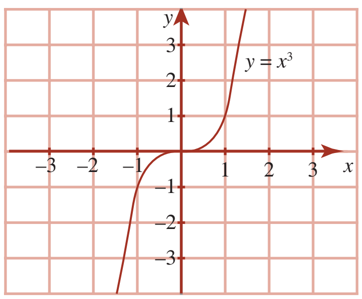
1. For the curve $y = x^3$ given in figure draw,

(i) $y = -x^3$ (ii) $y = x^3 + 1$

(iii) $y = x^3 - 1$ (iv) $y = (x+1)^3$

with the same scale.

Solution : **(i) $y = -x^3$.** **[Qy - 2019]**



02 BASIC ALGEBRA

MUST KNOW DEFINITIONS

Rational numbers : Any number of the form $\frac{p}{q}$, where $q \neq 0$ is called a real number where $p, q \in$ R.

Irrational numbers : A number that cannot be expressed as a ratio between two integers and is not an imaginary number.

Intervals : If a, b are real numbers such that $a < b$, then the set $\{x: a < x < b\}$ is called the open interval from a to b i.e. (a, b).

- The set $\{x: a \leq x \leq b\}$ is called the closed interval from a to b and is written as $[a, b]$
- If a is any real number, then the sets of the type $\{x: x < a\}$, $\{x: x \leq a\}$, $\{x: x > a\}$ and $\{x: x \geq a\}$ are called infinite intervals and are respectively written as $(-\infty, a)$, $(-\infty, a]$, (a, ∞) and $[a, \infty)$.
- These are semi-open and semi-closed intervals.

Absolute value of *x* Absolute value of $x = |x|$ is defined as:

$$|x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$

Radical (Surd) : If 'a' is a positive rational number and n is a positive integer such that $\sqrt[n]{a}$ is an irrational number then $\sqrt[n]{a}$ is called a surd or a radical.

Mixed surd : A surd is called a mixed if its rational co-efficient is other than unity. If the product of two irrational numbers is rational, then each one is called the rationalizing.

Pure surd : A surd is a pure surd if its rational co-efficient is unity.

Polynomial : An expression of more than two algebraic terms, especially the sum of several terms that contain different powers of the same variable

Identity : An identity is a statement of equality between two expressions which is true for all values of the variable involved.

Equation : An equation is a statement of equality between two expressions which is not true for all values of the variable involved.

$$z = e^{k(a-b)} \quad \ldots(2)$$

(i)

$$\begin{aligned} xyz &= e^{k(b-c)} \cdot e^{k(c-a)} \cdot e^{k(a-b)} \\ &= e^{k(\cancel{b} - \cancel{c} + \cancel{c} - \cancel{a} + \cancel{a} - \cancel{b})} \\ &= e^{k(0)} \\ &= e^0 = 1 \\ \Rightarrow \quad xyz &= 1 \end{aligned}$$

(ii) $x^a y^b z^c = 1$

$$\begin{aligned} x^a y^b z^c &= [e^{k(b-c)}]^a \cdot [e^{k(c-a)}]^b \cdot [e^{k(a-b)}]^c \\ &= e^{k(b-c)\cdot a} \cdot e^{k(c-a)\cdot b} \cdot e^{k(a-b)\cdot c} \\ &= e^{k(\cancel{ab} - \cancel{ac} + \cancel{bc} - \cancel{ab} + \cancel{ac} - \cancel{bc})} \\ &= e^{k(0)} = e^0 = 1 \\ \Rightarrow \quad x^a y^b z^c &= 1 \end{aligned}$$

Hence proved

❁❁❁❁❁

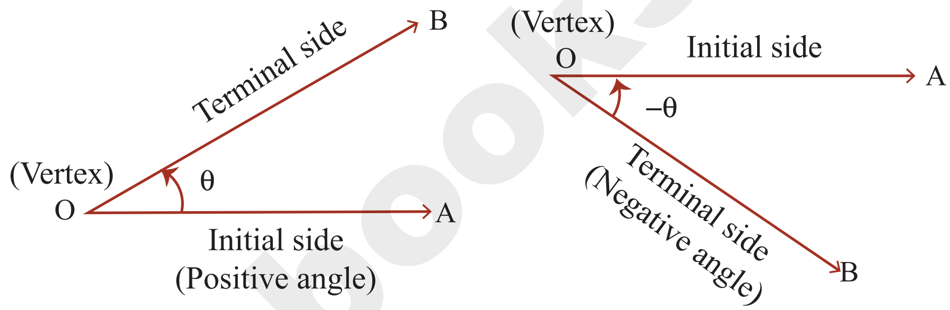
POINTS TO REMEMBER

- π and $\sqrt{p}$, where p is a prime number, are some irrational numbers.
- $|x - a| = r$ if and only if $r \geq 0$ and $x - a = \pm r$.
- $|x - a| \leq r$ if and only if $-r \leq x - a \leq r$ or $a - r \leq x \leq a + r$.
- $|x - a| > r$ implies $x < a - r$ and $x > a + r$ (or) $x \in (-\infty, a - r) \cup (a + r, \infty)$
- Inequalities, in general, have more than one solution.
- The nature of roots of $ax^2 + bx + c = 0$ is determined by the discriminant $D = b^2 - 4ac$.
- A real number a is a zero of a polynomial function $f(x)$ if and only if $(x - a)$ is a factor of $f(x)$.
- If degree of $f(x)$ is less than the degree of $g(x)$, then $\frac{f(x)}{g(x)}$ can be written as sum of its partial fractions.
- In general exponential functions and logarithmic functions are inverse functions to each other.

03 TRIGONOMETRY

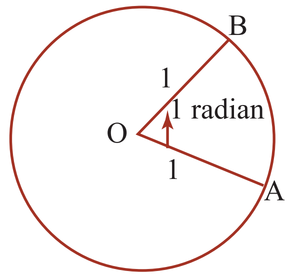
MUST KNOW DEFINITIONS

Angles : Angle is a measure of rotation of a given ray about its initial point.



Degree measure : If a rotation from the initial side to terminal side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, the angle is one degree (i) Also 1° = 60′ and 1′ = 60″.

Radian measure : Angle subtended at the center by an arc of length 1 unit of a unit circle is 1 radian.



Trigonometric equations : Equations involving trigonometric functions of a variable are called **trigonometric equations.**

Principal solutions : Among all solutions, the solution which is in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ for sine ratio, $[0, \pi]$ for cosine ratio, $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for tan ratio is the principal solution.

TEXTUAL QUESTIONS

Exercise 3.1

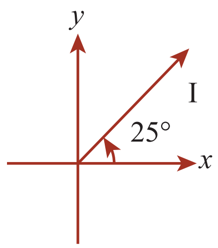
1. Identify the quadrant in which an angle of each given measure lies:

(i) 25° (ii) 825° (iii) –55° (iv) 328° (v) –230°

Solution :

(i) 25°

sin 25°, is an acute angle, 25° lies in the I quadrant.

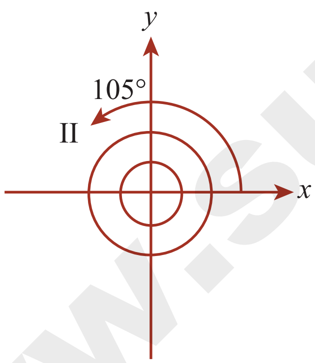


(ii) 825°

$825° = 2 \times 360 + \underline{105}°$

After two complete rounds the angle is 105° which lies between 90° and 180°

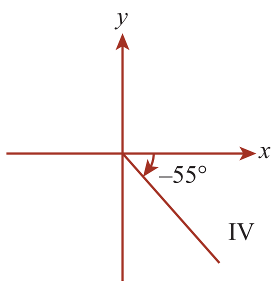
∴ 825° lies in the II quadrant



(iii) –55°

Since the given angle is negative, it moves in the clockwise direction.

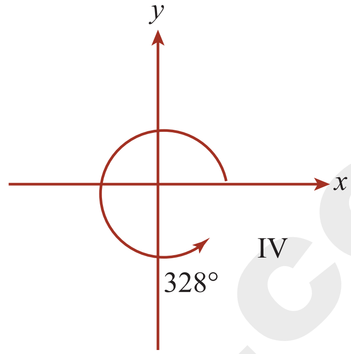
∴ – 55° lies in the IV quadrant



(iv) 328°

$328° = 270° + 58°$.

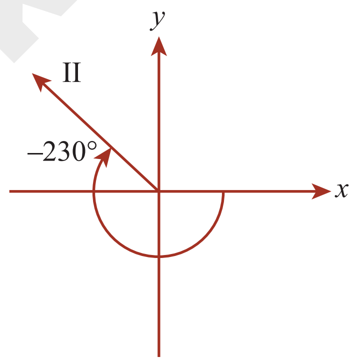
∴ 328° lies in the IV quadrant



(v) –230°

$-230° = -180° + (-50°)$

∴ –230° lies in the II quadrant.



2. For each given angle, find a co-terminal angle with measure of θ such that 0° ≤ θ ≤ 360°

(i) 395° (ii) 525° (iii) 1150° (iv) –270° (v) –450°

Solution :

(i) 395°

$$395° = 360° + 35°$$

$$\Rightarrow \quad 395 - 35° = 360°$$

∴ Co-terminal angle for 395° is 35°.

(ii) 525°

$$525° = 360 + 165°$$

$$\Rightarrow \quad 525° - 165° = 360°$$

∴ Co-terminal angle of 525° is 165°

(iii) 1150°

$$1150° = 360 + 360 + 360° + 70°$$

$$= 3 \times 360° + 70°$$

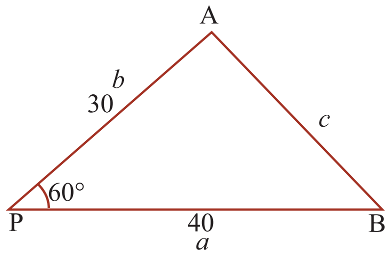
$$\Rightarrow 1150° - 70° = 3 \times 360°$$

∴ Co-terminal angle of 1150° is 70°

$$= \left(\cancel{60}^{30} \times \frac{1}{\cancel{2}}\right) = 30 \text{ km}$$

$\Rightarrow$ PA $= 30$ km.

Speed take by the IInd vehicle = 80 km/hr



$$\text{Time} = \frac{1}{2} \text{ hr}$$

$$\therefore \text{Distance} = \text{speed} \times \text{time}$$

$$= \cancel{80}^{40} \times \frac{1}{\cancel{2}} = 40 \text{ km} \Rightarrow \text{PB} = 40$$

Given $\angle APB = 60°$

Using cosine formula,

$$c^2 = a^2 + b^2 - 2ab \cos C.$$

$$\Rightarrow \quad c^2 = 40^2 + 30^2 - 2(40)(30) \cos 60°$$

$$c^2 = 1600 + 900 - \cancel{2}(40)(30)\left(\frac{1}{\cancel{2}}\right)$$

$$= 2500 - (40)(30) = 1300$$

$$\Rightarrow \quad c = \sqrt{1300} = \sqrt{13 \times 100}$$

$$= 10\sqrt{13} \text{ km.}$$

16. Suppose that a satellite in space, an earth station and the centre of earth all lie in the same plane. Let r be the radius of earth and R be the distance from the centre of earth to the satellite. Let d be the distance from the earth station to the satellite. Let 30° be the angle of elevation from the earth station to the satellite. If the line segment connecting earth station and satellite subtends angle α at the centre of earth, then prove that $d = \sqrt{1+\left(\frac{r}{R}\right)^2 - 2\frac{r}{R}\cos\alpha}$.

Solution : Les S be the position of the satellite, E be the position of the earth station and C be the centre of the earth.

Given CE $= r$, CS = R and SE = d

Given $\angle SCE = \alpha$

In ΔSCE, applying cosine rule, we get

$$d^2 = r^2 + R^2 - 2(r)(R)\cos\alpha$$

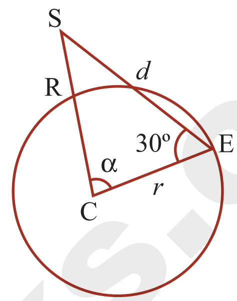
$$[\because c^2 = a^2 + b^2 - 2ab \cos c]$$

$$\Rightarrow \quad d^2 = r^2 + R^2 - 2r.\,R.\cos\alpha$$

Dividing by R^2 throughout we get,

$$\frac{d^2}{R^2} = \frac{r^2}{R^2} + \frac{\cancel{R}^2}{\cancel{R}^2} - \frac{2r.R}{R^2}\cos\alpha$$

$$\Rightarrow \quad \frac{d^2}{R^2} = \frac{r^2}{R^2} + 1 - \frac{2r}{R}\cos\alpha$$



$$\Rightarrow \quad d^2 = R^2\left[1 + \frac{r^2}{R^2} - \frac{2r}{R}\cos\alpha\right]$$

Taking positive square root both sides we get,

$$d = R\sqrt{1 + \frac{r^2}{R^2} - \frac{2r}{R}\cos\alpha}$$

$$\Rightarrow \quad d = R\sqrt{1 + \left(\frac{r}{R}\right)^2 - 2\left(\frac{r}{R}\right)\cos\alpha}$$

Hence proved.

Exercise 3.11

1. Find the principal value of

(i) $\sin^{-1}\frac{1}{\sqrt{2}}$ **(ii)** $\cos^{-1}\frac{\sqrt{3}}{2}$

(iii) $\text{cosec}^{-1}(-1)$ **(iv)** $\sec^{-1}(-\sqrt{2})$

(v) $\tan^{-1}(\sqrt{3})$

Solution :

(i) $\sin^{-1}\frac{1}{\sqrt{2}}$

$$\text{Let } \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = y, \text{ where } -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

$$\Rightarrow \quad \sin y = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin y = \sin\frac{\pi}{4} \Rightarrow y = \frac{\pi}{4}$$

Thus the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

04 Combinatorics and Mathematical Induction

MUST KNOW DEFINITIONS

Factorial : The continued product of first 'n' natural numbers is called the "n factorial" and is denoted by $n!$ or $\lfloor n$

Fundamental principle of multiplication : If there are two jobs such that one of them can be completed in m ways, second job can be completed in n ways, then the two jobs together can be completed in mn ways.

Fundamental principle of addition : If there are two jobs such that they can be performed independently in m and n ways, then either of the two jobs can be performed in $(m + n)$ ways.

Permutations : Each of the arrangements which can be made by taking some or all of a number of things is called a permutation.

Combinations : Each of the different selections made by taking some or all of a number of objects, irrespective of their arrangements is called a combination.

Mathematical statements : Statements involving mathematical relations are known as the mathematical statements.

Mathematical Induction : Proving mathematical statements using first principle of induction.

Formulae to remember :

1. $n! = n(n-1)(n-2)\ldots 3,2,1$
2. $n! = n(n-1)!$
3. $0! = 1$ and $1! = 1$
4. $p(n, r) = nP_r = n(n-1)(n-2)\ldots(n-(r-1))!$
5. $nP_r = P(n, r) = \dfrac{n!}{(n-r)!}$
6. The number of permutations of n distinct things, taken all at a time is $n!$

20. Find the sum of all 4-digit numbers that can be formed using digits 0, 2, 5, 7, 8 without repetition?

Solution :

tho	hun	tens	unit
4	4	3	2

The gives digits are 0, 2, 5, 7, 8

Number of 4 digit numbers that can be formed is $4 \times 4 \times 3 \times 2 = 96$.

Out of 96, there will be 24 numbers ending with 0

18 numbers ending with 2

18 numbers ending with 5

18 numbers ending with 7

18 numbers ending with 8

$\therefore$ Total for unit place is $(24 \times 0) + (18 \times 2) + (18 \times 5) + (18 \times 7) + (18 \times 8)$

$= 18(2+5+7+8) = 18 \times 22 = 396$

$\therefore$ Sum of all the 4 digit numbers $= 396 + 3960 + 39600 + (24 \times 22) \times 1000 = 571956$

Exercise 4.3

1. If ${}^nC_{12} = {}^nC_9$ find ${}^{21}C_n$

Solution : We have ${}^nCx = {}^nCy \Rightarrow x = y$ or $x + y = n$

$\Rightarrow 12 + 9 = n$

$\Rightarrow n = 21$

$\Rightarrow {}^{21}C_n = {}^{21}C_{21} = 1$ $[\because nC_n = 1]$

2. If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$, find r. **[Qy - 2019]**

Solution : Given ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$

Since ${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$

We get $2r - 1 = 2r + 4$

$\Rightarrow -1 \neq 4$ which is not possible (or)

$$2r - 1 + 2r + 4 = 15$$
$$4r + 3 = 15$$
$$4r = 12$$
$$\Rightarrow r = \frac{\cancel{12}^{3}}{\cancel{4}} = 3$$
$$\therefore r = 3$$

3. If ${}^nP_r = 720$. If ${}^nC_r = 120$, find n, r . **[Hy - 2019]**

Solution : Given ${}^nP_r = 720$ and ${}^nC_r = 120$

$$\Rightarrow \frac{n!}{(n-r)!} = 720 \quad ...(1)$$
$$\frac{n!}{r!(n-r)!} = 120 \quad ...(2)$$

$$\Rightarrow \frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{\cancel{720}^{60}}{\cancel{120}}$$

[Dividing (1) by (2)]

$$\Rightarrow \frac{\cancel{n!}}{\cancel{(n-r)!}} \times \frac{r!\cancel{(n-r)!}}{\cancel{n!}} = 6$$
$$\Rightarrow r! = 6$$
$$\Rightarrow r! = 3 \times 2 \times 1 = 3!$$
$$\Rightarrow r = 3.$$

Substituting $r = 3$ in (1) we get,

$$\frac{n!}{(n-r)!} = 720$$
$$\Rightarrow \frac{n!}{(n-3)!} = 720$$
$$\Rightarrow \frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 720$$
$$\Rightarrow n(n-1)(n-2) = 720$$
$$\Rightarrow n(n-1)(n-2) = 10 \times 9 \times 8$$
$$\Rightarrow n = 10.$$

4. Prove that ${}^{15}C_3 + 2 \times {}^{15}C_4 + {}^{15}C_5 = {}^{17}C_5$

Solution :

$$\text{LHS} = \frac{15\times14\times13}{1\times2\times3} + 2\times\frac{15\times14\times13\times12}{1\times2\times3\times4} + \frac{15\times14\times13\times12\times11}{1\times2\times3\times4\times5}$$
$$= \frac{15\times14\times13}{1\times2\times3}\left[1 + \frac{24}{4} + \frac{132}{20}\right]$$
$$= \frac{15\times14\times13}{1\times2\times3}\left[\frac{20+120+132}{20}\right]$$
$$= \frac{15\times14\times13\times272}{1\times2\times3\times4\times5} = \frac{17\times16\times15\times14\times13}{1\times2\times3\times4\times5}$$

$= {}^{17}C_5 = \text{RHS}$ Hence proved.

5. Prove that ${}^{35}C_5 + \sum_{r=0}^{4} {}^{(39-r)}C_4 = {}^{40}C_5$

Solution : $\text{LHS} = {}^{35}C_5 + \sum_{r=0}^{4} {}^{(39-r)}C_4$

$$= {}^{35}C_5 + {}^{39}C_4 + {}^{38}C_4 + {}^{37}C_4 + {}^{36}C_4 + {}^{35}C_4$$
$$= ({}^{35}C_5 + {}^{35}C_4) + {}^{39}C_4 + {}^{38}C_4 + {}^{37}C_4 + {}^{36}C_4$$
$$= {}^{36}C_5 + {}^{39}C_4 + {}^{38}C_4 + {}^{37}C_4 + {}^{36}C_4$$

$[\because nC_{r-1} + nC_r = n + 1C_r]$

$$= ({}^{36}C_5 + {}^{36}C_4) + {}^{39}C_4 + {}^{38}C_4 + {}^{37}C_4$$

21. The number of 10 digit number that can be written by using the digits 2 and 3 is

(1) ${}^{10}C_2 + {}^{9}C_2$ (2) 2^{10}

(3) $2^{10} - 2$ (4) $10!$

Hint : Number of 10 digit number that can be written by using the digits 2 and 3 is 2^{10}

[Ans : (2) 2^{10}]

22. If P_r stands for ${}^rP_r$ then the sum of the series $1 + P_1 + 2P_2 + 3P_3 + ... + nPn$ is

(1) P_{n+1} (2) $P_{n+1} - 1$

(3) $P_{n-1} + 1$ (4) ${}^{(n+1)}P_{(n-1)}$

Hint : $1+1\underline{|1}+2\underline{|2}+3\underline{|3}....+n\underline{|n}=\underline{|n+1}$

$\Rightarrow$ Proof : Let $n = 1$,

LHS = 1 + 1 = 2; RHS = $\underline{|2} = 2$

It is true for $n = 1$, in fact it is true for $n = 0$ also let us assume that it is true for $n = k$

$1+1\underline{|1}+2\underline{|2}+3\underline{|3}....+n\underline{|n}=\underline{|k+1}$

$\Rightarrow$ Then $1+1\underline{|1}+2\underline{|2}+3\underline{|3}+....+k\underline{|k}+(k+1)\underline{|k+1}$

$\underline{|k+1} + (k+1)\underline{|k+1} = \underline{|k+1}\,[1+k+1]$

$\Rightarrow$ $\underline{|k+1}\,(k+2) = \underline{|k+2}$

It is true for $(k + 1)$ also by mathematical induction, it is true for all values of $n \geq 0$, $n \in \mathbb{Z}$. **[Ans : (2)$P_{n+1} - 1$]**

23. The product of first n odd natural numbers equals **[Qy - 2018]**

(1) ${}^{2n}C_n \times {}^{n}P_n$ (2) $\left(\frac{1}{2}\right)^n {}^{2n}C_n \times {}^{n}P_n$

(3) $\left(\frac{1}{4}\right)^n \times {}^{2n}C_n \times {}^{2n}P_n$ (4) ${}^{n}C_n \times {}^{n}P_n$

Hint : $1 . 3 . 5 ...(2n-1) = \dfrac{1\cdot 2\cdot 3\cdot 4...(2n-1)(2n)}{2\cdot 4...(2n)}$

$$\Rightarrow \frac{\underline{|2n}}{2^n\underline{|n}} = \left(\frac{1}{2}\right)^n . {}^{2n}C_n \times {}^{n}P_n$$

[Ans : (2) $\left(\frac{1}{2}\right)^n {}^{2n}C_n \times {}^{n}P_n$]

24. If ${}^nC_4$, ${}^nC_5$, ${}^nC_6$ are in AP the value of n can be **[Govt. MQP - 2018]**

(1) 14 (2) 11 (3) 9 (4) 5

Hint : Given ${}^nC_4$, ${}^nC_5$, ${}^nC_6$ are in AP

$\Rightarrow$ $2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$$\Rightarrow \frac{2\underline{|n}}{\underline{|n-5}\underline{|5}} = \frac{\underline{|n}}{\underline{|n-4}\underline{|4}} + \frac{\underline{|n}}{\underline{|n-6}\underline{|6}}$$

$$\Rightarrow \frac{2}{\underline{|n-5}\underline{|5}} = \frac{1}{\underline{|n-4}.\underline{|4}} + \frac{1}{\underline{|n-6}\underline{|6}}$$

$$\Rightarrow \frac{2(n-4)6}{(n-4)\underline{|n-5}\underline{|5}.\,6} = \frac{5.6}{\underline{|n-4}.5.6\underline{|4}} + \frac{(n-4)(n-5)}{\underline{|6}(n-4)(n-5)\underline{|n-6}}$$

$$\Rightarrow \frac{12(n-4)}{\underline{|n-4}\underline{|6}} = \frac{30}{\underline{|n-4}\underline{|6}} + \frac{(n-4)(n-5)}{\underline{|n-4}\underline{|6}}$$

$\Rightarrow$ $12n \cdot 48 = 30 + n^2 - 9n + 20$

$\Rightarrow$ $n^2 - 21n + 98 = 0 \Rightarrow (n - 14)(n - 7) = 0$

$\Rightarrow$ $n = 14$ (or) $n = 7$. **[Ans : (1) 14]**

25. 1 + 3 + 5 + 7 +.....+ 17 is equal to

(1) 101 (2) 81 (3) 71 (4) 61

Hint : 1 + 3 + 5 + 7 + ... + 17 is equal to $9^2 = 81$

[Ans : (2) 81]

GOVERNMENT EXAM QUESTIONS

SECTION - A (1 MARK)

CHOOSE THE CORRECT OR THE MOST SUITABLE ANSWER.

1. The number of diagonals of octogen will be **[Qy - 2019]**

(1) 28 (2) 20 (3) 10 (4) 16

Hint : Number of diagonals = $8C_2 - 8 = \dfrac{8\times 7}{1\times 2}$

$= 28 - 8 = 20$ **[Ans : (3) 20]**

2. The number of words which can be formed from the letters of the word "MAXIMUM", if two consenents cannot occur together is **[Qy - 2019]**

(1) 4! (2) 3!× 4!(3) 7! (4) 5!

Hint : In word MAXIMUM, there are 4 consonants (M, X, M, M) and three vowels (A, I, U)

.A.I.U. The dotted places to be filled by MXMM.

Hence required number of ways $3! \times \dfrac{4!}{3!} = 4!$

[Ans : (1) 4!]

3. Value of $\dfrac{7!}{2!}$ is **[Qy - 2018]**

(1) 2520 (2) 2250 (3) 2205 (4) 2052

Hint : $\dfrac{7!}{2!} = \dfrac{1\times \cancel{2}\times 3\times 4\times 5\times 6\times 7}{1\times \cancel{2}}$

$= 3\times 4\times 5\times 6\times 7 = 2520$ **[Ans : (1) 2520]**

05 BINOMIAL THEOREM, SEQUENCES AND SERIES

MUST KNOW DEFINITIONS

Binomial theorem for positive integral index	:	If x and a are real numbers, then for all $n \in \mathbb{N}$, $(x+a)^n = nC_0x^na^0 + nC_1x^{n-1}a^1 + nC_2x^{n-2}a^2 + \ldots + nC_rx^{n-r}a^r + \ldots + a^n$.
Sequence	:	A sequence is a function whose domain is the set N of natural numbers.
Series	:	If $a_1, a_2, \ldots a_n$ is a sequence, then the expression $a_1 + a_2 + a_3 + \ldots + a_n$ is a series.
A.P	:	A sequence is called an arithmetic progression (A.P) if the difference of a term and the previous term is always same.
G.P	:	A sequence of non-Zero numbers is called a Geometric progression (G.P) if the ratio of a term and the term proceeding to it is always a constant.
H. P:	:	The reciprocals of the terms of an, A.P form a H.P.
Arithmetico – geometric progression (AGP)	:	An AGP is a progression in which each term can be represented as the product of the terms of an AP and a G.P.

$$= \frac{2a+(2k-2)d}{2}$$

$$= a+(k-1)d = ak.$$

Therefore, a_k is the arithmetic mean of a_{k-1} and a_{k+1}.

7. Prove that $\sqrt{x^2+25}-\sqrt{x^2+9} = \frac{8}{x}$ nearly when x is large. **[Hy - 2019]**

Solution : $\sqrt{x^2+25}-\sqrt{x^2+9}$

$$= \sqrt{x^2\left(1+\frac{25}{x^2}\right)}-\sqrt{x^2\left(1+\frac{9}{x^2}\right)}$$

$$= x\left(1+\frac{25}{x^2}\right)^{\frac{1}{2}} - x\left(1+\frac{9}{x^2}\right)^{\frac{1}{2}}$$

$$= x\left(1+\frac{1}{2}\cdot\frac{25}{x^2}+\ldots\right)-x\left(1+\frac{1}{2}\cdot\frac{9}{x^2}+\ldots\right)$$

$$= \frac{8}{x}+\ldots..,$$

ADDITIONAL PROBLEMS

SECTION - A (1 MARK)

CHOOSE THE CORRECT OR THE MOST SUITABLE ANSWER.

1. The first and last term of an A. P. are 1 and 11. If the sum of its terms is 36, then the number of terms will be

(1) 5 (2) 6 (3) 7 (4) 8

Hint : $\frac{n}{2}(a+l) = 36 \Rightarrow \frac{n}{2}(1+11) = 36$

$\Rightarrow \quad n = 6$ **[Ans :** (2) 6**]**

2. Choose the incorrect pair :

1.	$\frac{d}{dx}(\sin x)$	$\cos x$
2.	$\frac{d}{dx}(\tan x)$	$\sec^2 x$
3.	$\frac{d}{dx}(\cos x)$	$\sin x$
4.	$\frac{d}{dx}\log x$	$\frac{1}{x}$

Hint : $\frac{d}{dx}(\cos x) = -\sin x$ **[Ans :** (3) $\frac{d}{dx}(\cos x) = \sin x$**]**

3. Match List - I with List II.

	List I		List II
i.	Sum to n terms of an A.P	(a)	$a\left(\frac{r^n-1}{r-1}\right)$
ii.	Sum to n terms of a G.P	(b)	$\frac{n}{2}(a+l)$
iii.	Sum of an n infinite G.P	(c)	$\frac{1}{\frac{n}{2}(a+(n-1)d)}$
iv.	Sum to n terms of a H.P	(d)	$\frac{a}{1-r}$

The Correct match is

	(i)	(ii)	(iii)	(iv)
(1)	b	a	d	c
(2)	c	d	b	a
(3)	d	c	a	b
(4)	c	a	d	b

[Ans : (1) i – b ii – a iii – d iv – c**]**

4. Match List - I with List II.

	List I		List II
i.	$1+2+3+\ldots+n$	(a)	$\frac{n(n+1)(2n+1)}{6}$
ii.	$1^2+2^2+3^2+\ldots+n^2$	(b)	$\left(\frac{n(n+1)}{2}\right)^2$
iii.	$1^3+2^3+3^3+\ldots+n^3$	(c)	$\frac{n}{2}(2a+(n-1)d)$
iv.	$a+(a+d)+a+2d+\ldots+ a+(n-1)a$	(d)	$\frac{n(n+1)}{2}$

The Correct match is

	(i)	(ii)	(iii)	(iv)
(1)	d	a	b	c
(2)	c	d	b	a
(3)	b	c	d	a
(4)	d	c	b	a

[Ans : (1) i – d ii – a iii – b iv – c**]**

06 Two dimensional Analytical Geometry

MUST KNOW DEFINITIONS

Locus	:	The curve described by a point which moves under given conditions or condition is called its locus.
Straight line	:	A straight line is a curve such that every point on the line segment joining any two points on it lies on it.
Slope or Gradient	:	The trigonometrical tangent of the angle that a line makes with the positive direction of the X – axis in the anti clock wise direction.

FORMULAE TO REMEMBER

* Slope (m)
  1. $m = \tan\theta$
  2. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
  3. $m = -\dfrac{\text{Co-efficient of } x}{\text{Co-efficient of } y}$
* **Angle between two lines:**

  $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$ where m_1, m_2 are slopes of the lines
* Condition for parallel lines is $m_1 = m_2$
* Condition for perpendicular lines is $m_1 m_2 = -1$
* **Equation of straight lines:**
  1. Equation of X-axis is $y = 0$ and equation of any line parallel to X-axis is $y = k$.
  2. Equation of Y-axis is $x = 0$ and equation of any line parallel to y-axis is $x = k$, where K is the distance between the line and the Y-axis
  3. **Slope point form**: $y - y_1 = m(x - x_1)$
  4. **Slope – intercept form:** $y = mx + C$.
  5. **Two points form:** $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$
  6. **Intercepts form:** $\dfrac{x}{a} + \dfrac{y}{b} = 1$
  7. **Normal form:** $x\cos\alpha + y\sin\alpha = p$
  8. **Symmetric form/parametric form:**

     $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$
  9. **General form:** $ax + by + c = 0$
  10. Equation of pair of straight lines passing through the origin is $ax^2 + 2hxy + by^2 = 0$
  11. The straight lines are real and distinct if $h^2 > ab$
  12. The straight lines are coincident if $h^2 = ab$
  13. The straight lines are imaginary if $h^2 < ab$
  14. If $m_1 m_2$ are slopes of the pair of straight lines then $m_1 + m_2 = -\dfrac{2h}{b}$ and $m_1 m_2 = \dfrac{a}{b}$

(iii) the slopes of the escalator at the turning points.

Slope of the escalator at the turning points

Let $\lfloor AOE = \theta$

In ΔOAE, $\tan\theta = \frac{opp}{adj} = \frac{AE}{DE} = \frac{\cancel{180}^{9}}{\cancel{800}_{40}} = \frac{9}{40}$

$\therefore$ Slope at the point A $= \frac{9}{40}$

Since $\Delta OAE \equiv \Delta ABB' \equiv \Delta BCC' \equiv \Delta CDD'$

Slope at the points B, C will be $\frac{9}{40}$

EXERCISE 6.3

1. Show that the lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$ are parallel lines.

Solution : If the equation of two lines are in general form as $a_1 x + b_1 y + c = 0$ and $a_2 x + b_2 y + c_2 = 0$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \text{ or } a_1 b_2 = a_2 b_1$$

Given lines are $3x + 2y + 9 = 0$ and $12x + 8y - 15 = 0$

$$\frac{3}{12} = \frac{2}{8} \Rightarrow \frac{1}{4} = \frac{1}{4}$$

Hence the given lines are parallel.

2. Find the equation of the straight line parallel to $5x - 4y + 3 = 0$ and having x-intercept 3.

Solution : Since x - intercept is 3, A (3,0) will be a point on the required line.

Any line parallel to $5x - 4y + 3 = 0$ will be of the form $5x - 4y + k = 0$

Substituting the point (3, 0) we get

$+15 - 0 + k = 0 \qquad \Rightarrow k = -15$

$\therefore$ Required equation of the line is $5x - 4y - 15 = 0$

3. Find the distance between the line $4x + 3y + 4 = 0$ and a point (i) (–2, 4) (ii) (7,–3)

Solution : **(i) (–2,4)**

Distance from the point (x_1, y_1) to the line $ax + by + c = 0$ is $\pm \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$

Distance from the point (–2, 4) to the line $4x + 3y + 4 = 0$ is $\left|\frac{4(-2) + 3(4) + 4}{\sqrt{4^2 + 3^2}}\right|$

$$= \frac{(-8 + 12 + 4)}{\sqrt{25}} = \frac{8}{5} \text{ unit}$$

(ii) (7, –3)

Distance from the point (7, –3) to the line $4x + 3y + 4 = 0$ is

$$\pm \frac{4(7) + 3(-3) + 4}{\sqrt{4^2 + 3^2}} = \pm \frac{(28 - 9 + 4)}{\sqrt{25}} = \pm \left(\frac{23}{5}\right) = \frac{23}{5} \text{ units.}$$

4. Write the equation of the lines through the point (1,–1)

(i) parallel to $x + 3y - 4 = 0$ **[June - 2019]**

(ii) perpendicular to $3x + 4y = 6$

Solution : (i) Any line parallel to $x + 3y - 4 = 0$ will be of the form $x + 3y + k = 0$.

This line passes through (1, –1)

$\therefore 1 + 3(-1) + k = 0$

$\Rightarrow 1 - 3 + k = 0$

$\Rightarrow k - 2 = 0 \quad \Rightarrow k = 2$

$\therefore$ The required line is $x + 3y + 2 = 0$

(ii) Any line perpendicular to $3x + 4y - 6 = 0$ will be of the form $4x - 3y + k = 0$.

This line passes through (1, –1)

$\therefore 4(1) - 3(-1) + k = 0$

$\Rightarrow 4 + 3 + k = 0$

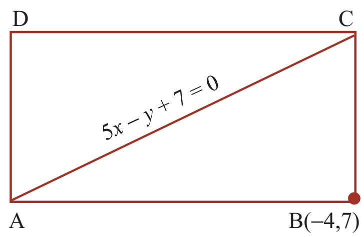
$\Rightarrow k = -7$

$\therefore$ The required line is $4x - 3y - 7 = 0$.

5. If (–4, 7) is one vertex of a rhombus and if the equation of one diagonal is $5x - y + 7 = 0$, then find the equation of another diagonal.

Solution : Let the vertex B is (–4, 7) and the equation of the diagonal AC is $5x - y + 7 = 0$

In rhombus, the diagonals are perpendicular to each other.



$\therefore$ Equation of the diagonal BD is $x + 5y + k = 0$ which is $\perp^r$ to AC.

Since BD passes through the point B (–4, 7) we get

$-4 + 5(7) + k = 0$

$\Rightarrow -4 + 35 + k = 0 \Rightarrow 31 + k = 0$

$\Rightarrow k = -31$

$\therefore$ Equation of the all another diagonal is $x + 5y - 31 = 0$.

6. Find the equation of the lines passing through the point of intersection lines $4x - y + 3 = 0$ and $5x + 2y + 7 = 0$ (i) through the point (–1, 2) (ii) Parallel to $x - y + 5 = 0$ (iii) Perpendicular to $x - 2y + 1 = 0$.

Solution : The family of equations of straight lines is of the form $(a_1 x + b_1 y + c_1) + \lambda (a_2 x + b_2 y + c_2) = 0$.

Volume - II

MATHEMATICS

11th Standard

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07 MATRICES AND DETERMINANTS

MUST KNOW DEFINITIONS

Matrix	:	A matrix is a rectangular array or arrangement of entries or elements displayed in rows and columns put within a square bracket [ ].
Order of Matrix	:	If a matrix A has m rows and n columns then the order or size of the matrix A is defined to be $m \times n$.
Column Matrix	:	A matrix having only one column is called a column matrix.
Row matrix	:	A matrix having only one row is called a row matrix.
Square matrix	:	A matrix in which number of rows is equal to the number of columns, is called a square matrix.
Diagonal matrix	:	A square matrix $A = [a_{ij}]_{m\times n}$ is called a diagonal matrix. If $a_{ij} = 0$ whenever $i \neq j$
Scalar matrix	:	A diagonal matrix whose entries along the principal diagonal are equal is called a scalar matrix.
Unit matrix	:	A square matrix in which all the diagonal entries are 1 and the rest are all zero is called a unit matrix.
Triangular matrix	:	A square matrix which is either upper triangular or lower triangular is called a triangular matrix.
Singular and Non - Singular Matrix	:	A square matrix A is said to be singular if $\|A\| = 0$. A square matrix A is said to be non-singular if $\|A\| \neq 0$.

Properties of Determinants :

1. The value of the determinant remains unchanged if its rows and columns are interchanged.
2. If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.
3. If any two rows (or columns) of a determinant are identical, then the value of the determinant is zero.
4. If each element of a row (or column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.
5. If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.
6. The value of the determinant remain same if we apply the operation. $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$

Minor of an element

- The concept of determinant can be extended to the case of square matrix or order n, $n \geq 4$. Let $A = [a_{ij}]_{m \times n}$, $n \geq 4$.
- If we delete the i^{th} row and j^{th} column from the matrix of $A = [a_{ij}]_{n \times m}$, we obtain a determinant of order $(n - 1)$, which is called the minor of the element a_{ij}.

Adjoint

- Adjoint of a square matrix $A = [a_{ij}]_{n \times n}$ is defined as the transpose of the matrix $[A_{ij}]_{n \times n}$ where A_{ij} is the co-factor of the element a_{ij}.

Solving linear equations by Gaussian Elimination method

- Gaussian elimination is a method of solving a linear system by bringing the augmented matrix to an upper triangular form.

FORMULAE TO REMEMBER

- $kA = [a_{ij}]_{m \times n}$ $[ka_{ij}]_{m \times n}$ where k is a scalar.
- $-A = (-1)A$, $A - B = A + (-1)B$
- $A + B = B + A$, (Commutative property for addition)
- $(A + B) + C = A + (B + C)$, (Associative property for addition)
- $k(A + B) = kA + kB$ where A, B are of same order, k is a constant.
- $(k + l)\, A = kA + lA$ where k and l are constants.
- $A\,(BC) = (AB)\,C$, $A(B + C) = AB + AC$, $(A + B)\,C = AC + BC$. (Distributive law)
- If $A = (a_{ij})_{m \times n}$, then $A^T = (a_{ji})_{n \times m}$
- Elementary operations of a matrix are as follows
  (i) $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$ (ii) $R_i \to kR_i$ or $C_i \to kC_j$ (iii) $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$
- Evaluation of determinant $A = [a_{11}]_{1 \times 1} = |A| = a_{11}$
- Evaluation of determinant $A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$
- Evaluation of determinant $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is $|A| = a_1 \begin{vmatrix} b_2 & c_3 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$
- If $A = [a_{ij}]_{3 \times 3}$, then $|k \cdot A| = k^3\,|A|$.
- $A\,(adj\,A) = (adj\,A)\,A = |A|.\,I$ where A is a square matrix of order n.
- A square matrix A is said to be singular or non-singular according as $|A| = 0$ or $|A| \neq 0$.
- **Transpose of a matrix:** $(A^T)^T = A$, $(kA)^T = kA^T$. $(A + B)^T = A^T + B^T$, $(AB)^T = B^T A^T$.
- Co-factor of a_{ij} of $A_{ij} = (-1)^{i+j}\, m_{ij}$ where m_{ij} is the minor of a_{ij}.
- $|AB| = |A| \cdot |B|$ where A and B are square matrices of same order.

EXERCISE 7.5

CHOOSE THE CORRECT OR THE MOST SUITABLE ANSWER FROM THE GIVEN FOUR ALTERNATIVES.

1. If $a_{ij} = \frac{1}{2}(3i - 2j)$ and $A = [a_{ij}]_{2 \times 2}$ is

(1) $\begin{bmatrix} \frac{1}{2} & 2 \\ -\frac{1}{2} & 1 \end{bmatrix}$ (2) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 2 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 2 & 2 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ (4) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ 1 & 2 \end{bmatrix}$

Hint : $a_{ij} = \frac{1}{2}(3i - 2j)$

$a_{11} = \frac{1}{2}(3-2) = \frac{1}{2}, \; a_{12} = \frac{1}{2}(3-4) = -\frac{1}{2}$

$a_{21} = \frac{1}{2}(6-2) = 2, \; a_{22} = \frac{1}{2}(6-4) = \frac{2}{2} = 1$

$\therefore A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 2 & 1 \end{bmatrix}$ **[Ans: (2) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ 2 & 1 \end{bmatrix}$]**

2. What must be the matrix X, if $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$?

(1) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

(3) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$ (4) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$

Hint : $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$

$\Rightarrow 2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$

$X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

[Ans: (1) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$]

3. Which one of the following is true about the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}$?

(1) a scalar matrix
(2) a diagonal matrix
(3) an upper triangular matrix
(4) a lower triangular matrix

[Ans: (2) a diagonal matrix]

4. If A and B are two matrices such that A + B and AB are both defined, then

(1) A and B are two matrices not necessarily of same order
(2) A and B are square matrices of same order
(3) Number of columns of A is equal to the number of rows of B
(4) A = B.

Hint : For addition both A and B must be of same order to get AB, number of columns of A should be equal to number of rows of B.

If A and B ae square matrices of same order both condition are satisfied.

[Ans: (2) A and B are square matrices of same order]

5. If $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$, then for what value of λ^2, $A^2 = 0$? **[June - 2019]**

(1) 0 (2) ±1 (3) –1 (4) 1

Hint : $A = \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}$

$A^2 = 0$

$\Rightarrow \begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix}\begin{bmatrix} \lambda & 1 \\ -1 & -\lambda \end{bmatrix} = \begin{bmatrix} \lambda^2 - 1 & 0 \\ 0 & -1 + \lambda^2 \end{bmatrix} = 0$

$\Rightarrow \lambda^2 - 1 = 0$

$\Rightarrow \lambda^2 = 1$

$\lambda = \pm 1$ **[Ans: (2) ±1]**

6. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then the values of a and b are

(1) $a = 4, b = 1$ (2) $a = 1, b = 4$
(3) $a = 0, b = 4$ (4) $a = 2, b = 4$

08 VECTOR ALGEBRA-I

MUST KNOW DEFINITIONS

- **Scalar:** A Scalar is a quantity that is determined by its magnitude.
- **Vector:** A vector is a quantity that is determined by both its magnitude and its direction and hence it is a directed line segment.
- **Position Vector:** Let O be the origin and P be any point (in the plane or space) Then the vector $\overrightarrow{OP}$ is called the position vector.
- **Magnitude of a Vector:** Magnitude of $\overrightarrow{AB} = |\overrightarrow{AB}|$ is a positive number which is a measure of its length.
- The arrow indicates the direction of the vector.
- **Types of vectors:**

1. **Zero or null vector:** A vector whose initial and terminal points are coincident.
2. **Unit Vector:** A vector whose modulus is unity.
3. **Like and unlike vectors:** Like vectors have the same sense of direction and unlike vectors have opposite directions.
4. **Co-initial vectors:** Vectors having the same initial point.
5. **Co-terminus vectors:** Vectors having the same terminal point.
6. **Collinear or parallel vectors:** Vectors having the same line of action or have the lines of action parallel to one another.
7. **Co-planar vectors:** Vectors parallel to the same plane or they lie in the same plane.
8. **Negative vector:** Vector which has the same magnitude as that of $\vec{a}$ but opposite direction is called the negative of $\vec{a}$.
9. **Reciprocal of a vector:** vector which has the same direction as that of $\vec{a}$ but has magnitude reciprocal to that of $\vec{a}$

$$|(\vec{a})^{-1}| = \frac{1}{a}$$

10. **Free and localised vector:**

    When the origin of the vector is any point it is called as a **free vector**, but when it is restricted to a certain specific point it is said to be a **localised vector.**

$$\Rightarrow \quad 1 = \lambda^2 [1 - \cos^2 \frac{\pi}{3}] \qquad [\because |\vec{b}| = |\vec{c}| = 1]$$

$$\Rightarrow \quad 1 = \lambda^2 [1 - \frac{1}{4}] \Rightarrow 1 = \lambda^2 \left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \lambda^2 = \frac{4}{3} \Rightarrow \lambda = \pm\frac{2}{\sqrt{3}}$$

Substituting $\lambda = \pm\frac{2}{\sqrt{3}}$ in (1) we get,

$$\vec{a} = \pm\frac{2}{\sqrt{3}}(\vec{b} \times \vec{c})$$

10. Find the angle between the vectors $2\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ using vector product.

Solution : Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1+2) - \hat{j}(2+1) + \hat{k}(4-1)$$

$$= 3\hat{i} - 3\hat{j} + 3\hat{k} = 3(\hat{i} - \hat{j} + \hat{k})$$

$$|\vec{a} \times \vec{b}| = 3\sqrt{1^2 + 1^2 + (-1)^2} = 3\sqrt{3}$$

$$|\vec{a}| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}$$

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$$

$$= \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{3\sqrt{3}}{\sqrt{6}\sqrt{6}} = \frac{\cancel{3}\sqrt{3}}{\cancel{6}} = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

EXERCISE 8.5

CHOOSE THE CORRECT OR THE MOST SUITABLE ANSWER FROM THE GIVEN FOUR ALTERNATIVES.

1. The value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{DA} + \overrightarrow{CD}$ is

(1) $\overrightarrow{AD}$ (2) $\overrightarrow{CA}$ (3) $\vec{0}$ (4) $-\overrightarrow{AD}$

Hint : $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{DA} + \overrightarrow{CD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{AA} = \vec{0}$

[Ans : (3) $\vec{0}$]

2. If $\vec{a} + 2\vec{b}$ and $3\vec{a} + m\vec{b}$ are parallel, then the value of m is

(1) 3 (2) $\frac{1}{3}$ (3) 6 (4) $\frac{1}{6}$

Hint :
$$\vec{a} + 2\vec{b} = 3(\vec{a} + 2\vec{b})$$
$$= 3\vec{a} + 6\vec{b} = 3\vec{a} + m\vec{b}$$
$$m = 6$$

[Ans: (3) 6]

3. The unit vector parallel to the resultant of the vectors $\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$ is **[March - 2019]**

(1) $\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{5}}$ (2) $\frac{2\hat{i} + \hat{j}}{\sqrt{5}}$

(3) $\frac{2\hat{i} - \hat{j} + \hat{k}}{\sqrt{5}}$ (4) $\frac{2\hat{i} - \hat{j}}{\sqrt{5}}$

Hint : Resultant vector of $\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} - 2\hat{j} + \hat{k}$ is $2\hat{i} - \hat{j}$

Its magnitude is $\sqrt{2^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$

∴ Required unit vector $= \frac{2\hat{i} - \hat{j}}{\sqrt{5}}$ **[Ans: (4) $\frac{2\hat{i} - \hat{j}}{\sqrt{5}}$]**

4. A vector $\overrightarrow{OP}$ makes 60° and 45° with the positive direction of the x and y axes respectively. Then the angle between $\overrightarrow{OP}$ and the z-axis is

(1) 45° (2) 60° (3) 90° (4) 30°

Hint : Given α = 60°, β = 45°

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\Rightarrow \cos^2 60 + \cos^2 45 + \cos^2\gamma = 1$$
$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\gamma = 1$$
$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1 \Rightarrow \frac{3}{4} + \cos^2\gamma = 1$$
$$\Rightarrow \cos^2\gamma = 1 - \frac{3}{4}$$
$$= \frac{1}{4} = \left(\frac{1}{2}\right)^2 = (\cos 60)^2$$
$$\Rightarrow \cos\gamma = \cos 60$$
$$\therefore \gamma = 60^\circ$$

[Ans: (2) 60°]

2. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = \sqrt{24}$ and sum of any two vectors is orthogonal to the third vector, then find $|\vec{a}+\vec{b}+\vec{c}|$.

Solution : Given $(\vec{a}+\vec{b}).\vec{c} = 0$

$$\Rightarrow \vec{a}.\vec{c}+\vec{b}.\vec{c} = 0$$
$$(\vec{b}+\vec{c}).\vec{a} = 0$$
$$\Rightarrow \vec{b}.\vec{a}+\vec{c}.\vec{a} = 0$$
$$(\vec{c}+\vec{a}).\vec{b} = 0$$
$$\Rightarrow \vec{c}.\vec{b}+\vec{a}.\vec{b} = 0$$
$$\text{Adding, } 2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$$
$$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = 0 \quad \ldots (1)$$
$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a})$$
$$= 9+16+24+2(0)$$
$$= 49$$
$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = 7$$

3. If $|\vec{a}| = |\vec{b}| = |\vec{a}+\vec{b}| = 1$ then prove that $|\vec{a}-\vec{b}| = \sqrt{3}$.

Solution : Given $|\vec{a}+\vec{b}| = 1$

$$|\vec{a}+\vec{b}|^2 = 1$$
$$|\vec{a}|^2+|\vec{b}|^2+2(\vec{a}.\vec{b}) = 1$$
$$1+1+2|\vec{a}||\vec{b}|\cos\theta = 1$$

where θ is the angle between $\vec{a}$ and $\vec{b}$.

$$2+2(1)(1)\cos\theta = 1$$
$$2\cos\theta = 1-2 = -1$$
$$\cos\theta = -\frac{1}{2}$$
$$\text{Consider } |\vec{a}-\vec{b}|^2 = |\vec{a}|^2+|\vec{b}|^2-2(\vec{a}.\vec{b})$$
$$= 1+1-2|\vec{a}||\vec{b}|\cos\theta$$
$$= 2-\cancel{2}(1)(1)\left(-\frac{1}{\cancel{2}}\right)$$
$$= 2+1 = 3$$
$$\therefore |\vec{a}-\vec{b}| = \sqrt{3}$$

✵ ✵ ✵ ✵ ✵

POINTS TO REMEMBER

In this chapter we have acquired the knowledge of the following :

- A scalar is a quantity that is determined by its magnitude.
- A vector is a quantity that is determined by both its magnitude and its direction
- If we have a liberty to choose the origins of the vector at any point then it is said to be a **free vector**, whereas if it is restricted to a certain specified point then the vector is said to be a **localized vector.**
- Two or more vectors are said to be **coplanar** if they lie on the same plane or parallel to the same plane.
- Two vectors are said to be equal if they have equal length and the same direction.
- A vector of magnitude 0 is called the **zero vector.**
- A vector of magnitude 1 is called a **unit vector**.
- Let a $\vec{a}$ be a vector and m be a scalar. Then the vector $m\vec{a}$ is called the scalar multiple of a vector $\vec{a}$ by the scalar m.
- Two vectors $\vec{a}$ and $\vec{b}$ are said to be parallel if $\vec{a} = \lambda\vec{b}$, where λ is a scalar.

21. At $x = \frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is

(1) continuous (2) discontinuous
(3) differentiable (4) non-zero

Hint : $f(x) = (1, \text{ if } x)\ 1\ (-1) \text{ if } x < 1$

[Ans: (2) discontinuous]

22. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x) = \begin{cases} x & x \text{ is irrational} \\ 1-x & x \text{ is rational} \end{cases}$ then f is [June - 2019]

(1) discontinuous at $x = \frac{1}{2}$
(2) continuous at $x = \frac{1}{2}$
(3) continuous everywhere
(4) discontinuous everywhere

[Ans: (2) continuous at $x = \frac{1}{2}$]

23. The function $f(x) = \begin{cases} \frac{x^2-1}{x^3+1} & x \neq -1 \\ P & x = -1 \end{cases}$ is not defined for $x = -1$.

The value of $f(-1)$ so that the function extended by this value is continuous is

(1) $\frac{2}{3}$ (2) $-\frac{2}{3}$ (3) 1 (4) 0

Hint : $\Rightarrow f(x) = \begin{cases} \frac{x^2-1}{x^3+1} & x \neq -1 \\ P & x = -1 \end{cases} \Rightarrow f(-1) = \frac{-2}{3}$

[Ans: (2) $-\frac{2}{3}$]

24. Let f be a continuous function on [2, 5]. If f takes only rational values for all x and $f(3) = 12$, then $f(4.5)$ is equal to

(1) $\frac{f(3)+f(4.5)}{7.5}$ (2) 12
(3) 17.5 (4) $\frac{f(4.5)-f(3)}{1.5}$

Hint : f is a constant function [Ans: (2) 12]

25. Let a function f be defined by $f(x) = \frac{x-|x|}{x}$ for $x \neq 0$ and $f(0) = 2$. Then f is

(1) continuous nowhere
(2) continuous everywhere
(3) continuous for all x except $x = 1$
(4) continuous for all x except at $x = 0$

Hint : $f(x) = \begin{cases} 0, x > 0 \\ 2, x \leq 0 \end{cases}$

[Ans: (4) continuous for all x except at $x = 0$]

GOVERNMENT EXAM QUESTIONS

SECTION - A (1 MARK)

1. Find $\lim\limits_{x\to 0} \frac{\sin 2x}{x}$. [Hy - 2019]

(1) 2 (2) 1/2 (3) 1 (4) 0

Hint : $\lim\limits_{x\to 0} \frac{\sin 2x}{x} = \lim\limits_{2x\to 0} \frac{2\sin 2x}{2x}$

$= 2 \lim\limits_{2x\to 0} \frac{\sin 2x}{2x} = 2 \quad \left[\lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right]$

[Ans: (1) 2]

SECTION - B (2 MARKS)

1. Evaluate : $\lim\limits_{x\to 0} \frac{e^{5x}-1}{x}$. [Hy - 2018]

Solution : $\lim\limits_{5x\to 0} \frac{e^{5x}-1}{5x} \times 5 = 5\,(1)$

2. Define a continuous function on the closed interval $[a, b]$. [March - 2019]

Solution : A function $f: [a, b] \to R$ is said to be continuous on the closed interval $[a, b]$ if it is continuous on the open interval (a, b) and

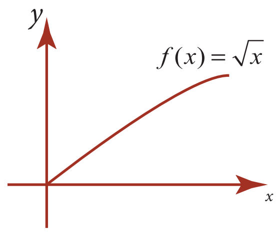
$\lim\limits_{x\to a^+} f(x) = f(a)$ and $\lim\limits_{x\to b^-} f(x) = f(b)$

3. Consider the function $f(x) = \sqrt{x}, x \geq 0$. Does $\lim\limits_{x\to 0} f(x)$ exist? [March - 2019]

Solution : $\lim\limits_{x\to 0^+} \sqrt{x} = 0$

$\lim\limits_{x\to 0^-} \sqrt{x}$ does not exist

$\therefore \lim\limits_{x\to 0} f(x)$ does not exists



4. Show that $\lim\limits_{x\to\infty} \left(\frac{x+2}{x-2}\right)^x = e^4$. [Hy - 2019]

Solution : $\lim\limits_{x\to\infty} \left(\frac{x+2}{x-2}\right)^x = \lim\limits_{x\to\infty} \left(\frac{x-2+4}{x-2}\right)^{x-2+2}$

$= \lim\limits_{x\to\infty} \left(1 + \frac{4}{x-2}\right)^{(x-2)+2}$

Let $y = x-2$, as $x \to \infty, y \to \infty$ and (Let $y = x-2$, Then as $x \to \infty, y \to \infty$)

$\lim\limits_{x\to\infty} \left(\frac{x+2}{x-2}\right)^x = \lim\limits_{y\to\infty} \left(1+\frac{4}{y}\right)^{y+2}$

$= \lim\limits_{y\to\infty} \left(1+\frac{4}{y}\right)^y . \lim\limits_{y\to\infty} \left(1+\frac{4}{y}\right)^2$

$= e^4.1 = e^4$

10 Differential Calculus - Differentiability and Methods of Differentiation

MUST KNOW DEFINITIONS

- **Slope know definitions :** The slope of the tangent line $(x_0, f(x_0))$ is also called the slope of the curve.
- **Position functions :** Suppose an object moves along a straight line according to an equation of motion $s = f(t)$ where s is the displacement (directed distance) of the object from the origin at time t. The function f that describes the motion is called the position function.
- **Differentiation :** The process of finding the derivative of a function is called differentiation.
- **Leibnitz symbol :** The notation $\frac{dy}{dx}$ is read as "derivative of y with respect to x" or simply "dy-dx", or we should rather read it as "Dee y Dee x" or "Dee Dee x of y". But it is cautioned that we should not regard $\frac{dy}{dx}$ as the quotient $dy \div dx$ and should not refer it as "dy by dx". The symbol $\frac{dy}{dx}$ is know as Leibnitz symbol.
- **Derivatives from first Principle :** The process of finding the derivative of a function using the conditions stated in the definition of derivatives is known as derivatives from first principle.
- **Intermediate Argument :** We have to take the derivative of the outer function f regarding the argument $g(x) = u$, and multiply the derivative of the inner function $g(x)$ with respect to the independent variable x. The variable u is known as intermediate argument.
- **Logarithmic differentiation:** The operation consists of first taking the logarithm of the function $f(x)$ (to base e) then differentiating is called logarithmic differentiation.
- **Parameter :** If two variables x and y are defined separately as a function of an intermediating (auxiliary) variable t, then the specification of a functional relationship between x and y is described as parametric and the auxiliary variable is known as parameter.

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28. If $y = (\cos^{-1}x)^2$ prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$ [June - 2019]

Solution : Given $y = (\cos^{-1}x)^2$

Differentiating with respect to 'x' we get,

$$y' = 2.\ \cos^{-1}x.\ \frac{d}{dx}(\cos^{-1}x)$$

$$\Rightarrow \quad y' = 2.\ \cos^{-1}x\ \frac{-1}{\sqrt{1-x^2}} \quad ...(1)$$

$$\Rightarrow \quad y'\sqrt{1-x^2} = -2\cos^{-1}x$$

Squaring both sides we get, $(y')^2(1-x^2) = 4(\cos^{-1}x)^2$

Differentiating again with respect to 'x' we get,

$$(y')^2(-2x) + (1-x^2)\,2y'.y'' = 4(2)\cos^{-1}x.\ \frac{d}{dx}(\cos^{-1}x)$$

$$\Rightarrow -2x(y')^2 + 2(1-x^2)y'.y'' = 8\cos^{-1}x.\ \frac{-1}{\sqrt{1-x^2}}$$

$$\Rightarrow -2x(y')^2 + 2(1-x^2)y'.y'' = 4\left(\frac{-2\cos^{-1}x}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow -2x(y')^2 + 2(1-x^2)y'.y'' = 4y' \quad \text{[From (1)]}$$

Dividing throughout by $2y'$ we get, $-xy' + (1-x^2)y'' = 2$

$$\Rightarrow \quad (1-x^2)y'' - xy' - 2 = 0$$

$$\Rightarrow \text{ i.e., } (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$ Hence proved.

Exercise 10.5

Choose the correct or the most suitable answer from the given four alternatives.

1. $\frac{d}{dx}\left(\frac{2}{\pi}\sin x^o\right)$ is

(1) $\frac{\pi}{180}\cos x^o$ (2) $\frac{1}{90}\cos x^o$

(3) $\frac{\pi}{90}\cos x^o$ (4) $\frac{2}{\pi}\cos x^o$

Hint : $\frac{d}{dx}\left(\frac{2}{\pi}\sin x\right) = \frac{2}{\pi}\cos x^o = \frac{2}{180}\cos x^o = \frac{1}{90}\cos x^o$

[Ans : (2) $\frac{1}{90}\cos x^o$]

2. If $y = f(x^2+2)$ and $f'(3) = 5$, then $\frac{dy}{dx}$ at $x = 1$ is

(1) 5 (2) 25 (3) 15 (4) 10

Hint :

$$y = f(x^2+2)$$

$$\Rightarrow \quad y' = f'(x^2+2)(2x)$$

$$\Rightarrow \quad f'(x^2+2) = \frac{y'}{2x} \quad ...(1)$$

Given $f'(3) = 5$ ... (2)

$\therefore$ Comparing (1) and (2),

$$x^2 + 2 = 3$$

$$\Rightarrow \quad x^2 = 1 \Rightarrow x = \pm 1 \text{ and } \frac{y'}{2x} = 5$$

$$\Rightarrow \quad y' = 10x \Rightarrow y = 10(1) = 10$$

[Ans : (4) 10]

3. If $y = \frac{1}{4}u^4$ and $u = \frac{2}{3}x^3 + 5$, then $\frac{dy}{dx}$ is

(1) $\frac{1}{27}x^2(2x^3+15)^3$ (2) $\frac{2}{27}x(2x^3+5)^3$

(3) $\frac{2}{27}x^2(2x^3+15)^3$ (4) $\frac{2}{27}x(2x^3+5)^3$

Hint : Given $y = \frac{1}{4}u^4$ and $u = \frac{2}{3}x^3 + 5$ ... (1)

$$\Rightarrow \quad \frac{dy}{du} = \frac{1}{4}\times 4u^3 = u^3 \text{ and } \frac{du}{dx} = \frac{2}{3}\times 3x^2 = 2x^2$$

$$\therefore \frac{dy}{dx} = y' = \frac{dy}{du}\times\frac{du}{dx} = u^3\times 2x^2$$

$$= \left(\frac{2}{3}x^3+5\right)^3 2x^2 = \frac{(2x^3+15)^3}{3^3}\times 2x^2$$

$$= \frac{2}{27}x^2(2x^3+15)^3$$

[Ans : (3) $\frac{2}{27}x^2(2x^3+15)^3$]

4. If $f(x) = x^2 - 3x$, then the points at which $f(x) = f'(x)$ are [June - 2019]

(1) both positive integers
(2) both negative integers
(3) both irrational
(4) one rational and another irrational

Hint :

$$\text{Given } f(x) = x^2 - 3x$$

$$f'(x) = 2x - 3$$

$$f(x) = f'(x)$$

$$\Rightarrow \quad x^2 - 3x = 2x - 3$$

$$\Rightarrow \quad x^2 - 5x + 3 = 0 \Rightarrow x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{5\pm\sqrt{25-12}}{2} = \frac{5\pm\sqrt{13}}{2}$$

[Ans : (3) both irrational]

5. If $y = \frac{1}{a-z}$, then $\frac{dz}{dy}$ is

(1) $(a-z)^2$ (2) $-(z-a)^2$

(3) $(z+a)^2$ (4) $-(z+a)^2$

11 Integral Calculus

MUST KNOW DEFINITIONS

Anti-derivative function : A function F(x) is called an antiderivative (Newton - Leibnitz integral or primitive) of a function $f(x)$ on an interval I if F(x) = $f(x)$, for every value of x in I.

Integration : The process of finding the anti-derivative of a given function $f(x)$ is called **integration**.

FORMULAE TO REMEMBER

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
- $\int \frac{1}{x} dx = \log|x| + c$
- $\int \sin x \, dx = -\cos x + c$
- $\int \cos x \, dx = \sin x + c$
- $\int \tan x \, dx = -\log|\cos x| + c = \log|\sec x| + c$
- $\int \cot x \, dx = \log|\sin x| + c = -\log|\operatorname{cosec} x| + c$.
- $\int \sec x \, dx = \log|\sec x + \tan x| + c$
- $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c$

(iii) $\dfrac{1}{\sqrt{9+8x-x^2}}$

Let $I = \int \dfrac{dx}{\sqrt{9+8x-x^2}} = \int \dfrac{dx}{\sqrt{-(x^2-8x-9)}}$

$= \int \dfrac{dx}{\sqrt{-(x^2-8x+16-16-9)}} = \int \dfrac{dx}{\sqrt{-[(x-4)^2-25]}}$

$\left[\begin{array}{l}\text{Adding and subtracting} \\ \left[\frac{1}{2}(\text{Co-efficient of } x)\right]^2 \\ = \left[\frac{1}{2}(-8)\right]^2 = (-4)^2 = 16\end{array}\right]$

$= \int \dfrac{dx}{\sqrt{25-(x-4)^2}} = \int \dfrac{dx}{\sqrt{5^2-(x-4)^2}}$

since $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}(x/a)$ we get,

$I = \sin^{-1}\left(\dfrac{x-4}{5}\right) + c$

Exercise 11.11

1. Integrate the following with respect to x:

(i) $\dfrac{2x-3}{x^2+4x-12}$ **(ii)** $\dfrac{5x-2}{2+2x+x^2}$

(iii) $\dfrac{3x+1}{2x^2-2x+3}$

Solution :

(i) $\dfrac{2x-3}{x^2+4x-12}$

Let $I = \int \dfrac{2x-3}{x^2+4x-12}\,dx$

$2x-3 = A.\dfrac{d}{dx}(x^2+4x-12) + B$

$\Rightarrow 2x-3 = A.(2x+4) + B$

Equating the co-efficients of x term and constant term we get

$2 = 2A \Rightarrow A = 1$

$-3 = 4A + B \Rightarrow -3 = 4 + B \Rightarrow B = -7$

$\therefore 2x-3 = 1(2x+4) - 7$

$\therefore I = \int \dfrac{2x-3}{x^2+4x-12}dx = \int \dfrac{(2x+4)-7}{x^2+4x-12}dx$

$= \int \dfrac{(2x+4)dx}{x^2+4x-12} - 7\int \dfrac{dx}{x^2+4x-12}$

$= \log|x^2+4x+12| - 7\int \dfrac{dx}{x^2+4x+4-4-12}$

$= \log|x^2+4x-12| - 7\int \dfrac{dx}{(x+2)^2-16}$

$= \log|x^2+4x-12| - 7\int \dfrac{dx}{(x+2)^2-4^2}$

$= \log|x^2+4x-12| - 7\times\dfrac{1}{2\times 4}\log\left|\dfrac{x+2-4}{x+2+4}\right| + c$

$I = \log|x^2+4x-12| - \dfrac{7}{8}\log\left|\dfrac{x-2}{x+6}\right| + c$

(ii) $\dfrac{5x-2}{2+2x+x^2}$

Let $I = \int \dfrac{(5x-2)dx}{2+2x+x^2}$

Now, $5x-2 = A.\dfrac{d}{dx}(x^2+2x+2) + B$

$\Rightarrow 5x-2 = A(2x+2) + B$

Equating the co-efficients of x term and constant term we get

$5 = 2A \Rightarrow A = \dfrac{5}{2}$

$-2 = 2A + B \Rightarrow -2 = \cancel{2}\left(\dfrac{5}{\cancel{2}}\right) + B$

$\Rightarrow -2 = 5 + B \Rightarrow B = -7$

$\therefore 5x-2 = \dfrac{5}{2}(2x+2) - 7$

$\therefore I = \int \dfrac{(5x-2)dx}{x^2+2x+2} = \dfrac{5}{2}\int \dfrac{2x+2}{x^2+2x+2}dx - 7\int \dfrac{dx}{x^2+2x+2}$

$= \dfrac{5}{2}\log|x^2+2x+2| - 7\int \dfrac{dx}{x^2+2x+1-1+2}$

$= \dfrac{5}{2}\log|x^2+2x+2| - 7\int \dfrac{dx}{(x+1)^2+1^2}$

$= \dfrac{5}{2}\log|x^2+2x+2| - 7\tan^{-1}\left(\dfrac{x+1}{1}\right) + C$

$= \dfrac{5}{2}\log|x^2+2x+2| - 7\tan^{-1}(x+1) + c$

(iii) $\dfrac{3x+1}{2x^2-2x+3}$

Let $I = \int \dfrac{(3x+1)dx}{2x^2-2x+3}$

POINTS TO REMEMBER

Derivatives	Anti derivatives
$\frac{d}{dx}(c) = 0$, where c is a constant	$\int 0\,dx = c$, where c is a constant
$\frac{d}{dx}(kx) = k$, where k is a constant	$\int k\,dx = kx + c$ where k is a constant
$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$ (power rule)
$\frac{d}{dx}\log x = \left(\frac{1}{x}\right)$	$\int \frac{1}{x}\,dx = \log\|x\| + c$
$\frac{d}{dx}(-\cos x) = \sin x$	$\int \sin x\,dx = -\cos x + c$
$\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x\,dx = \sin x + c$
$\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x\,dx = \tan x + c$
$\frac{d}{dx}(-\cot x) = \text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + c$
$\frac{d}{dx}(\sec x) = \sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + c$
$\frac{d}{dx}(-\text{cosec}\,x) = \text{cosec}\,x \cot x$	$\int \text{cosec}\,x \cot x\,dx = -\text{cosec}\,x + c$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$	$\int a^x dx = \frac{a^x}{\log a} + c$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + c$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + c$

12 Introduction of Probability Theorem

MUST KNOW DEFINITIONS

- **Sample Space :** A sample space (S) is the set of all possible outcomes of a random experiment.
- **Sure event :** The sample space S is called the **Sure event** or certain event.
- The null set ϕ is called an impossible event.
- For every event A, there corresponds another event $\overline{A}$ is called the **Complementary event** to A. It is also called the event 'not A'
- **Mutually exclusive events : (disjoint events)** Two events cannot occur simultaneously are **mutually exclusive events** $\Rightarrow A_i \cap A_j = \phi$ for $i \neq j$
- **Exhaustive events :** Events $A_1\ A_2\ ....\ A_k$ are exhaustive events if $A_1 \cup A_2 \cup .... \cup A_k = S$.
- **Equally likely events :** Two events having the same chance of occurrences are called **equally likely events.**
- **Odds :** Odds relate the chances in favour of an event A to the chances against it.
- **Independent events :** Events are said to be independent if occurrence or non-occurrence of any one of the event does not affect the probability of occurrence or non-occurrence of the other events

By Bayes' theorem

$$P(A_2/B) = \frac{P(A_2).P(B/A_2)}{P(A_1).P(B/A_1)+P(A_2).P(B/A_2)+P(A_3).P(B/A_3)}$$

$$= \frac{\frac{3}{10}\times 0.5}{\frac{5}{10}\times 0.4+\frac{3}{10}\times 0.5+\frac{2}{10}\times 0.3} = \frac{\frac{1.5}{10}}{\frac{2}{10}+\frac{1.5}{10}+\frac{.6}{10}} = \frac{\frac{1.5}{10}}{\frac{4.1}{10}}$$

$$P(A_2/B) = \frac{1.5}{10}\times\frac{10}{4.1} = \frac{1.5}{4.1} = \frac{15}{41}$$

5. An advertising executive is studying television viewing habits of married men and women during prime time hours. Based on the past viewing records he has determined that during prime time wives are watching television 60% of the time. It has also been determined that when the wife is watching television, 40% of the time the husband is also watching. When the wife is not watching the television, 30% of the time the husband is watching the television. Find the probability that (i) the husband is watching the television during the prime time of television (ii) if the husband is watching the television, the wife is also watching the television.

Solution : Let the events be defined as follows :

A_1 : Event of wife and watching the television

A_2 : Wife not watching the television

B : Husband is watching the television.

$$\text{Given } P(A_1) = 0.60$$
$$P(B/A_1) = 0.40$$
$$P(A_2) = 1 - 0.60 = 0.40$$
$$P(B/A_2) = 0.30$$

(i) P (Husband watching the television)

$\Rightarrow \quad P(B) = P(A_1).\ P(B/A_1) + P(A_2).P(B/A_2)$

$\Rightarrow \quad P(B) = (0.40)(0.60) + (0.30)(0.40)$

$\Rightarrow \quad P(B) = 0.24 + 0.12$

$\Rightarrow \quad P(B) = 0.36$

$\Rightarrow \quad P(B) = \frac{36}{100} = \frac{9}{25}$

(ii) P (if the husband is watching, the wife is also watching the television)

$$\Rightarrow \quad P(A_1/B) = \frac{P(A_1).P(B/A_1)}{P(A_1).P(B/A_1)+P(\bar{A}_2).P(B/\bar{A}_2)}$$

$$\Rightarrow \quad P(A_1/B) = \frac{(0.40)(0.60)}{P(B)} = \frac{\cdot 24}{\frac{9}{25}}$$

$$\Rightarrow \quad P(A/B) = \frac{\cancel{24}^{6}}{\cancel{100}_{4}}\times\frac{\cancel{25}}{9} = \frac{2}{3}$$

EXERCISE 12.5

CHOOSE THE CORRECT OR MOST SUITABLE ANSWER FROM THE GIVEN FOUR ALTERNATIVES.

1. Four persons are selected at random from a group of 3 men, 2 women and 4 children. The probability that exactly two of them are children is

(1) $\frac{3}{4}$ (2) $\frac{10}{23}$ (3) $\frac{1}{2}$ (4) $\frac{10}{21}$

Hint : Number of ways of choosing the 4 people group $= {}^9C_4 = 126$

Number of ways of choosing the 4 people group with exactly 2 children $= {}^4C_2 \times {}^5C_2 = 60$

$$\text{Probability} = \frac{\cancel{60}^{10}}{\cancel{126}_{21}} = \frac{10}{21}$$

[Ans : (4) $\frac{10}{21}$]

2. A number is selected from the set {1, 2,3,..., 20} . The probability that the selected number is divisible by 3 or 4 is **[March - 2019]**

(1) $\frac{2}{5}$ (2) $\frac{1}{8}$ (3) $\frac{1}{2}$ (4) $\frac{2}{3}$

Hint :

$S = \{1, 2, 3, \dots 20\} \Rightarrow n(S) = 20$

A = No is divisible by 3

$= \{3, 6, 9, 12, 15, 18\} \Rightarrow n(A) = 6$

B = No. is divisible by 4

$= \{4, 8, 12, 16, 20\} \Rightarrow n(B) = 5$

$A \cap B = \{12\} \Rightarrow n(A \cap B) = 1$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{6}{20}+\frac{5}{20}-\frac{1}{20} = \frac{6+5-1}{20} = \frac{\cancel{10}}{\cancel{20}_{2}} = \frac{1}{2}$$

[Ans : (3) $\frac{1}{2}$]

3. A, B, and C try to hit a target simultaneously but independently. Their respective probabilities of hitting the target are $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$. The probability that the target is hit by A or B but not by C is

(1) $\frac{21}{64}$ (2) $\frac{7}{32}$ (3) $\frac{9}{64}$ (4) $\frac{7}{8}$

Hint : Given $P(A) = \frac{3}{4}$, $P(B) = \frac{1}{2}$, $P(C) = \frac{5}{8}$

$$\therefore P(\bar{C}) = 1 - \frac{5}{8} = \frac{3}{8}$$

On 23.02.2019, Model Question Paper is released by the Govt.

11th STD

GOVT. MODEL QUESTION PAPER (2018 -29)

Reg. No.

Mathematics

TIME ALLOWED : 2.30 Hours] [MAXIMUM MARKS : 90

Section - I

Note : (i) All questions are compulsory **[20 × 1 = 20]**

(ii) Choose the correct or most suitable answer from the given four alternatives. Write the option code and the corresponding answer.

1. If two sets A and B have 17 elements in common, then the number of elements common to the set A × B and B × A is

(1) 2^{17} (2) 17^2
(3) 34 (4) insufficient data

2. If $\mathbb{R}$ is the set of all real numbers and if $f : \mathbb{R} - \{3\} \to \mathbb{R}$ is defined by $f(x) = \frac{3+x}{3-x}$ for $x \in \mathbb{R} - \{3\}$, then the range of f is

(1) $\mathbb{R}$ (2) $\mathbb{R} - \{1\}$
(3) $\mathbb{R} - \{-1\}$ (4) $\mathbb{R} - \{-3\}$

3. If the sum and product of the roots of the equation $2x^2 + (a - 3)x + 3a - 5 = 0$ are equal, then the value of a is

(1) 1 (2) 2 (3) 0 (4) 4

4. Which one of the following is not true?

(1) $|\sin x| \leq 1$ (2) $|\sec x| < 1$
(3) $|\cos x| \leq 1$
(4) $\operatorname{cosec} x \geq 1$ or $\operatorname{cosec} x \leq -1$

5. $\cos 1° + \cos 2° + \cos 3° + ... + \cos 179°$ is

(1) 0 (2) 1 (3) – 1 (4) 89

6. If 10 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, then the total number of points of intersection are

(1) 45 (2) 40 (3) 10! (4) 2^{10}

7. The remainder when 2^{2020} is divided by 15 is

(1) 4 (2) 8 (3) 1 (4) 2

8. The harmonic mean of two positive numbers whose arithmetic mean and geometric mean are 16, 8 respectively is

(1) 10 (2) 6 (3) 5 (4) 4

9. In the equation of a straight line $ax + by + c = 0$, if a, b, c are in arithmetic progression then the point on the straight line is

(1) (1, 2) (2) (1, –2)
(3) (2, –1) (4) (2, 1)

10. If two straight lines $x + (2k - 7)y + 3 = 0$ and $3kx + 9y - 5 = 0$ are perpendicular to each other then the value of k is

(1) 3 (2) $\frac{1}{3}$ (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

11. If $|\vec{a}| = 13$, $|\vec{b}| = 5$ and $\vec{a}.\vec{b} = 60°$ then $|\vec{a} \times \vec{b}|$ is

(1) 15 (2) 35 (3) 45 (4) 25

12. A vector $\overline{OP}$ makes 60° and 45° with the positive direction of the x and y axes respectively. Then the angle between $\overline{OP}$ and the z-axis is

(1) 45° (2) 60° (3) 90° (4) 30°

13. A vector perpendicular to both $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$ is,

(1) $2\hat{i} + \hat{j} - \hat{k}$ (2) $2\hat{i} - \hat{j} - \hat{k}$
(3) $3\hat{i} + \hat{j} + 2\hat{k}$ (4) $3\hat{i} + \hat{j} - 2\hat{k}$

14. $\lim_{x \to 0} \frac{\sin|x|}{x}$ is

(1) 1 (2) – 1
(3) 0 (4) does not exist

15. if $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \lfloor x - 3 \rfloor + |x - 4|$, $x \in \mathbb{R}$ then $\lim_{x \to 3^-} f(x)$ is equal to

(1) –2 (2) –1 (3) 0 (4) 1

11th STD

COMMON QUARTERLY EXAMINATION - 2019

Mathematics

Reg. No.

Time Allowed : 2.30 Hours] [Maximum Marks : 90

Instructions :

(a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams

PART - I

Note : (i) Answer the all questions. **[20 × 1 = 20]**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. The range of the function $\frac{1}{1-2\sin x}$ is

(a) $(-\infty, -1) \cup \left(\frac{1}{3}, \infty\right)$ (b) $\left(-1, \frac{1}{3}\right)$

(c) $\left[-1, \frac{1}{3}\right]$ (d) $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$

2. The function $f : [0, 2\pi] \to [-1, 1]$ defined by $f(x) = \sin x$ is

(a) one-to-one (b) onto

(c) bijection (d) cannot be defined

3. The solution set of the following inequality $|x-1| \geq |x-3|$ is

(a) $[0, 2]$ (b) $[2, \infty)$ (c) $(0, 2)$ (d) $(-\infty, 2)$

4. If 3 is the logarithm of 343, then the base is

(a) 5 (b) 7 (c) 6 (d) 9

5. In a Δ ABC, $\tan\left(\frac{A}{2}\right) =$

(a) $\sqrt{\frac{(s-b)(s-c)}{bc}}$ (b) $\sqrt{\frac{s(s-a)}{bc}}$

(c) $\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ (d) $\sqrt{s(s-a)(s-b)(s-c)}$

6. There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two points is

(a) 45 (b) 40 (c) 39 (d) 38

7. The n^{th} term of the sequence 1, 2, 4, 7, 11,... is

(a) $n^3 + 3n^2 + 2n$ (b) $n^3 - 3n^2 + 3n$

(c) $\frac{n(n+1)(n+2)}{3}$ (d) $\frac{n^2 - n + 2}{2}$

8. The slopes of the line which makes an angle 45° with the line $3x - y = -5$ are

(a) $1, -1$ (b) $\frac{1}{2}, -2$ (c) $1, \frac{1}{2}$ (d) $2, \frac{-1}{2}$

9. If the pair of straight lines $6x^2 + 41xy - 7y^2 = 0$ makes angle α and β with x-axis, then $\tan\alpha \tan\beta =$

(a) $\frac{-6}{7}$ (b) $\frac{6}{7}$ (c) $\frac{-7}{6}$ (d) $\frac{7}{6}$

10. If $n(A) = 5$ and $n(B) = 7$ then the number of subsets of $A \times B$ is

(a) 2^{35} (b) 2^{49} (c) 2^{25} (d) 2^{70}

11. The relation "less than" in the set of natural numbers is

(a) only symmetric (b) only transitive

(c) only reflexive (d) Equivalence

12. If $\left(\frac{2}{3}\right)^{x+2} = \left(\frac{3}{2}\right)^{2-2x}$ then $x =$

(a) 1 (b) 3 (c) 4 (d) 0

13. The value of $\frac{2(3^{n+1}) + 7(3^{n-1})}{3^{n+2} - 2\left(\frac{1}{3}\right)^{1-n}}$

(a) 1 (b) 3 (c) -1 (d) 0

14. If $\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2}$ then the value of θ is

(a) $\theta = n\pi + \frac{\pi}{3}$ (b) $\theta = 2n\pi + \frac{\pi}{3}$

(c) $\theta = 2n\pi \pm \frac{\pi}{6}$ (d) $\theta = n\pi \pm \frac{\pi}{6}$

11th STD

PUBLIC EXAMINATION MARCH - 2020

Reg. No.

Mathematics (with answers)

TIME ALLOWED : 3.00 Hours] [MAXIMUM MARKS : 90

PART - I

Note : (i) Answer all the questions. **[20 × 1 = 20]**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If $x = at^2$, $y = 2at$, then $\frac{dy}{dx} =$

(a) $-t$ (b) $\frac{1}{t}$ (c) $-\frac{1}{t}$ (d) 1

2. If $\vec{a}$ and $\vec{b}$ include an angle 120º and their magnitudes are 2, $\sqrt{3}$ and, then $\vec{a}\cdot\vec{b}$ is equal to

(a) $\frac{-\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) 2

3. If $|\vec{a}+\vec{b}| = 60$, $|\vec{a}-\vec{b}| = 40$ and $|\vec{b}| = 46$, then $|\vec{a}|$ is :

(a) 32 (b) 42 (c) 12 (d) 22

4. $\lim_{x\to\infty} \frac{a^x - b^x}{x} =$

(a) $\frac{a}{b}$ (b) $\log ab$

(c) $\log\left(\frac{a}{b}\right)$ (d) $\log\left(\frac{b}{a}\right)$

5. If A is a square matrix, then which of the following is not symmetric :

(a) $A - A^T$ (b) $A + A^T$

(c) AA^T (d) $A^T A$

6. If the function $f : [-3,3] \to S$ defined by $f(x) = x^2$ is onto, then S is :

(a) [0,9] (b) [–9,9]

(c) $\mathbb{R}$ (d) [–3,3]

7. If $A = \{(x, y); y = \sin x, x \in \mathbb{R}\}$ and $B = \{(x, y); y = \cos x, x \in \mathbb{R}\}$ then$(A \cap B)$ contains :

(a) cannot be determined

(b) no element

(c) infinitely many elements

(d) only one element

8. $f(x) = |x| + |x - 1|$ is :

(a) discontinuous at $x = 0, 1$

(b) continuous at $x = 0$ only

(c) continuous at $x = 1$ only

(d) continuous at both $x = 0$ and $x = 1$

9. If ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$, then n is :

(a) 7 (b) 5 (c) 6 (d) 11

10. $\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ is equal to

(a) $2\cos x$ (b) $\cos 2x$

(c) $\cos x$ (d) $\cos 3x$

11. Ten coin are tossed. The probability of getting at least 8 heads is :

(a) $\frac{7}{128}$ (b) $\frac{7}{64}$ (c) $\frac{7}{32}$ (d) $\frac{7}{16}$

12. The image of the point (2, 3) in the line $y = -x$ is :

(a) (3,2) (b) (–3,–2)

(c) (–3, 2) (d) (–2, –3)

13. $\int\left(\frac{x-1}{x+1}\right)dx =$

(a) $x + 2\log(x + 1) + c$

(b) $\frac{1}{2}\left(\frac{x-1}{x+1}\right)^2 + c$

(c) $x - 2\log(x + 1) + c$

(d) $\frac{(x-1)^2}{2}\log(x + 1) + c$

14. $\int \frac{dx}{e^x - 1} =$

(a) $\log(e^x + 1) - \log(e^x) + c$

(b) $\log(e^x) - \log(e^x - 1) + c$

(c) $\log(e^x) + \log(e^x - 1) + c$

(d) $\log(e^x - 1) - \log(e^x) + c$