SURA'S

Business Mathematics and Statistics

11th Standard

VOLUME - I & II

Based on the New Textbook

Salient Features

- Prepared as per the updated new textbook
- Exhaustive Additional MCQs (Questions, Match the following, Fill in the blanks, Choose the odd man out, Choose the incorrect/Correct pair, Assertion-Reason, Choose the correct or incorrect statement are given in each chapter.
- Comprehensive Additional VSA, SA, LA, HOTS questions with answers are given in each chapter.
- Govt. Model Question Paper-2018 [Govt. MQP-2018,2019], First Mid-Term Test (2018) [First Mid-2018], March Exam-2019 [MAR.-2019], June Exam-2019 [JUNE-2019], Quarterly Exam - 2018 [QY-2018,2019], Half Yearly Exam - 2018 [HY-2018, 2019] are incorporated at appropriate sections.
- Govt. Model - 2019 Question Paper.
- Common Quarterly - 2019 Question Paper.
- Common Half-yearly - 2019 Question Paper.

SURA PUBLICATIONS
Chennai

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Volume - I

01 Matrices and Determinants

TEXTUAL QUESTIONS

Exercise 1.1

1. Find the minors and cofactors of all the elements of the following determinants.

(i) $\begin{vmatrix} 5 & 20 \\ 0 & -1 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$

Solution :

(i) $\begin{vmatrix} 5 & 20 \\ 0 & -1 \end{vmatrix}$

Let A $= \begin{vmatrix} 5 & 20 \\ 0 & -1 \end{vmatrix}$

Minor of 5 $= M_{11} = -1$

Minor of 20 $= M_{12} = 0$

Minor of 0 $= M_{21} = 20$

Minor of –1 $= M_{22} = 5$

Co-factor of 5 $= A_{11} = (-1)^{1+1} M_{11}$

$= (-1)^2 (-1) = -1$

Co-factor of 20 $= A_{12} (-1)^{1+2} M_{12} = -0 = 0$

Co-factor of 0 $= A_{21} (-1)^{2+1} M_{21} = -20$

Co-factor of –1 $= A_{22} (-1)^{2+2} M_{22} = 5$

(ii) $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$

Let B $= \begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$

Minor of 1 $= M_{11} = \begin{vmatrix} -1 & 2 \\ 5 & 2 \end{vmatrix} = -2 - 10 = -12$

Minor of –3 $= M_{12} = \begin{vmatrix} 4 & 2 \\ 3 & 2 \end{vmatrix} = 8 - 6 = 2$

Minor of 2 $= M_{13} = \begin{vmatrix} 4 & -1 \\ 3 & 5 \end{vmatrix} = 20 + 3 = 23$

Minor of 4 $= M_{21} = \begin{vmatrix} -3 & 2 \\ 5 & 2 \end{vmatrix} = -6 - 10 = -16$

Minor of –1 $= M_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 2 - 6 = -4$

Minor of 2 $= M_{23} = \begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} = 5 + 9 = 14$

Minor of 3 $= M_{31} = \begin{vmatrix} -3 & 2 \\ -1 & 2 \end{vmatrix} = -6 + 2 = -4$

Minor of 5 $= M_{32} = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 2 - 8 = -6$

Minor of 2 $= M_{33} = \begin{vmatrix} 1 & -3 \\ 4 & -1 \end{vmatrix} = -1 + 12 = 11$

Co-factor of 1 $= A_{11} = (-1)^{1+1} M_{11} = -12$

Co-factor of –3 $= A_{12} = (-1)^{1+2} M_{12} = -2$

Co-factor of 2 $= A_{13} = (-1)^{1+3} M_{13} = 23$

Co-factor of 4 $= A_{21} = (-1)^{2+1} M_{21} = 16$

Co-factor of –1 $= A_{22} = (-1)^{2+2} M_{22} = -4$

Co-factor of 2 $= A_{23} = (-1)^{2+3} M_{23} = -14$

Co-factor of 3 $= A_{31} = (-1)^{3+1} M_{31} = -4$

Co-factor of 5 $= A_{32} = (-1)^{3+2} M_{32} = 6$

Co-factor of 2 $= A_{33} = (-1)^{3+3} M_{33} = 11$

2. Evaluate : $\begin{vmatrix} 3 & -2 & 4 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Solution :

Let A $= \begin{vmatrix} 3 & -2 & 4 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

$\therefore$ The technology matrix is $B = \begin{bmatrix} \frac{3}{7} & \frac{2}{13} \\ \frac{4}{7} & \frac{6}{13} \end{bmatrix}$

$$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{3}{7} & \frac{2}{13} \\ \frac{4}{7} & \frac{6}{13} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{7} & \frac{-2}{13} \\ \frac{-4}{7} & \frac{7}{13} \end{bmatrix}$$

$$|I - B| = \begin{vmatrix} \frac{4}{7} & \frac{-2}{13} \\ \frac{-4}{7} & \frac{7}{13} \end{vmatrix} = \left[\frac{4}{7}\right]\left[\frac{7}{13}\right] - \left[\frac{4}{7}\right]\left[\frac{2}{13}\right]$$

$$= \frac{28}{91} - \frac{8}{91} = \frac{20}{91}$$

Since the diagonal elements of (I – B) are positive and |I – B| is positive, the system is viable

$$(I - B)^{-1} = \frac{1}{|I - B|} \text{adj } (I - B)$$

$$= \frac{1}{\frac{20}{91}} \begin{bmatrix} \frac{7}{13} & \frac{2}{13} \\ \frac{4}{7} & \frac{4}{7} \end{bmatrix} = \frac{91}{20} \begin{bmatrix} \frac{7}{13} & \frac{2}{13} \\ \frac{4}{7} & \frac{4}{7} \end{bmatrix}$$

Now, $X = (I - B)^{-1} (D)$

$$\text{where } D = \begin{bmatrix} 12 \\ 18 \end{bmatrix} = \frac{91}{20} \begin{bmatrix} \frac{7}{13} & \frac{2}{13} \\ \frac{4}{7} & \frac{4}{7} \end{bmatrix} \begin{bmatrix} 12 \\ 18 \end{bmatrix}$$

$$= \frac{1}{20} \begin{bmatrix} 49 & 14 \\ 52 & 52 \end{bmatrix} \begin{bmatrix} 12 \\ 18 \end{bmatrix}$$

$$= \frac{1}{20} \begin{bmatrix} 49 \times 12 + 14 \times 18 \\ 52 \times 12 + 52 \times 18 \end{bmatrix}$$

$$= \frac{1}{20} \begin{bmatrix} 588 + 252 \\ 624 + 936 \end{bmatrix} = \frac{1}{20} \begin{bmatrix} 840 \\ 1560 \end{bmatrix}$$

$$= \begin{bmatrix} 42 \\ 78 \end{bmatrix}$$

$\therefore$ The gross output for two sectors X and Y are 42 and 78 respectively.

Exercise 1.5

Choose the correct answer:

1. The value of x if $\begin{vmatrix} 0 & 1 & 0 \\ x & 2 & x \\ 1 & 3 & x \end{vmatrix} = 0$ is

(a) 0, –1 (b) 0, 1
(c) –1, 1 (d) –1, –1 **Ans: (b) 0, 1**

Hint:
$$-1[x^2 - x] = 0$$
$$0 = x^2 - x$$
$$x(x - x) = 0$$
$$x = 0 \,;\, x = 1$$

2. The value of $\begin{vmatrix} 2x+y & x & y \\ 2y+z & y & z \\ 2z+x & z & x \end{vmatrix}$ is *[GMQP-2018, First Mid - 2018]*

(a) xyz (b) $x + y + z$
(c) $2x + 2y + 2z$ (d) 0 **Ans: (d) 0**

Hint: $= \begin{vmatrix} 2x & x & y \\ 2y & t & z \\ 2z & z & x \end{vmatrix} + \begin{vmatrix} y & x & y \\ z & y & z \\ x & z & x \end{vmatrix} \begin{matrix} [C_1 \equiv 2C_2] \\ [C_1 \equiv C_3] \end{matrix}$

$= 0 + 0 = 0$

3. The co-factor of –7 in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ is

(a) –18 (b) 18
(c) –7 (d) 7 **Ans: (b) 18**

Hint: $= (-1)^{3+3} \begin{vmatrix} 2 & -3 \\ 6 & 0 \end{vmatrix}$

$= 2 \times 0 - [(-3) \times 6] = 0 - (-18) = 18$

4. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix}$ then $\begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{vmatrix}$ is

(a) Δ (b) $-\Delta$
(c) 3Δ (d) -3Δ **Ans: (b) $-\Delta$**

Hint: $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} = -\begin{vmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{vmatrix} R_1 \rightleftharpoons R_2$

$= -\Delta$

$$\therefore X = A^{-1}B.$$

$$|A| = \begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix} = 4 - 15 = -11$$

$$A_{11} = 2, A_{12} = -3, A_{21} = -5, A_{22} = 2$$

$$\therefore \text{adj } A = \begin{bmatrix} 2 & -3 \\ -5 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{-1}{11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$$

$$X = A^{-1}B = \frac{-1}{11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix}$$

$$= \frac{-1}{11}\begin{bmatrix} +2-35 \\ -3+14 \end{bmatrix} = \frac{-1}{11}\begin{bmatrix} -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\therefore x = 3 \text{ and } y = -1.$$

11. The data below are about an economy of two industries P and Q. The values are in lakhs of rupees.

Producer	User		Final Demand	Total output
	P	Q		
P	16	12	12	40
Q	12	8	4	24

Find the technology matrix and check whether the system is viable as per Hawkins-Simon conditions.

Solution : With the usual notation we have

$$a_{11} = 16, a_{12} = 12, x_1 = 40$$

$$a_{21} = 12, a_{22} = 8, x_2 = 24$$

$$\text{Now } b_{11} = \frac{a_{11}}{x_1} = \frac{16}{40} = \frac{2}{5}, \quad b_{12} = \frac{a_{12}}{x_2} = \frac{12}{24} = \frac{1}{2}$$

$$b_{21} = \frac{a_{21}}{x_1} = \frac{12}{40} = \frac{3}{10}, \quad b_{22} = \frac{a_{22}}{x_2} = \frac{8}{24} = \frac{1}{3}$$

The technology matrix is

$$B = \begin{bmatrix} \frac{2}{5} & \frac{1}{2} \\ \frac{3}{10} & \frac{1}{3} \end{bmatrix}$$

$$I - B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{2}{5} & \frac{1}{2} \\ \frac{3}{10} & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & -\frac{1}{2} \\ -\frac{3}{10} & \frac{2}{3} \end{pmatrix}$$

The main diagonal elements in (I – B) namely $\frac{3}{5}$ and $\frac{2}{3}$ are positive.

$$\text{Also, } |I - B| = \begin{vmatrix} \frac{3}{5} & \frac{-1}{2} \\ \frac{-3}{10} & \frac{2}{3} \end{vmatrix} = \left(\frac{3}{5}\right)\left(\frac{2}{3}\right) - \left(\frac{1}{2}\right)\left(\frac{3}{10}\right)$$

$$= \frac{2}{5} - \frac{3}{20} = \frac{8-3}{20} = \frac{5}{20} = \frac{1}{4}$$

∴ |I – B| is positive.

∴ The two Hawkin's Simon conditions are satisfied. Hence the system is viable.

Middle Order Thinking Skills (MOTS)

1. Show that $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2(x+a+b+c)$

Solution :

$$\text{LHS} = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$ we get,

$$\text{LHS} = \begin{vmatrix} x+a+b+c & b & c \\ x+a+b+c & x+b & c \\ x+a+b+c & b & x+c \end{vmatrix}$$

Taking $(x + a + b + c)$ common from C_1 we get,

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & x+b & c \\ 1 & b & x+c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ we get,

$$= (x+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix}$$

Expanding along C_1 we get,

$$= (x+a+b+c)\left[1\begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} + 0 + 0\right]$$

$$= (x + a + b + c)\,(x^2) = \text{RHS}$$

Hence proved.

02 ALGEBRA

TEXTUAL QUESTIONS

EXERCISE 2.1

1. Resolve into partial fractions for the following:

1. $\frac{3x+7}{x^2-3x+2}$
2. $\frac{4x+1}{(x-2)(x+1)}$
3. $\frac{1}{(x-1)(x+2)^2}$
4. $\frac{1}{x^2-1}$
5. $\frac{x-2}{(x+2)(x-1)^2}$
6. $\frac{2x^2-5x-7}{(x-2)^3}$
7. $\frac{x^2-6x+2}{x^2(x+2)}$
8. $\frac{x^2-3}{(x+2)(x^2+1)}$
9. $\frac{x+2}{(x-1)(x+3)^2}$
10. $\frac{1}{(x^2+4)(x+1)}$

Solution :

1. $\frac{3x+7}{x^2-3x+2}$

$$\frac{3x+7}{x^2-3x+2} = \frac{3x+7}{(x-2)(x-1)}$$

$$= \frac{A}{x-2}+\frac{B}{x-1}$$

$$\Rightarrow \frac{3x+7}{(x-2)(x-1)} = \frac{A(x-1)+B(x-2)}{(x-2)(x-1)}$$

$$\Rightarrow 3x+7 = A(x-1)+B(x-2) \ldots (1)$$

Putting $x = 1$ in (1) we get,

$$3+7 = 0+B(1-2)$$

$$\Rightarrow 10 = -B$$

$$\Rightarrow \boxed{B = -10}$$

Putting $x = 2$ in (1) we get,

$$6+7 = A(2-1)+0$$

$$\Rightarrow 13 = A$$

$$\therefore \frac{3x+7}{x^2-3x+2} = \frac{13}{x-2}-\frac{10}{x-1}$$

2. $\frac{4x+1}{(x-2)(x+1)}$

$$\frac{4x+1}{(x-2)(x+1)} = \frac{A}{x-2}+\frac{B}{x+1}$$

$$\Rightarrow \frac{4x+1}{(x-2)(x+1)} = \frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$$

$$\Rightarrow 4x+1 = A(x+1)+B(x-2) \ldots (1)$$

Putting $x = -1$ in (1) we get,

$$-4+1 = 0+B(-1-2)$$

$$\Rightarrow -3 = -3B \Rightarrow \boxed{B = 1}$$

Putting $x = 2$ in (1) we get

$$8+1 = A(2+1)+0$$

$$\Rightarrow 9 = 3A \Rightarrow \boxed{A = 3}$$

$$\frac{4x+1}{(x-2)(x+1)} = \frac{3}{x-2}+\frac{1}{x+1}$$

3. $\frac{1}{(x-1)(x+2)^2}$

$$\frac{1}{(x-1)(x+2)^2} = \frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$$

$$\Rightarrow \frac{1}{(x-1)(x+2)^2} = \frac{A(x+2)^2+B(x-1)(x+2)+C(x-1)}{(x-1)(x+2)^2}$$

$$\Rightarrow 1 = A(x+2)^2+B(x-1)(x+2)+C(x-1) \ldots (1)$$

Putting $x = -2$ in (1) we get,

$1 = C(-2-1)$

Step - 2 : Let us assume that P(k) is true.

$\therefore 5^{2k} - 1$ is divisible by 24.

$\Rightarrow 5^{2k} - 1 = 24\lambda \Rightarrow 5^{2k} = 24\lambda + 1$ ... (1)

Where λ is a constant

Step - 3 : To prove that P $(k+1)$ is true. i.e. to P.T.

$5^{2(k+1)} - 1$ is divisible by 24

Consider $5^{2(k+1)} - 1$.

$= 5^{2k+2} - 1 = 5^{2k}.5^2 - 1$

$= 25\,(5^{2k}) - 1 = 25\,(24\lambda + 1) - 1$ [by (1)]

$= 25.24\lambda + 25 - 1 = 25.24\lambda + 24$

$= 24\,(25\lambda + 1)$

Which is divisible by 24

$\therefore$ P $(k+1)$ is true whenever P(k) is true.

$\therefore$ By mathematical induction, P(n) is true for all $n \in \mathbb{N}$.

8. $n\,(n+1)\,(n+2)$ is divisible by 6, for all $n \in \mathbb{N}$

Let P (n) denote the statement $n\,(n+1)\,(n+2)$ is divisible by 6.

Step - 1 :

Put $n = 1$

$1\,(1+1)\,(1+2) = 6$ is divisible by 6.

$\therefore$ P (1) is true.

Step - 2 :

Let us assume that P(k) is true.

i.e. $k\,(k+1)\,(k+2)$ is divisible by 6

$\Rightarrow k\,(k+1)\,(k+2) = 6\lambda$ ... (1)

Where λ is a constant

Step - 3 :

To prove that P $(k+1)$ is true. i.e. to P.T.

$(k+1)\,(k+2)\,(k+3)$ is divisible by 6.

Consider $(k+1)\,(k+2)\,(k+3)$

$= (k+1)\,(k+2)\,(k) + 3\,(k+1)\,(k+2)$

$= 6\lambda + 3\,(k+1)\,(k+2)$ [using (1)]

$= 6\lambda + 3\,(k+1)\,(k+2)$

$= 6\lambda + 3$(even number) [If k is odd or even 6 second term is even always]

= which is divisible by 6.

$\therefore$ P $(k+1)$ is true whenever P(k) is true.

$\therefore$ By mathematical induction, P(n) is true for all $n \in \mathbb{N}$.

9. $2^n > n$, for all $n \in \mathbb{N}$.

Let P (n) denote the statement $2^n > n$.

Step - 1 :

Put $n = 1$

$2^1 > 1 \Rightarrow 2 > 1$.

$\therefore$ P (1) is true.

Step - 2 :

Let us assume that P(k) is true.

i.e. $2^k > k$ ... (1)

Step - 3 :

To prove that P $(k+1)$ is true. i.e. to P.T.

$2^{k+1} > k+1$.

$$\begin{aligned}\text{Consider } 2^{k+1} &= 2^k \,.\, 2^1\\ &> k\,(2)\\ &> 2k\\ &> k+1\end{aligned}$$

$[\because k > 1 \Rightarrow k + k > 1 + k \Rightarrow 2k > k+1]$

$\therefore$ P $(k+1)$ is true whenever P(k) is true.

$\therefore$ By mathematical induction, P(n) is true for all $n \in \mathbb{N}$.

Exercise 2.6

1. Expand the following by using binomial theorem.

(i) $(2a - 3b)^4$ **(ii)** $\left(x + \frac{1}{y}\right)^7$ **(iii)** $\left(x + \frac{1}{x^2}\right)^6$

Solution :

(i) $(2a - 3b)^4$

$$\begin{aligned}(x+a)^n &= x^n + nC_1\, x^{n-1}\, a^1 + nC_2\, x^{n-2}\, a^2 + \ldots + a^n\\ \therefore (2a-3b)^4 &= (2a)^4 - 4C_1\,(2a)^3\,(3b)^1 + 4C_2\,(2a)^2\,(3b)^2 - 4\,C_3\,(2a)^1\,(3b)^3 + 4C_4\,(3b)^4\\ &= 16a^4 - 4\,(8a^3)\,(3b) + 6\,(4a^2)\,(9b^2) - 4\,(2a)(27b^3) + 81b^4\\ &= 16a^4 - 96\,a^3\,b + 216\,a^2\,b^2 - 216\,ab^3 + 81b^4\end{aligned}$$

(ii) $\left(x + \frac{1}{y}\right)^7$

$$= x^7 + 7C_1\, x^6 \left(\frac{1}{y}\right)^1 + 7C_2\, x^5 \left(\frac{1}{y}\right)^2 + 7C_3\, x^4 + 7C_4\, x^3 \left(\frac{1}{y}\right)^4 + 7C_5\, x^2 \left(\frac{1}{y}\right)^5 + 7C_6\, x \left(\frac{1}{y}\right)^6 + \left(\frac{1}{y}\right)^7$$

$$= x^7 + 7\frac{x^6}{y} + 21\frac{x^5}{y^2} + 35\frac{x^4}{y^3} + 35\frac{x^3}{y^4} + 21\frac{x^2}{y^5} + \frac{7x}{y^6} + \left(\frac{1}{y}\right)^7$$

7. From 20 raffle tickets in a hat, four tickets are to be selected in order. The holder of the first ticket wins a car, the second a motor cycle, the third a bicycle and the fourth a skateboard. In how many different ways can these prizes be awarded?

Solution : The first prize can be awarded in 20 different ways.

- The second prize can be awarded in 19 ways.
- The third prize can be awarded in 18 ways.
- The fourth prize can be awarded in 17 ways.

$\therefore$ By fundamental principle of multiplication, the total number of ways the prizes can be awarded is

= 20 + 19 + 18 + 17 = 74

8. In how many different ways, 2 Mathematics, 2 Economics and 2 History books can be selected from 9 Mathematics, 8 Economics and 7 History books?

Solution : The number of ways of selecting 2 maths books from 9 maths books is $9C_2$.

Similarly number of ways of selecting Economics, and History books are $8C_2$ and $7C_2$.

$\therefore$ Total number of selecting books.

$$= 9C_2 \times 8C_2 \times 7C_2 = \frac{9\times \overset{4}{\cancel{8}}}{\cancel{2}\times 1} + \frac{\overset{4}{\cancel{8}}\times 7}{\cancel{2}\times 1} + \frac{7\times \overset{3}{\cancel{6}}}{\cancel{2}\times 1}$$

$$= 9 \times 4 \quad + \quad 4 \times 7 \quad + \quad 7 \times 3 = 36 + 28 + 21 = 85$$

9. Let there be 3 red, 2 yellow and 2 green signal flags. How many different signals are possible if we wish to make signals by arranging all of them vertically on a staff?

Solution : We have to arrange totally 7 flags out of which 3 are one kind (Red) 2 are of another kind (yellow) and 2 are of third kind (green)

So, total number of signals = $\frac{7!}{3!\ 2!\ 2!}$

$$= \frac{7\times 6\times 5\times \cancel{4}\times \cancel{3!}}{\cancel{3!}\times \cancel{2}\times \cancel{2}} = 7 \times 6 \times 5 = 210$$

10. Find the Co-efficient of x^{11} in the expansion of $\left(x+\frac{2}{x^2}\right)^{17}$.

Solution : In $\left(x+\frac{2}{x^2}\right)^{17}$, $n = 17, x = x$, $a = \frac{2}{x^2}$

$\therefore$ The general terms is

$$t_{r+1} = nC_r\ x^{n-r} a^r$$

$$= 17C_r\, x^{17-r}\left(\frac{2}{x^2}\right)^r$$

$$= 17C_r\, x^{17-r} . \frac{2^r}{x^{2r}}$$

$$= 17C_r . 2^r\, x^{17-3r} \quad \ldots (1)$$

To get the co-efficient of x^{11},

$\Rightarrow \quad 17 - 3r = 11 \Rightarrow 17 - 11 = 3r$

$\Rightarrow \quad 3r = 6 \Rightarrow r = 2$

Put $r = 2$ in (1) we get,

$$t_3 = 17C_2 2^2 x^{17-3(2)} = 17C_2(4)\, x^{11}$$

$$= \frac{17\times 16}{\cancel{2}\times 1}\times \overset{2}{\cancel{4}} . x^{11} = 544\, x^{11}$$

$\therefore$ Co-efficient of x^{11} is 544.

Govt. Exam Questions

1 Marks

1. Number of chords that can be drawn through 48 points on a circle is ________. **[MAR.-2019]**

(a) 47 (b) 210 (c) 1128 (d) 24

Ans: (c) 1128

2. Out of 10 things, 3 things can be selected in _______ ways. **[JUNE-2019]**

(a) $3C_{10}$ (b) $10C_3$ (c) $4C_{10}$ (d) $10C_4$

Ans: (b) $10C_3$

3. The no. of permutations of English words A,E,I,O,U taking two at a time: **[HY-2019]**

(a) 20 (b) 120 (c) 5 (d) 2

Ans: (a) 20

2 Marks

1. If $nC_4 = 495$, find n. **[MAR.-2019]**

Solution : $n\,(n-1)\,(n-2)\,(n-3) = 495 \times 4!$

$= 11 \times 5 \times 3 \times 3 \times 4 \times 3 \times 2$

$= (4 \times 3) \times 11 \times (5 \times 2) \times (3 \times 3)$

$\therefore n = 12$

5	495
11	99
3	9
	3

Solution :

2. Find the value of A and B if $\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$

Solution : $\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$ **[QY-2019]**

$$1 = A(x+1) + B\,(x-1)$$

$$A = \frac{1}{2} \text{ and } B = \frac{-1}{2}$$

03 ANALYTICAL GEOMETRY

POINTS TO REMEMBER

- Angle between the two intersecting lines $y = m_1x + c$ and $y = m_2 x + c_2$ is $\tan^{-1}\left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$.
- Condition for the three straight lines $a_1x + b_1 y + c_1=0$, $a_2x + b_2y + c_2= 0$, and $a_3x + b_3y + c_3 = 0$ to be concurrent is $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$
- The condition for a general second degree equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to represent a pair of straight lines is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.
- Pair of straight lines passing through the origin is $ax^2 + 2hxy + by^2 = 0$.
- Let $ax^2 + 2hxy + by^2 = 0$ be the straight lines passing through the origin then the product of the slopes is $\frac{a}{b}$ and sum of the slopes is $\frac{-2h}{b}$.
- If θ is the angle between the pair of straight lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, then $\theta = \tan^{-1}\left[\left|\frac{\pm 2\sqrt{h^2 - ab}}{a + b}\right|\right]$.
- The equation of a circle with centre (h , k) and the radius r is $(x - h)^2 + (y - k)^2 = r^2$.
- The equation of the circle with, (x_1, y_1) and, (x_2 , y_2) as end points of a diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.
- The parametric equations of the circle $x^2 + y^2 = r^2$ are $x = r \cos\theta$, $y = r \sin\theta$, $\theta \le \theta \le 2\pi$
- The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.
- The equation of the tangent at, (x_1 , y_1) to the circle $x^2 + y^2 = a^2$ is $xx_1 + yy_1 = a^2$.
- Length of the tangent to the circle $x^2+y^2+2gx+2fy+c=0$ from a point, (x_1,y_1) is $\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$
Condition for the straight line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = a^2$ is $c^2 = a^2(1 + m^2)$.
- The standard equation of the parabola is $y^2 = 4ax$

$\Rightarrow \quad x^2 - 30x + 225 = 5y - 500 + 225$

[Square of $\frac{1}{2}$ (Co-efficient of x)=$\left[\frac{1}{2}(-30)\right]^2$=$(-15)^2$=225]

$\Rightarrow \quad (x-15)^2 = 5y - 275$

$\Rightarrow \quad (x-15)^2 = 5(y-55)$

$\Rightarrow \quad X^2 = 5Y$

where $X = x - 15, Y = y - 55$.

Thus, the average variable cost curve is a parabola with vertex (15, 55)

$\therefore$ At the vertex of the parabola, the output is 15 tonnes and the average cost is ₹ 55.

6. The profit ₹y accumulated in thousand in x months is given by $y = -x^2 + 10x - 15$. Find the best time to end the project.

Solution : Let y be the profit in x months.

Given $y = -x^2 + 10x - 15$

$\Rightarrow \quad x^2 - 10x = -y - 15$

$\Rightarrow \quad x^2 - 10x + 25 = -y - 15 + 25$

[Square of $\frac{1}{2}$ $(-10) = (-5)^2 = 25$]

$\Rightarrow \quad (x-5)^2 = -y + 10$

$\Rightarrow \quad (x-5)^2 = -(y-10)$

$\Rightarrow \quad X^2 = -y$

where $X = x - 5$ and $Y = y - 10$ Vertex is (5, 10)

Since x represents the month, the best time to end the project = 5 months.

Exercise 3.7

Choose the correct answer :

1. If m_1 and m_2 are the slopes of the pair of lines given by $ax^2 + 2hxy + by^2 = 0$, then the value of $m_1 + m_2$ is

a) $\frac{2h}{b}$ b) $-\frac{2h}{b}$

c) $\frac{2h}{a}$ d) $-\frac{2h}{a}$ **Ans: a) $\frac{2h}{b}$**

2. The angle between the pair of straight lines $x^2 - 7xy + 4y^2 = 0$ is

(a) $\tan^{-1}\left(\frac{1}{3}\right)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}\left(\frac{\sqrt{33}}{5}\right)$ (d) $\tan^{-1}\left(\frac{5}{\sqrt{33}}\right)$

Ans: (c) $\tan^{-1}\left(\frac{\sqrt{33}}{5}\right)$

3. If the lines $2x - 3y - 5 = 0$ and $3x - 4y - 7 = 0$ are the diameters of a circle, then its centre is *[GMQP-2019]*

(a) (–1, 1) (b) (1,1)

(c) (1, –1) (d) (–1, –1)

Ans: (c) (1, –1)

Hint: $2x - 3y = 5$ ... (1)

$2x - 4y = 7$ ... (2)

(1) × 3 $6x - 9y = 15$

(2) × 2 $6x - 8y = 14$

$-y = 1$

$y = -1$

$2x - 3(-1) = 5$

$2x + 3 = 5$

$2x = 2$

$\therefore x = 1$

$\therefore$ (1, 1)

4. The x-intercept of the straight line $3x + 2y - 1=0$ is

(a) 3 (b) 2 (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Hint: $3x + 2(0) = 1 \Rightarrow x = \frac{1}{3}$ **Ans: (c) $\frac{1}{3}$**

5. The slope of the line $7x + 5y - 8 = 0$ is *[JUNE-2019]*

(a) $\frac{7}{5}$ (b) $-\frac{7}{5}$ (c) $\frac{5}{7}$ (d) $-\frac{5}{7}$

Hint: $y = \frac{-7}{5}x + \frac{8}{8} \Rightarrow m = -\frac{7}{5}$ **Ans: (b) $-\frac{7}{5}$**

6. The locus of the point P which moves such that P is at equidistance from their coordinate axes is

(a) $y = \frac{1}{x}$ (b) $y = -x$ *[GMQP-2018]*

(c) $y = x$ (d) $y = -\frac{1}{x}$

Ans: (c) $y = x$

7. The locus of the point P which moves such that P is always at equidistance from the line $x + 2y + 7 = 0$ is

(a) $x + 2y + 2 = 0$ (b) $x - 2y + 1 = 0$

(c) $2x - y + 2 = 0$ (d) $3x + y + 1 = 0$

Ans: (a) $x + 2y + 2 = 0$

Hint: Parallel line to $x + 2y + 7 = 0$ is $x + 2y + 2 = 0$

8. If $kx^2 + 3xy - 2y^2 = 0$ represent a pair of lines which are perpendicular then k is equal to

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) –2

Ans: (c) 2

$$-2g = 3+4=7$$
$$g = \frac{-7}{2}$$
$$f = -1$$

Put $g = \frac{-7}{2}$ and $f = -1$ in (1)

$$2\left(\frac{-7}{2}\right) + 2(-1) + c = -2$$
$$-7 - 2 + c = -2$$
$$\therefore x^2 + y^2 + 2\left(\frac{-7}{2}\right)x + 2(-1)y + 7 = 0$$
$$x^2 + y^2 +- 7x - 2y + 7 = 0$$

2. Show that the equation $2x^2 + 7xy + 3y^2 + 5x + 5y + 2 = 0$ represent two straight lines and find their separate equations. *[QY-2019]*

Solution : $2x^2 + 7xy + 3y^2 + 5x + 5y + 2 = 0$

$a = 2; b = 3;\ h = \frac{7}{2};\ g = \frac{5}{2};\ f = \frac{5}{2};\ c = 2$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 2 & \frac{7}{2} & \frac{5}{2} \\ \frac{7}{2} & 3 & \frac{5}{2} \\ \frac{5}{2} & \frac{5}{2} & 2 \end{vmatrix} = 0$$

$2x^2 + 7xy + 3y^2 + 5x + 5y + 2$

$$= (x + 3y + l)(2x + y + m)$$

$$\left.\begin{aligned} 2l + m &= 5 \\ l + 3m &= 5 \end{aligned}\right\} \Rightarrow \begin{aligned} l &= 2 \\ m &= 1 \end{aligned}$$

$\therefore$ Separate equations are
$x + 3y + 2 = 0$ and $2x + y + 1 = 1$

3. Find the axis, vertex, focus, equation of directrix and length of latus rectum for the parabola $x^2 + 6x - 4y + 21 = 0$ *[QY-2019]*

Solution : $x^2 + 6x - 4y + 21 = 0$

$$4y = x^2 + 6x + 21$$
$$4y = (x+3)^2 + 12$$
$$(x+3)^2 = 4(y-3)$$
$$X^2 = 4Y \text{ where } X = x + 3$$
$$Y = y - 3$$
$$4a = 1 \Rightarrow a = \frac{1}{4}$$

Axis : $x = -3$
Vertiex : v(–3, 3)
Focus : F(3, 4)
Directrix : $y = 2$
L.LR : 4(1) = 4

4. Show that the pair of straight lines $4x^2 - 12xy + 9y^2 + 18x - 27y + 8 = 0$ represents a pair of parallel straight lines and find their separate equations. *[HY-2019]*

Solution : The given equation is
$4x^2 - 12xy + 9y^2 + 18x - 27y + 8 = 0$
Here $a = 4$, $b = 9$ and $h = -6$
$h^2 - ab = 36 - 36 = 0$
Hence the given equation represents a pair of parallel straight lines.

Now $4x^2 - 12xy + 9y^2 = (2x - 3y)^2$

Consider $4x^2 - 12xy + 9y^2 + 18x - 27y + 8 = 0$
$\Rightarrow (2x - 3y)2 + 9(2x - 3y) + 8 = 0$

Put $2x - 3y = z$

$z^2 + 9z + 8 = 0$

$(z + 1)(z + 8) = 0$

$z + 1 = 0$	$z + 8 = 0$
$2x - 3y + 1 = 0$	$2x - 3y + 8 = 0$

Hence the separate equations are
2x – 3y + 1 = 0 and 2x – 3y + 8 = 0

ADDITIONAL PROBLEMS

3 MARKS

1. The line joining A(2,0) and B(3,2) rotates 15º about A in anti-clockwise direction hence find the equation of the line in new position. *[GMQP-2019]*

Solution :

Slope of the line joining he points A(2,0) and B(3,1) is.

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\tan\theta = \frac{1-0}{3-2}$$
$$\tan\theta = 1 \Rightarrow \theta = 45^\circ$$

Angle formed rotating 15º anti-clockwise direction about (45º + 15º) = 60º

$$\therefore \text{ new slope } (m) = \tan\theta = \tan 60^\circ = \sqrt{3}$$

04 TRIGONOMETRY

POINTS TO REMEMBER

- If in a circle of radius r, an are arc of length l subtends an angle of θ radians, then $l = r\,\theta$.
- $\sin^2 x + \cos^2 x = 1$
- $\tan^2 x + 1 = \sec^2 x$
- $\cot^2 x + 1 = \operatorname{cosec}^2 x$
- $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
- $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
- $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\cos\beta$
- $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
- $\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
- $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$
- $\sin 2\alpha = 2\sin\alpha\cos\alpha$
- $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$
- $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$
- $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$
- $\tan 3\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{1 - 3\tan^2\alpha}$
- $\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$
- $\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$
- $\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$
- $\cos x - \cos y = -2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$
- $\sin x\cos y = \dfrac{1}{2}[\sin(x+y) + \sin(x-y)]$
- $\cos x\sin y = \dfrac{1}{2}[\sin(x+y) - \sin(x-y)]$
- $\cos x\cos y = \dfrac{1}{2}[\cos(x+y) + \cos(x-y)]$
- $\sin x\sin y = \dfrac{1}{2}[\cos(x-y) - \cos(x+y)]$

TEXTUAL QUESTIONS

Exercise 4.1

1. Convert the following degree measure into radian measure

(i) 60° (ii) 150° (iii) 240° (iv) – 320°

Solution :

(i) 60°

$$60° = 60 \times \frac{\pi}{180} \text{ radians} = \frac{\pi}{3} \text{ radians.}$$

(ii) 150°

$$150° = 150 \times \frac{\pi}{180} \text{ radians}$$
$$= \frac{5\pi}{6} \text{ radians.}$$

(iii) 240°

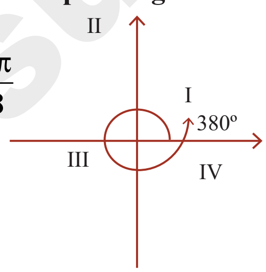
$$240° = 240 \times \frac{\pi}{180} \text{ radians}$$
$$= \frac{4\pi}{3} \text{ radians.}$$

(iv) – 320°

$$-320° = -320° \times \frac{\pi}{180} \text{ radians}$$
$$= \frac{-16\pi}{9} \text{ radians.}$$

2. Find the degree measure corresponding to the following radian measure.

(i) $\frac{\pi}{8}$ (ii) $\frac{9\pi}{5}$ (iii) –3 (iv) $\frac{11\pi}{18}$



Solution :

(i) $\frac{\pi}{8}$

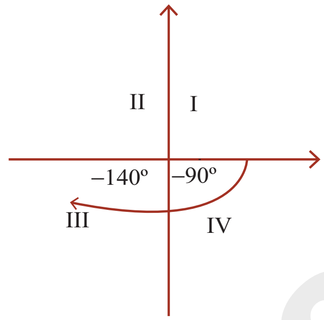
$$\frac{\pi}{8} \text{ radians} = \frac{\pi}{8} \times \frac{180°}{\pi} = 22\tfrac{1}{2}° = 22° \, 30'$$

[one degree = 60 minutes (60′) and $\frac{1}{2}$ degree = 30′]

(ii) $\frac{9\pi}{5}$

$$\frac{9\pi}{5} \text{ radians} = \frac{9\pi}{5} \times \frac{180°}{\pi} = 324°$$

(iii) –3



$$-3 \text{ radians} = -3 \times \frac{180}{\pi} \text{ degrees}$$
$$= -3 \times \frac{180}{22} \times 7$$
$$= -\frac{3780°}{22} = -171.81°$$
$$= -171°48' [0.81° = 0.81 \times 60' = 48']$$

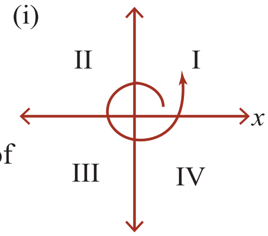
(iv) $\frac{11\pi}{18}$

$$\frac{11\pi}{18} \text{ radians} = \frac{11\pi}{18} \times \frac{180}{\pi} \text{ degrees}$$
$$= 11 \times 10 = 110°$$

3. Determine the quadrants in which the following degree lie. (i) 380° (ii) – 140° (iii) 1195°

Solution :

(i)



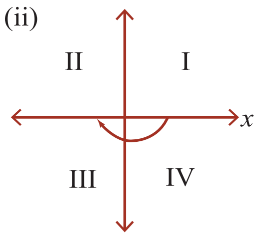
(i) 380°

380° = 360° + 20°

After one completion of round, the angle is 20°

380° lie's in the I quadrant.

(ii)



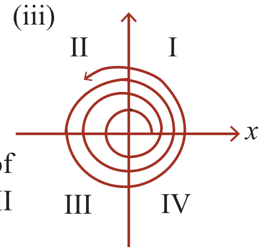
(ii) –140°

140° = – 90° + (–50°)

Since the angle is negative it moves in the anti clockwise direction.

∴ –140° lies in the III quadrant.

(iii)



(iii) 1195°

1195° = 3 × 360° + 115°

= 3 × 360° + 90° + 25°

∴ After 3 Completion of round, the angle will lie in II quadrant

∴ 1195° lies in the II quadrant.

$$= \sin^{-1}\sqrt{\frac{169-144}{169}} + \sin^{-1}\left(\frac{3}{5}\right)$$

$$= \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right)$$

$$= \sin^{-1}\left[\frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2} + \frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2}\right]$$

$$\left[\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)\right]$$

$$= \sin^{-1}\left[\frac{5}{13}\times\sqrt{\frac{25-9}{25}} + \frac{3}{5}\sqrt{\frac{169-25}{169}}\right]$$

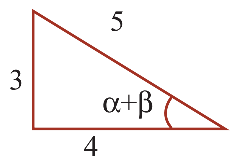
$$= \sin^{-1}\left[\frac{5}{13}\times\frac{4}{5}+\frac{3}{5}\times\frac{12}{13}\right]$$

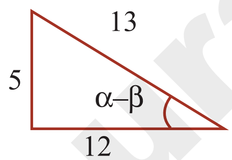
$$= \sin^{-1}\left[\frac{4}{13}+\frac{36}{65}\right] = \sin^{-1}\left[\frac{20+36}{65}\right]$$

$$= \sin^{-1}\left(\frac{56}{65}\right) = \text{RHS.}$$ Hence proved.

9. If $\cos(\alpha+\beta) = \frac{4}{5}$ and $\sin(\alpha-\beta) = \frac{5}{13}$ where $(\alpha+\beta)$ and $(\alpha-\beta)$ are acute, then find $\tan 2\alpha$.

Solution :





$\cos(\alpha+\beta) = \frac{4}{5}$ | $\sin(\alpha-\beta) = \frac{5}{13}$

$\sin(\alpha+\beta) = \frac{3}{5}$ | $\cos(\alpha-\beta) = \frac{12}{13}$

$\tan(\alpha+\beta) = \frac{3}{4}$ | $\tan(\alpha-\beta) = \frac{5}{12}$

Now $\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)]$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$\left[\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}\right]$$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\left(\frac{3}{4}\right)\left(\frac{5}{12}\right)} = \frac{\frac{9+5}{12}}{\frac{48-15}{48}}$$

$$= \frac{\frac{14}{12}}{\frac{33}{48}} = \frac{14}{12}\times\frac{48}{33} = \frac{14\times4}{33} \qquad \therefore \tan(2\alpha) = \frac{56}{33}$$

10. Express $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$ in the simplest form.

Solution : Consider $\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$

Dividing the numerator and denominator by $\cos x$,

we get
$$\left(\frac{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}}\right) = \frac{1-\tan x}{1+\tan x} = \frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}$$

$$\left[\because \tan\frac{\pi}{4} = 1\right]$$

$$= \tan\left(\frac{\pi}{4}-x\right) \qquad \left[\because \tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}\right]$$

$$\therefore \tan^{-1}\left[\tan\left(\frac{\pi}{4}-x\right)\right] = \frac{\pi}{4}-x$$

which is the simplest form.

Govt. Exam Questions

1 Marks

1. The value of $\cot 300°$ is ***[MAR.-2019]***

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{-1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$

Ans: (d) $-\sqrt{3}$

2 Marks

1. Convert the following into the product of trigonometric functions $\cos 8A + \cos 12A$. ***[QY-2019]***

Solution : $\cos 8A + \cos 12A$

$$= 2\cos\left(\frac{8A+12A}{2}\right)\cos\left(\frac{8A-12A}{2}\right)$$

$$= 2\cos 10A \cos 2A$$

2. In any quadrilateral ABCD, prove that $\sin(A+B) + \sin(C+D) = 0$. ***[QY-2019]***

Solution : Since A, B, C, D are angles of a quadrilateral,

$\therefore$ $A+B+C+D = 2\pi$

$A+B+C+D = 2\pi$

05 Differential Calculus

POINTS TO REMEMBER

- Let A and B be two non empty sets, then a function f from A to B, associates every element of A to an unique element of B.
- $\lim_{x \to a} f(x)$ exists $\Rightarrow \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$
- For the function $f(x)$ and a real number a, $\lim_{x \to a} f(x)$ and $f(a)$ may not be same.
- A function $f(x)$ is a continuous function at $x = a$ if only if $\lim_{x \to a} f(x) = f(a)$
- A function $f(x)$ is continuous, if it is continuous at every point of its domain.
- If $f(x)$ and $g(x)$ are continuous on their common domain, then $f \pm g, f.g, kf$ (k is a constant) are continuous and if $g \neq 0$ then $\frac{f}{g}$ is also continuous.
- A function $f(x)$ is differentiable at $x = c$ if and only if $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists finitely and denoted by $f'(c)$.
- $L[f'(c)] = \lim_{h \to 0} \frac{f(c-h) - f(c)}{-h}$ and $R[f'(c)] = \lim_{h \to 0} \frac{f(c+h) - f(c)}{-h}$
- A function $f(x)$ is said to be differentiable at $x = c$ if and only if $L[f'(c)] = R[f'(c)]$.
- Every differentiable function is continuous but, the converse is not necessarily true.
- If $y = f(x)$ then $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ is called second order derivative of y with respect to x.
- If $x = f(t)$ and $y = g(t)$ then $\frac{d^2y}{dx} = \frac{d}{dx}\left\{\frac{g'(t)}{f'(t)}\right\}$ (or) $\frac{d^2y}{dx} = \frac{d}{dt}\left\{\frac{g'(t)}{f'(t)}\right\} \cdot \frac{dt}{dx}$.

20. $\dfrac{d}{dx}\left(\dfrac{1}{x}\right)$ **is equal to** *[JUNE-2019]*

(a) $-\dfrac{1}{x^2}$ (b) $-\dfrac{1}{x}$

(c) $\log x$ (d) $\dfrac{1}{x^2}$ **Ans: (a)** $-\dfrac{1}{x^2}$

Hint: $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = \dfrac{d}{dx}\left(x^{-1}\right) = (-1)x^{-2} = \dfrac{-1}{x^2}$

21. $\dfrac{d}{dx}$ **$(5e^x - 2\log x)$ is equal to**

(a) $5e^x - \dfrac{2}{x}$ (b) $5e^x - 2x$

(c) $5e^x - \dfrac{1}{x}$ (d) $2\log x$

Ans: (a) $5e^x - \dfrac{2}{x}$

Hint: $\dfrac{d}{dx}\left(5e^x\right) - \dfrac{d}{dx}(2\log x) = 5e^x - \dfrac{2}{x}$

22. **If $y = x$ and $z = \dfrac{1}{x}$ then $\dfrac{dy}{dz} =$**

(a) x^2 (b) 1 (c) $-x^2$ (d) $-\dfrac{1}{x^2}$

Ans: (c) $-x^2$

Hint:

$$\frac{dy}{dx} = 1; \frac{dz}{dx} = (-1)x^{-2} = \frac{-1}{x^2}$$

$$\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{1}{-\frac{1}{x^2}} = -x^2$$

23. **If $y = e^{2x}$ then $\dfrac{d^2y}{dx^2}$ at $x = 0$ is** *[GMQP-2018 & 19]*

(a) 4 (b) 9 (c) 2 (d) 0

Ans: (a) 4

Hint:

$$\frac{dy}{dx} = 2e^{2x}$$

$$\frac{d^2y}{dx^2} = 4e^{2x} \quad \text{At } x = 0 \text{ is } 4$$

24. **If $y = \log x$ then $y_2 =$** *[MAR.-2019] [Qy-2018]*

(a) $\dfrac{1}{x}$ (b) $-\dfrac{1}{x^2}$ (c) $-\dfrac{2}{x^2}$ (d) e^2

Hint:

$$\frac{dy}{dx} = \frac{1}{x}$$

$$y_2 = \frac{-1}{x^2}$$

Ans: (b) $-\dfrac{1}{x^2}$

25. $\dfrac{d}{dx}$ **$(a^x) =$**

(a) $\dfrac{1}{x\log_e^a}$ (b) a^a (c) $x\log_e^a$ (d) $a^x \log_e^a$

Ans: (d) $a^x \log_e^a$

Miscellaneous Problems

1. **If $f(x) = \dfrac{1}{2x+1}, x \neq -\dfrac{1}{2}$, then show that $f(f(x)) = \dfrac{2x+1}{2x+3}$, provided that $x \neq -\dfrac{3}{2}$.**

Solution : Given $f(x) = \dfrac{1}{2x+1}$

$$f(f(x)) = f\left(\frac{1}{2x+1}\right)$$

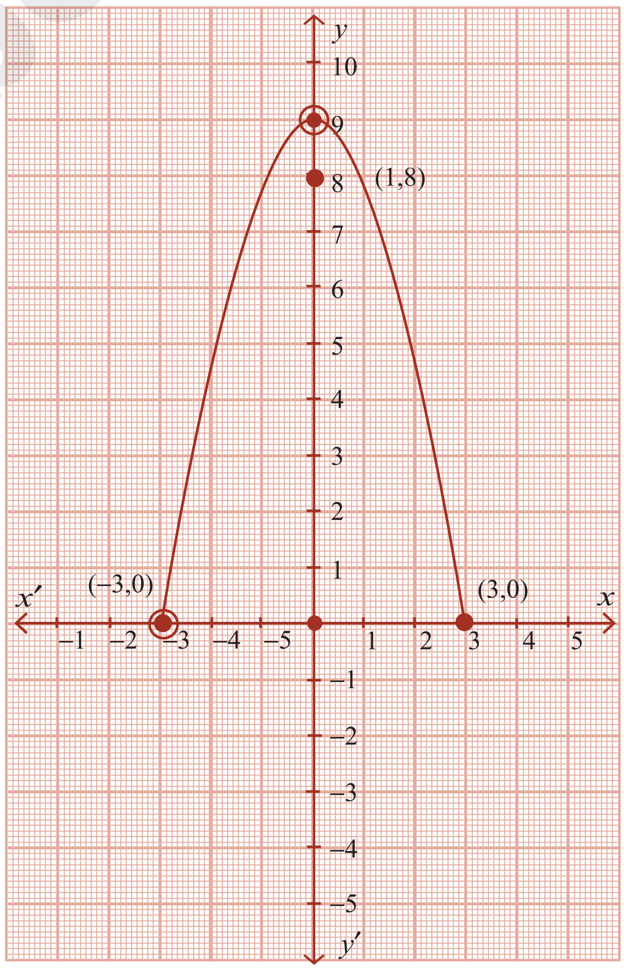
$$= \frac{1}{2\left(\frac{1}{2x+1}\right)+1} = \frac{1}{\frac{2}{2x+1}+1} = \frac{1}{\frac{2x+2+1}{2x+1}} = \frac{2x+1}{2x+3}$$

since $x \neq -\dfrac{3}{2}$

2. **Draw the graph of $y = 9 - x^2$.**

Solution : Given $y = 9 - x^2$

x	0	3	–3	1
y	9	0	0	8



Volume - II

06 Applications of Differentiation

TEXTUAL QUESTIONS

Exercise 6.1

1. A firm produces x tonnes of output at a total cost of $C(x) = \frac{1}{10}x^3 - 4x^2 - 20x + 7$. Find the

(i) average cost function *[GMQP - 2019]*
(ii) average variable cost function *[GMQP - 2019]*
(iii) average fixed cost function *[GMQP - 2019]*
(iv) marginal cost and function
(v) marginal average cost function

Solution :

Given $C(x) = \frac{1}{10}x^3 - 4x^2 - 20x + 7$

(i) Average Cost (AC)

$$= \frac{C}{x} = \frac{\frac{1}{10}x^3 - 4x^2 - 20x + 7}{x}$$

$$= \frac{1}{10}x^2 - 4x - 20 + \frac{7}{x}$$

(ii) Average variable cost (AVC)

$$= \frac{\frac{1}{10}x^3 - 4x^2 - 20x}{x}$$

$$= \frac{1}{10}x^2 - 4x - 20$$

(iii) Average fixed cost (AFC)

$$= \frac{k}{x} = \frac{7}{x}$$

(iv) Marginal cost (MC)

$$= \frac{dC}{dx} = \frac{d}{dx}\left(\frac{1}{10}x^3 - 4x^2 - 20x + 7\right)$$

$$= \frac{1}{10}(3x^2) - 4(2x) - 20$$

$$= \frac{3}{10}x^2 - 8x - 20$$

(v) Marginal Average cost (MAC)

$$= \frac{d}{dx}(AC)$$

$$= \frac{d}{dx}\left(\frac{1}{10}x^2 - 4x - 20 + \frac{7}{x}\right)$$

$$= \frac{1}{10}(2x) - 4 + 7\left(\frac{-1}{x^2}\right)$$

$$= \frac{x}{5} - 4 - \frac{7}{x^2}$$

2. The total cost of x units of output of a firm is given by $C = \frac{2}{3}x + \frac{35}{2}$. Find the

(i) cost, when output is 4 units
(ii) average cost, when output is 10 units
(iii) marginal cost, when output is 3 units

Solution :

(i) Given $C = \frac{2}{3}x + \frac{35}{2}$

When $x = 4$ units,

$$C = \frac{2}{3}(4) + \frac{35}{2} = \frac{8}{3} + \frac{35}{2}$$

$$= \frac{16 + 105}{6} = \frac{121}{6}$$

(ii) Average cost when the output is 10 units.

Average cost (AC)

$$= \frac{C}{x} = \frac{\frac{2}{3}x + \frac{35}{2}}{x}$$

(iv) EOQ in years of supply

$$= \frac{q_0}{R} = \frac{200}{400} = 0.5$$

(v) Number of orders per year

$$= \frac{R}{q_0} = \frac{400}{200} = 2$$

(C) $R = 13,800, C_3 = 5, C_1 = \frac{10}{100} \times 0.20 = 0.02$

(i) EOQ in units

$$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2\times5\times13800}{0.02}} = \sqrt{\frac{138000}{0.02}}$$

$$= 2626.78 = 2627$$

(ii) Minimum Average Cost

$$= \sqrt{2C_1C_3R} = \sqrt{2\times.02\times5\times13800}$$

$$= \sqrt{138000\times.02} = 52.54$$

(iii) EOQ in rupees

$$= 2627 \times 0.20 = 525.40 \text{ Rupees}$$

(iv) EOQ in years of supply

$$= \frac{q_0}{R} = \frac{2627}{13800} = 0.19$$

(v) Number of orders per year

$$= \frac{R}{q_0} = \frac{13800}{2627} = 5.253$$

2. A dealer has to supply his customer with 400 units of a product per every week. The dealer gets the product from the manufacturer at a cost of ₹ 50 per unit. The cost of ordering from the manufacturers is ₹75 per order. The cost of holding inventory is 7.5 % per year of the product cost. Find (i) EOQ (ii) Total optimum cost.

Solution : R = 400 units per week

C_3 = ₹ 75

C_1 = 7.5% of 50 per year

$$= \frac{7.5}{100\times52}\times50 \text{ per week}$$

$$= \frac{0.75\times50}{52} = 0.072$$

(i) EOQ $= \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2\times400\times75}{0.072}}$

$$= \sqrt{\frac{60,000}{0.072}} = \sqrt{833,333.33}$$

= 912.87 units per order.

(ii) Total Optimum Cost = Purchasing cost + minimum annual cost

$$= 400 \times 50 + \sqrt{2RC_3C_1}$$

$$= 20,000 + \sqrt{2\times400\times75\times0.7211} \quad \left[\because C_1 = \frac{7.5}{5200}\times50\right]$$

= 20,000 + 65.75 = 20,065.75 per week

Exercise 6.4

1. If $z = (ax + b)(cy + d)$ then find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Solution : Given $Z = (ax + b)(cy + d)$

Differentiating partially with respect to 'x',

$$\frac{\partial z}{\partial x} = (cy + d)(a) = a(cy + d)$$

Differentiating partially with respect to 'y',

$$\frac{\partial z}{\partial y} = (ax + b)(c) = c(ax + b)$$

2. If $u = e^{xy}$, then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = u(x^2 + y^2)$.

Solution :

Given $u = e^{xy}$ ... (1)

Differentiating partially with respect to 'x' we get

$$\frac{\partial u}{\partial x} = e^{xy}[y]$$

Also, $\frac{\partial^2 u}{\partial x^2} = y.e^{xy}[y] = y^2e^{xy} = y^2u$ ... (2)

$u = e^{xy}$

Differentiating partially with respect to 'y', we get

$$\frac{\partial u}{\partial y} = e^{xy}[x]$$

and $\frac{\partial^2 u}{\partial y^2} = x.e^{xy}[x] = x^2e^{xy} = x^2u$ ... (3)

Adding (2) and (3) we get,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = y^2u + x^2u = u(x^2 + y^2)$$

Hence Proved.

3. Let $u = x \cos y + y \cos x$, Verify $\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y \partial x}$.

Solution :

Given $u = x \cos y + y \cos x$

Differentiating partially with respect to 'y' we get,

$$\frac{\partial u}{\partial y} = x(-\sin y) + \cos x$$

5 Marks

1. A company uses 48000 units of a raw material costing ₹ 2.5 per unit. Placing each order costs ₹45 and the carrying cost is 10.8 % per year of the average inventory. Find the EOQ, total number of orders per year and time between each order. Also verify that at EOQ carrying cost is equal to ordering cost. *[HY-2019]*

Solution : Here demand rate R = 48000

Inventory cost C1 = 10.8% of 2.5 = $\frac{10.8}{100} \times 2.5$

= 0.27

Ordering cost $C_3 = 45$

Economic order quantity q0 = $\sqrt{\frac{2C_3R}{C_1}}$

= $\sqrt{\frac{2 \times 45 \times 48000}{0.27}}$ = 4000 units

Number of orders per year = $\frac{R}{q_0} = \frac{48000}{4000} = 12$

Time between order $t_0 = \frac{q_0}{R} = \frac{1}{2} = 0.083$ year

At EOQ, carrying cost = $\frac{q_0}{2} \times C_1 = \frac{4000}{2} \times 0.27$

= ₹ 540

Ordering cost = $\frac{R}{q_0} \times C_3 = \frac{48000}{4000} \times 45$ = ₹ 540

So at EOQ carrying cost is equal to ordering cost.

ADDITIONAL PROBLEMS

2 MARKS

1. If $u = \log(x^2 + y^2)$, find $\frac{\partial^2 u}{\partial x^2}$. *[GMQP-2019]*

Solution :

$$u = \log(x^2 + y^2)$$

$$\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2}(2x) = \frac{2x}{x^2+y^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^2+y^2).2 - 2x.2x}{(x^2+y^2)^2} = \frac{2(y^2-x^2)}{(x^2+y^2)^2}$$

2. Find the minimum value of $e^x + e^{-x}$. *[GMQP-2019]*

Solution :

$$y = e^x + e^{-x}$$

$$\frac{dy}{dx} = e^x - e^{-x} = 0 \Rightarrow e^x = \frac{1}{e^x}$$

$$e^{2x} = 1 \Rightarrow x = 0$$

$$y(0) = e^0 + e^{-(0)} = 1 + 1 = 2$$

The minimum value is 2.

Fill in the Blanks.

1. The stationary points of the function $f(x) = x^3 - 3x^2 - 9x + 5$ is ____________

(a) – 1, 2 (b) – 1, 3 (c) 0, 2 (d) 1, 2

Hint:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 3x^2 - 6x - 9 = 0$$

$$3(x^2 - 2x - 3) = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3, -1$$

Ans: (b) – 1, 3

2. Euler's theorem is ____________

(a) $x\frac{\partial u}{dx} + y\frac{\partial u}{\partial y} = nu$ (b) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = nu$

(c) $u\frac{\partial u}{\partial x} y\frac{\partial u}{\partial y} = nu$ (d) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$

Ans: (d) $x\frac{\partial u}{dx} + y\frac{\partial u}{\partial y} = nu$

3. If $f'(c) = 0$ and $f''(c) < 0$ then at c f has a _____

(a) Local minimum (b) Local maximum

(c) Constant value (d) Infinities

Ans: (b) Local maximum

Find the odd one out of the following:

1. (a) $\frac{dy}{dx} = x + 5$ (b) $y' + y'' + 3y = 0$

(c) $x' + y'' + 3y = 0$ (d) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$

Ans: (d) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$

Hint: (d) is partial differential equation others are ordinary differential equation.

Match List I with List II

1.

List I	List II
(1) $f(x)$ is strictly increasing in (a, b)	(A) $f'(x) < 0$ for every $x \in (a, b)$
(2) $f(x)$ is strictly decreasing in (a, b)	(B) $f'(x) > 0$ for every $x \in (a, b)$
(3) $f(x)$ is said to be a constant	(C) f has a local minimum at c function.
(4) $f'(x) = 0$ and $f''(c) > 0$	(D) $f'(x) = 0$

The correct match is

(a) 1 – B 2 – A 3 – D 4 – C

(b) 1 – C 2 – B 3 – A 4 – D

(c) 1 – D 2 – A 3 – B 4 – D

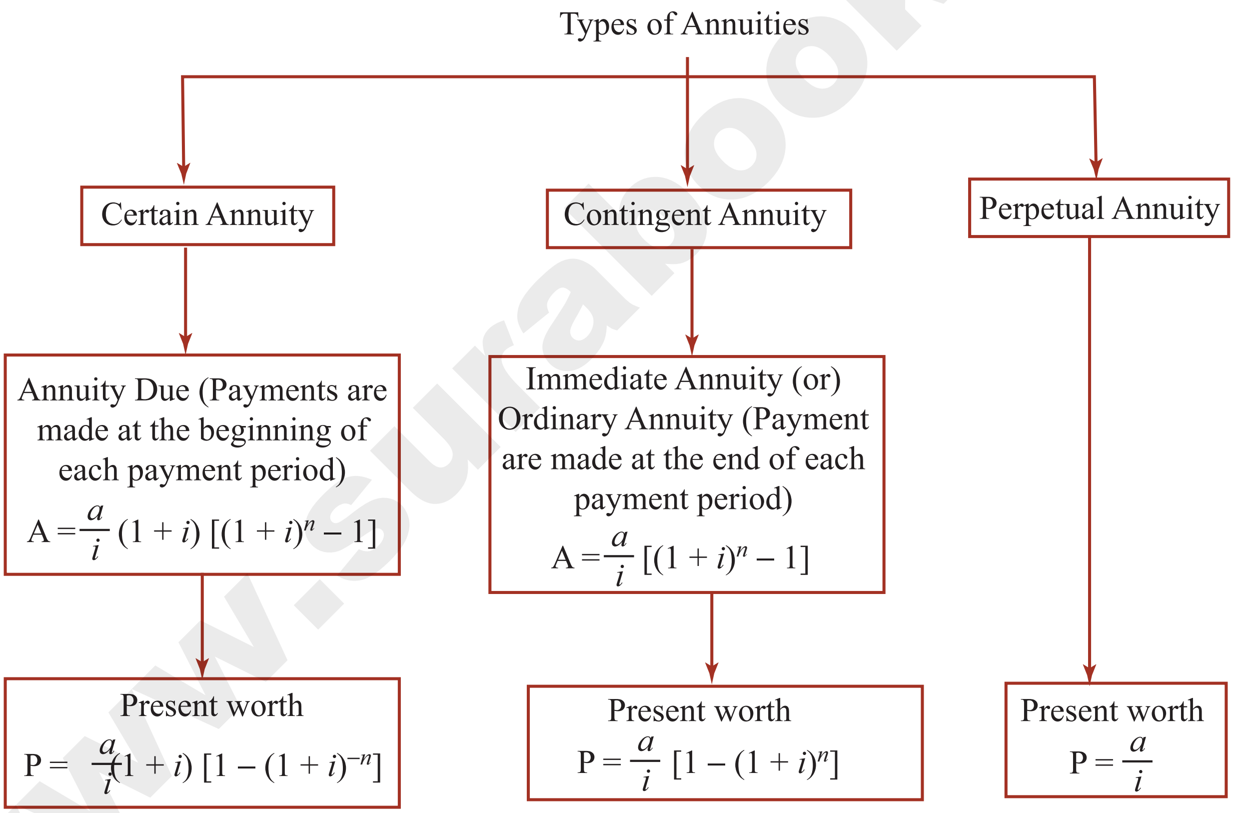
(d) 1 – B 2 – C 3 – D 4 – B

Ans: (a) 1 – B; 2 – A; 3 – D; 4 – C

07

FINANCIAL MATHEMATICS

POINTS TO REMEMBER



Let the dividend rate = x %

Dividend income from ordinary shares

$$= \text{Ordinary stock} \times \frac{x}{100}$$

$$50{,}000 = 2{,}50{,}000 \times \frac{x}{100}$$

$$\Rightarrow x = \frac{50{,}000}{2500} \Rightarrow x = 20\%$$

Govt. Exam Questions

1 Marks

1. When a stock is purchased, brokerage is ______ to the cost price. *[HY-2019]*
   (a) added (b) subtracted
   (c) multiplied (d) dividend

Ans: (b) subtracted

3 Marks

1. Find the amount of annuity of ₹ 5,000 Payable at the end of each year for 4 years of money is worth 10% compounded annually. $[(1.1)^4 = 1.4641]$ *[HY-2019]*

Solution : $a = 500$, $n = 4$

$$r = \frac{10}{100} = 0.1$$

$$A = \frac{a}{i}[(1+i)^n - 1] = \frac{5000}{0.1}\left[(1+0.1)^4 - 1\right]$$

$= 23200$

5 Marks

1. A person sells a 20% stock or face value ₹ 5,000 at a premium of 62%. With the money obtained he buys a 15% stock at a discount of 22%. What is the change in his income if the brokerage paid is 2%? *[MAR.-2019]*

Solution : Stock = ₹ 5000

FV = ₹ 100

Dividend Rate = 20%

Income from 20% stock

$$= \frac{\text{Stock}}{\text{FV}} \times \text{Rate} = \frac{5000}{100} \times 20 = ₹\ 1000 \quad \text{... (1)}$$

Selling price of one share = 100 + 62 – 2 = 160

$$\text{Sale proceeds} = \frac{\text{Stock}}{\text{FV}} \times \text{S.P. of one share}$$

$$= \frac{5000}{100} \times 160 = ₹\ 8000$$

Investment = ₹ 8000

Market Price = 100 – 22 + 2 = 80

Dividend Rate = 15%

$$\therefore \text{Income from 15\% stock} = \frac{\text{Investment}}{\text{Market Price}} \times \text{Rate}$$

$$= \frac{80{,}000}{80} \times 15 = ₹\ 1500 \quad \text{... (2)}$$

From (1) and (2),

Change in his income = 1500 – 1000 = ₹ 500

2. Kamal sold ₹ 9,000 worth 7% stock at 80 and invested the proceeds in 15% stock at 120. Find the change in income. *[HY-2019]*

Solution : 7% stock

Stock (₹)	Income (₹)
100	7
9000	?

$$\text{Income} = \frac{9000}{100} \times 7 = ₹600 \quad \text{....(1)}$$

Stock (₹)	Sale proceeds (₹)
100	80
9000	?

$$\text{Sale proceeds} = \frac{9000}{100} \times 80 = ₹\ 7{,}200$$

15% stock

Investment (₹)	Income (₹)
120	15
7,200	?

$$\text{Income} = \frac{7200}{120} \times 15 = ₹\ 900 \quad \text{....(2)}$$

comparing (1) and (2), we conclude that the change in income (increase) = ₹ 270

ADDITIONAL PROBLEMS

3 MARKS

1. **Define face value and Market value.** *[GMQP-2019]*

Solution :

Face Value : The original value of a share at which the company sells it to investors is called a face value or nominal value or par value. It is to be noted that the original value of share is printed on the share certificate.

Market value : The price at which the stock is bought or sold in the market is called the market value.

08 DESCRIPTIVE STATISTICS AND PROBABILITY

TEXTUAL QUESTIONS

EXERCISE 8.1

1. Find the first quartile and third quartile for the given observations.
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22

Solution :

Arranging the observations in ascending order, we get
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22

Here $n = 11$

First quartile $Q_1 = \text{size of } \dfrac{(n+1)^{th}}{4} \text{ value}$

$= \text{size of } \left(\dfrac{11+1}{4}\right)^{th} \text{ value}$

$= \text{size of } 3^{rd} \text{ value}$

$Q_1 = 6$

Third quartile $Q_3 = \text{Size of } \left(\dfrac{3(n+1)}{4}\right)^{th} \text{ value}$

$= \text{size of } \left(3\left(\dfrac{11+1}{4}\right)\right)^{th} \text{ value}$

$= \text{size of } 9^{th} \text{ value}$

$Q_3 = 18$

2. Find Q_1, Q_3, D_8 and P_{67} of the following data :

Size of Shares	4	4.5	5	5.5	6	6.5	7	7.5	8
Frequency	10	18	22	25	40	15	10	8	7

Solution :

Size of shares	Frequency	Cumulative Frequency
4	10	10
4.5	18	28
5	22	50
5.5	25	75
6	40	115
6.5	15	130
7	10	140
7.5	8	148
8	7	N = 155

$Q_1 = \text{size of } \left(\dfrac{N+1}{4}\right)^{th} \text{ value}$

$= \text{size of } \left(\dfrac{155+1}{4}\right)^{th} \text{ value}$

$= \text{size of } 39^{th} \text{ value} = 5$

$Q_3 = \text{size of } 3\left(\dfrac{N+1}{4}\right)^{th} \text{ value}$

$= \text{size of } 3(39)^{th} \text{ value}$

$= \text{size of } 117^{th} \text{ value} = 6.5$

$D_8 = \text{size of } 8\left(\dfrac{N+1}{10}\right)^{th} \text{ value}$

$= \text{size of } 8\left(\dfrac{156}{10}\right)^{th} \text{ value}$

$= \text{size of } 124.8^{th} \text{ value} = 6.5$

$P_{67} = \text{size of } \left(\dfrac{67(N+1)}{100}\right)^{th} \text{ value}$

$= \text{size of } 67\left(\dfrac{156}{100}\right)^{th} \text{ value}$

$= \text{size of } 104.52^{th} \text{ value}$

$P_{67} = 6$

17. **The two events A and B are mutually exclusive if**

(a) $P(A\cap B)=0$ (b) $P(A\cap B)=1$ *[JUNE-2019]*
(c) $P(A\cup B)=0$ (d) $P(A\cup B)=1$

Ans: (b) $P(A\cap B)=1$

18. **The events A and B are independent if**

(a) $P(A\cap B)=0$
(b) $P(A\cap B)=P(A)\times P(B)$
(c) $P(A\cap B)=P(A)+P(B)$
(d) $P(A\cup B)=P(A)\times P(B)$

Ans: (b) $P(A\cap B)=P(A)\times P(B)$

19. **If two events A and B are dependent then the conditional probability of P(B/A) is**

(a) P(A) P(B/A) (b) $\frac{P(A\cap B)}{P(B)}$
(c) $\frac{P(A\cap B)}{P(A)}$ (d) P(A) P(A/B)

Ans: (c) $\frac{P(A\cap B)}{P(A)}$

20. **The probability of drawing a spade from a pack of card is** *[MAR.-2019]*

(a) $\frac{1}{52}$ (b) $\frac{1}{13}$ (c) $\frac{4}{13}$ (d) $\frac{1}{4}$

Hint: P(Spade Card) = $\frac{13}{52}=\frac{1}{4}$ **Ans: (d) $\frac{1}{4}$**

21. **If the outcome of one event does not influence another event then the two events are**

(a) Mutually exclusive (b) Dependent
(c) Not disjoint (d) Independent

Ans: (d) Independent

22. **Let a sample space of an experiment be**

$S=\{E_1,E_2, ..., E_n\}$ then $\sum_{i=1}^{n} P(E_i)$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Ans: (b) 1

23. **The probability of obtaining an even prime number on each die, when a pair of dice is rolled is**

(a) $\frac{1}{36}$ (b) 0 (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

Hint: $n(s) = 36$

P(A) = P(Even Prime number on each die)

A = $\{(2, 2)\} \Rightarrow n(A) = 1$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$ **Ans: (a) 1/36**

24. **Probability of an impossible event is**

(a) 1 (b) 0 (c) 0.2 (d) 0.5

Ans: (b) 0

25. **Probability that at least one of the events A, B occur is**

(a) $P(A\cup B)$ (b) $(A\cap B)$
(c) P (A/B) (d) $(A\cup B)$

Ans: (a) $P(A\cup B)$

Miscellaneous Problems

1. **Find out the GM for the following**

Yield of Rice (tonnes)	No. of farms
7.5 - 10.5	5
10.5 - 13.5	9
13.5 - 16.5	19
16.5 - 19.5	23
19.5 - 22.5	7
22.5 - 25.5	4
25.5 - 28.5	1

Solution :

Yield of Rice (tonnes)	Mid x	f	$\log x$	$f\log x$
7.5 - 10.5	9	5	0.9542	4.771
10.5 - 13.5	12	9	1.0792	9.7128
13.5 - 16.5	15	19	1.1761	22.3459
16.5 - 19.5	18	23	1.2553	28.8719
19.5 - 22.5	21	7	1.3222	9.2554
22.5 - 25.5	24	4	1.3802	5.5208
25.5 - 28.5	27	1	1.4314	1.4314
		68		81.9092

$$GM = \text{Antilog}\left(\frac{\Sigma f\log x}{N}\right)$$

$$= \text{Antilog}\left(\frac{81.9092}{68}\right)$$

$$= \text{Antilog}(1.2045) = 16.02 \text{ tons}$$

10. A bag contains 6 black and 5 red balls. Two balls are drawn at random. What is the probability that they are of the same colour?

Solution :

Total number of balls $= 6 + 5 = 11$

$$n(S) = 11C_2 = \frac{11\times10}{2\times1} = 55$$

Let A be the event of getting a black ball annd B be the event of getting a red ball.

$$n(A) = 6C_2 = \frac{6\times5}{2\times1} = 15$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{55}$$

$$n(B) = 5C_2 = \frac{5\times4}{2\times1} = 10$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{10}{55}$$

Since the two balls must be of same colour $A\cap B = \phi$

$$\therefore P(A\cup B) = P(A) + P(B) = \frac{15}{55}+\frac{10}{55} = \frac{25}{55} = \frac{5}{11}$$

High Order Thinking Skills (HOTS)

1. Two identical boxes containing respectively 4 white and 3 red balls, 3 white and 7 red balls. A box is chosen at random and a ball is drawn from it. Find the probability that the ball is white. If the ball is white, what is the probability that it is from first box?

Solution :

Let A_1 and A_2 are two boxes.

$$P(A_1) = P(A_2) = 1/2$$

$$P(B/A_1) = P(\text{White ball from I box}) = 4/7$$

$$P(B/A_2) = P(\text{White ball from II box}) = 3/10$$

$$P(B) = P(\text{White ball from } A_1 \text{ or } A_2)$$

$$= P(A_1).\,P(B/A_1) + P(A_2).P(B/A_2)$$

$$= \frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{3}{10} = \frac{2}{7}+\frac{3}{20}$$

$$= \frac{40+21}{140} = \frac{61}{140}$$

By Baye's theorem,

$$P(B_1/A) = \frac{P(A_1)\cdot P\left(B/A_1\right)}{P(A_1)\cdot P\left(B/A_1\right)+P(A_2)\cdot P\left(B/A_2\right)}$$

$$= \frac{\frac{1}{2}\times\frac{4}{7}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{3}{10}} = \frac{\frac{2}{7}}{\frac{61}{140}} = \frac{2}{7}\times\frac{140}{61}$$

$$\therefore P(B_1/A) = \frac{40}{61}$$

2. Three events A, B and C have probabilities $\frac{2}{5}$, $\frac{1}{3}$ and $\frac{1}{2}$ respectively. Given that $P(A\cap C) = \frac{1}{5}$ and $P(B\cap C) = \frac{1}{4}$, find P(C/B) and $P\left(\bar{A}\cap\bar{C}\right)$.

Solution :

Given $P(A) = \frac{2}{5}$, $P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{2}$

$$P(A\cap C) = \frac{1}{5} \text{ and } P(B\cap C) = \frac{1}{4}$$

$$\therefore P(C/B) = \frac{P(C\cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4}$$

and $P\left(\bar{A}\cap\bar{C}\right) = P\left(\overline{A\cup C}\right) = 1 - P(A\cup C)$

$$= 1 - [P(A) + P(C) - P(A\cap C)]$$

$$= 1-\left(\frac{2}{5}+\frac{1}{2}-\frac{1}{5}\right) = 1-\left(\frac{4+5-2}{10}\right)$$

$$= 1-\frac{7}{10} = \frac{10-7}{10} = \frac{3}{10}$$

3. Two dice are thrown together. Let A be the event "getting 6 on the first die" and B be the event "getting 2 on the second die". Are the events A and B independent?

Solution : The events favourable to A are {(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)} and to B are {(1, 2) (2, 2) (3, 2) (4, 2) (5, 2) (6, 2)}

$$\Rightarrow n(A) = 6,\ n(B) = 6$$

$$\therefore A\cap B = \{ (6, 2) \} \Rightarrow \therefore n(A\cap B) = 1$$

$$P(A) = \frac{6}{36} = \frac{1}{6}$$

$$P(B) = \frac{6}{36} = \frac{1}{6} \text{ and } P(A\cap B) = \frac{1}{36} = \frac{1}{6}\times\frac{1}{6}$$

$$= P(A).\,P(B)$$

$$\therefore P(A\cap B) = P(A).\,P(B)$$

Hence, A and B are independent events.

4. In two successive throws of a pair of dice, determine the probability of getting a total of 8 each time.

Solution : Let A denote the event of getting a total of 8 in first throw and B be the event of getting a total of 8 in second throw.

09 CORRELATION AND REGRESSION ANALYSIS

TEXTUAL QUESTIONS

EXERCISE 9.1

1. Calculate the correlation co-efficient for the following data

X	5	10	5	11	12	4	3	2	7	1
Y	1	6	2	8	5	1	4	6	5	2

Solution :

x	y	x^2	y^2	xy
5	1	25	1	5
10	6	100	36	60
5	2	25	4	10
11	8	121	64	88
12	5	144	25	60
4	1	16	1	4
3	4	9	16	12
2	6	4	36	12
7	5	49	25	35
1	2	1	4	2
$\Sigma x = 60$	$\Sigma y = 40$	494	212	288

Correlation Co-efficient

$$r = \frac{N\Sigma xy - (\Sigma x)(\Sigma y)}{\sqrt{N\Sigma x^2 - (\Sigma x)^2} \times \sqrt{N\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{10(288) - (60)(40)}{\sqrt{10(494) - (60)^2} . \sqrt{10(212) - (40)^2}}$$

$$= \frac{2880 - 2400}{\sqrt{1340} \cdot \sqrt{520}}$$

$$= \frac{480}{(36.61)(22.80)} = \frac{480}{834.71}$$

$$r = 0.575$$

2. Find coefficient of correlation for the following:

Cost (Rs.)	14	19	24	21	26	22	15	20	19
Sales (Rs.)	31	36	48	37	50	45	33	41	39

Solution :

Cost (₹) X	Sales (₹) Y	x = X – 20	y = Y – 40	x^2	y^2	xy
14	31	– 6	– 9	36	81	54
19	36	– 1	– 4	1	16	4
24	48	4	8	16	64	32
21	37	1	– 3	1	9	– 3
26	50	6	10	36	100	60
22	45	2	5	4	25	10
15	33	– 5	– 7	25	49	35
20	41	0	1	0	1	0
19	39	– 1	– 1	1	1	1
ΣX = 180	ΣY = 360	$\Sigma x = 0$	$\Sigma y = 0$	$\Sigma x^2 = 120$	$\Sigma y^2 = 346$	$\Sigma xy = 193$

$$N = 9,\ \bar{X} = \frac{\Sigma X}{N} = \frac{180}{9} = 20;\ \bar{Y} = \frac{\Sigma Y}{N} = \frac{360}{9} = 40$$

Correlation Co-efficient

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \cdot \Sigma y^2}}$$

where $x = X - \bar{X}$ and $y = Y - \bar{Y}$

$$r = \frac{193}{\sqrt{(120)(346)}} = \frac{193}{\sqrt{41520}} = \frac{193}{203.76}$$

$$r = 0.947$$

$$\frac{4}{5}X + \frac{33}{5} = Y$$

$$Y = \frac{4}{5}X + \frac{33}{5}$$

∴ Regression Co-efficient of Y on X is

$$b_{yx} = \frac{4}{5}, \text{ which is less than one}$$

Let the regression line of X on Y be

$$20X - 9Y = 107$$

$$20X = 9Y + 107$$

$$X = \frac{9}{20}X + \frac{107}{20}$$

Regression Co-efficient of X on Y is

$$b_{xy} = \frac{9}{20} \text{ which is also less than one.}$$

Since the two regression co-efficients are positive, then the correlation co-efficient is also positive and it is given by

$$r = \sqrt{b_{yx} \cdot b_{xy}} = \sqrt{\frac{4}{5} \times \frac{9}{20}} = \sqrt{\frac{36}{100}}$$

$$= \frac{6}{10} = 0.6$$

$$\therefore r = 0.6$$

12. The equations of two lines of regression obtained in a correlation analysis are the following 2X = 8 – 3Y and 2Y = 5 – X . Obtain the value of the regression coefficients and correlation coefficient.

Solution :

Given regression lines are 2X = 8 – 3Y and 2Y = 5 – X

Let the regression line of X on Y be

$$2X = 8 - 3Y$$

$$\Rightarrow \quad X = \frac{8}{2} - \frac{3}{2}Y$$

$$\Rightarrow \quad X = -\frac{3}{2}Y + 4$$

∴ Regression Co-efficient of X on Y is

$$b_{xy} = -3/2 \text{ which is less than one.}$$

Let the regression line of Y on X be

$$2Y = 5 - X$$

$$Y = \frac{5}{2} - \frac{1}{2}X$$

$$Y = -\frac{1}{2}X + \frac{5}{2}$$

Regression Co-efficients of Y on X is

$$b_{yx} = -1/2 \text{ which is also less than one.}$$

Since both the regression Co-efficients are negative Correlation Co-efficient

$$r = -\sqrt{b_{xy} \cdot b_{yx}} = -\sqrt{\left(\tfrac{1}{2}\right)\left(\tfrac{3}{2}\right)}$$

$$= -\sqrt{\tfrac{3}{4}} = -\frac{1.732}{4}$$

$$r = -0.866$$

Exercise 9.3

Choose the correct answer:

1. Example for positive correlation is

(a) Income and expenditure
(b) Price and demand
(c) Repayment period and EMI
(d) Weight and Income

Ans: (a) Income and expenditure

2. If the values of two variables move in same direction then the correlation is said to be

(a) Negative (b) Positive
(c) Perfect positive (d) No correlation

Ans: (b) positive

3. If the values of two variables move in opposite direction then the correlation is said to be

(a) Negative (b) Positive
(c) Perfect positive (d) No correlation

Ans: (a) Negative

4. Correlation co-efficient lies between ***[MAR.-2019]***

(a) 0 to ∞ (b) –1 to +1 ***[JUNE-2019]***
(c) –1 to 0 (d) –1 to ∞

Ans: (b) –1 to +1

5. If *r*(X,Y) = 0 the variables X and Y are said to be

(a) Positive correlation ***[HY-2019]***
(b) Negative correlation
(c) No correlation
(d) Perfect positive correlation

Ans: (c) No correlation

6. The correlation coefficient from the following data N = 25, ΣX = 125, ΣY = 100, ΣX^2 = 650, ΣY^2 = 436, ΣXY = 520

(a) 0.667 (b) –0.006
(c) –0.667 (d) 0.70 **Ans: (a) 0.667**

ADDITIONAL PROBLEMS

1 MARK

1. The correlation coefficient from the following data $\bar{x} = \bar{y} = 0, \Sigma x_i y_i = 1.2, \sigma_x = 2, \sigma_y = 2$ and $n = 10$ *[GMQP-2019]*

(a) 0.4 (b) 0.3 (c) 0.2 (d) 0.1

Ans: (c) 0.2

3 MARKS

1. The following table shows the sales and advertisement expenditure of a form *[GMQP-2019]*

	Sales	Advertisement expenditure (₹in Crores)
Mean	40	6
SD	10	1.5

Coefficient of correlation $r = 0.9$. Estimate the likely sales for a proposed advertisement expenditure of Rs. 10 crores.

Solution :

Given $\bar{X} = 40, \ \bar{Y} = 6$

$\sigma_X = 10, \ \sigma_Y = 1.5$

and $r = 0.9$

Equation of line of regression x on y is

$$X - \bar{X} = r\frac{\sigma_x}{\sigma_y}(Y - \bar{Y})$$

$$X - 40 = (0.9)\frac{10}{1.5}(Y - 6)$$

$$X - 40 = 6Y - 36$$

$$X = 6Y + 4$$

5 MARKS

1. Find out the coefficient of correlation in the following case and interpet. *[GMQP-2019]*

Height of father (in inches)	65	66	67	67	68	69	71	73
Height of son (in inches)	67	68	64	68	72	70	69	70

Solution :

Let us consider Height of father (in inches) is represented as X and Height of son (in inches) is represented as Y.

X	dx = (X –67)	dx^2	Y	dy = (Y – 68)	dy^2	dxdy
65	–2	4	67	–1	1	2
66	–1	1	68	0	0	0
67	0	0	64	–4	16	0
67	0	0	68	0	0	0
68	1	1	72	4	16	4
69	2	4	70	2	4	4
71	4	16	69	1	1	4
73	6	36	70	2	4	12
ΣX = 546	Σdx = 10	Σdx^2 = 62	ΣY = 548	Σdy = 4	Σdy^2 = 42	Σdxdy = 26

$$r = \frac{N\sum dx\,dy - (\sum dx)(\sum dy)}{\sqrt{N\sum dx^2 - (\sum dx)^2} \times \sqrt{N\sum dy^2 - (\sum dy)^2}}$$

Where

$\Sigma dx = 10, \Sigma dx^2 = 62, \Sigma dy = 4, \Sigma dy^2 = 42$ and $\Sigma dxdy = 26$

$$r = \frac{(8\times 26)-(10\times 4)}{\sqrt{(8\times 62)-(10)^2} \times \sqrt{(8\times 42)-(4)^2}}$$

$$r = \frac{168}{\sqrt{396}\times\sqrt{320}}$$

$$r = \frac{168}{355.98} = 0.472$$

$$r = +0.472$$

Heights of fathers and their respective sons are positively correlated.

2. Calculate the two regression equations of X on Y and Y on X from the data given below, taking deviations from a actual means of X and Y. *[GMQP-2019]*

Price (Rs.)	10	12	13	12	16	15
Amount demanded	40	38	43	45	37	43

Estimate the likely demand when the price is Rs.20.

X	x = (X – 13)	x^2	Y	y = (Y – 41)	y^2	xy
10	–3	9	40	–1	1	3
12	–1	1	38	–3	9	3
13	0	0	43	2	4	0

10 OPERATIONS RESEARCH

TEXTUAL QUESTIONS

Exercise 10.1

1. A company produces two types of pens A and B. Pen A is of superior quality and pen B is of lower quality. Profits on pens A and B are Rs 5 and Rs 3 per pen respectively. Raw materials required for each pen A is twice as that of pen B. The supply of raw material is sufficient only for 1000 pens per day. Pen A requires a special clip and only 400 such clips are available per day. For pen B, only 700 clips are available per day . Formulate this problem as a linear programming problem.

Solution :

(i) **Variables:** Let x_1 and x_2 represents the types of pen A and B respectively.

(ii) **Objective function:** Profit on pens of type A = $5x_1$

and profit on pens of type B = $3x_2$

$\therefore$ Total profit = $5x_1 + 3x_2$

Let Z = $5x_1 + 3x_2$ which is the objective function.

Since the total profit is to be maximized, we have to maximize Z = $5x_1 + 3x_2$

(iii) **Constraints:**

Raw materials required is

$(2x_1 + x_2)$ which is sufficient only for 1000 pens

$\therefore 2x_1 + x_2 \leq 1000$

Pen A requires 400 clips $\Rightarrow x_1 = 400$

Pen B requires 700 clips $\Rightarrow x_2 = 700$

(iv) **Non-negative restrictions:** Since the number of pens connot be negative, we have $x_1 = 0, x_2 = 0$.

Thus, the mathematical formulation of the LPP is

Maximize Z = $5x_1 + 3x_2$

Subject to the constraints

$$2x_1 + x_2 = 1000,$$
$$x_1 = 400,$$
$$x_2 = 700$$
$$x_1, x_2 = 0$$

2. A company produces two types of products say type A and B. Profits on the two types of product are Rs.30/- and Rs.40/- per kg respectively. The data on resources required and availability of resources are given below. *[HY-2019]*

	Requirements		Capacity available per month
	Product A	Product B	
Raw material (kgs)	60	120	12000
Machining hours / piece	8	5	600
Assembling (man hours)	3	4	500

Formulate this problem as a linear programming problem to maximize the profit.

Solution :

(i) Variables: Let x_1 and x_2 denote the types of products A and B respectively.

(ii) Objective function:

Profit on Type A product = $30x_1$

Profit on Type B product = $40x_2$

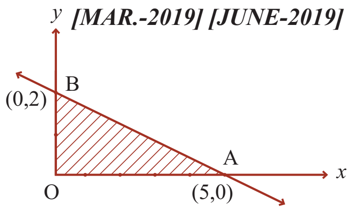
$\therefore$ Total profit = $30x_1 + 40x_2$

Let Z = $30x_1 + 40x_2$ which is the objective function.

Since the total profit is to be maximized, we have to maximize Z = $30x_1 + 40x_2$

8. The maximum value of the objective function $Z = 3x + 5y$ subject to the constraints, $x > 0, y > 0$ and $2x + 5y \leq 10$ is *[MAR.-2019] [JUNE-2019]*

(a) 6
(b) 15
(c) 25
(d) 31



Hint: $2x + 5y = 10$

x	0	5
y	2	0

Corner Points	$z = 3x + 5y$
O(0, 0)	0
A(5, 0)	15
B(0, 2)	10

∴ Maximum value is 15. **Ans: (b) 15**

9. The minimum value of the objective function $Z = x + 3y$ subject to the constraints, $2x + y \leq 20$, $x + 2y \leq 20$, $x > 0$ and $y > 0$ is

(a) 10 (b) 20 (c) 0 (d) 5

Hint:

$2x + y = 20$

x	0	10
y	20	0

$x + y = 20$

x	0	20
y	20	0

Corner Points	$z = x + 3y$
O(0, 0)	0
A(0, 20)	60
B(10, 0)	10
C(20, 0)	20

∴ Minimum value is 0 **Ans: (c) 0**

10. Which of the following is not correct?

(a) Objective that we aim to maximize or minimize
(b) Constraints that we need to specify
(c) Decision variables that we need to determine
(d) Decision variables are to be unrestricted.

Ans: (d) Decision variables are to be unrestricted.

11. In the context of network, which of the following is not correct

(a) A network is a graphical representation .
(b) A project network cannot have multiple initial and final nodes
(c) An arrow diagram is essentially a closed network
(d) An arrow representing an activity may not have a length and shape

Ans: (d) An arrow representing an activity may not have a length and shape

12. The objective of network analysis is to *[HY-2019]*

(a) Minimize total project cost
(b) Minimize total project duration
(c) Minimize production delays, interruption and conflicts
(d) All the above

Ans: (b) Minimize total project duration

13. Network problems have advantage in terms of project

(a) Scheduling (b) Planning
(c) Controlling (d) All the above

Ans: (d) All the above

14. In critical path analysis, the word CPM mean

(a) Critical path method
(b) Crash project management
(c) Critical project management
(d) Critical path management

Ans: (a) Critical path method

15. Given an L.P.P maximize $Z = 2x_1 + 3x_2$ subject to the constrains $x_1 + x_2 \leq 1$, $5x_1 + 5x_2 \geq 0$ and $x_1 \geq 0$, $x_2 \geq 0$ using graphical method, we observe

(a) No feasible solution
(b) unique optimum solution
(c) multiple optimum solution
(d) none of these

Hint: Since there is no common area between the lines $x_1 + x_2 \leq 1$ and $5x_1 + 5x_2 \geq 0$

Ans: (a) No feasible solution

Miscellaneous Problems

1. A firm manufactures two products A and B on which the profits earned per unit are Rs. 3 and Rs. 4 respectively. Each product is processed on two machines M_1 and M_2. Product A requires one minute of processing time on M_1 and two minutes on M_2, While B requires one minute on M_1 and one minute on M_2. Machine M_1 is available for not more than 7 hrs 30 minutes while M_2 is available for 10 hrs during any working day. Formulate this problem as a linear programming problem to maximize the profit.

Solution :

(i) Variables: Let x_1 represents the product A and x_2 represents the product B.

(ii) Objective function:
Profit earned from Product A = $3x_1$

(iii) Constraints:

$$x_1 + x_2 \le 20$$
$$360x_1 + 240x_2 \le 5760$$

(iv) Non-negative restrictions:

Since the number of fans and sewing machine cannot be negative, we have $x_1, x_2 \ge 0$

Hence, mathematical formulation of the LPP is

$$\text{Maximize } Z = 22x_1 + 18x_2$$

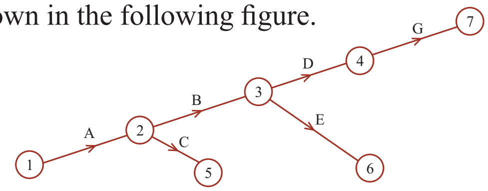
Subject to the constraints

$$x_1 + x_2 \le 20$$
$$360x_1 + 240x_2 \le 5760$$
$$\text{and } x_1, x_2 \ge 0$$

8. Construct a network diagram for the following situtation A < B; B < D, E; C < A and D < G.

Solution :

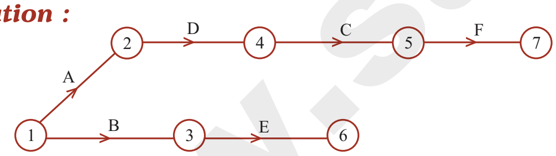
Using the precedence relationship and following the rules of network construction, the required network is shown in the following figure.



9. Draw a network diagram for the project whose activites and their predecessor relationships are given below

Activity	A	B	C	D	E	F
Predecessor activity	-	-	D	A	B	C

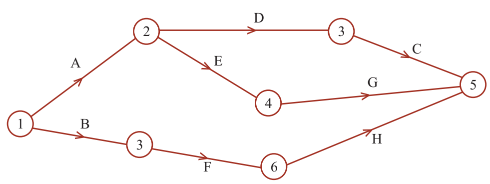
Solution :



10. Construct the network for the projects consisting of various activities and their precedence relationships are as given below: A, B can start simultaneously A < D, E; B < F; E < G, D < C, F < H.

Solution :

Using the precedence relationship and following the rules of network construction, the required network is shown in the following diagram.



Middle Order Thinking Skills (MOTS)

1. Solve the following LPP graphically.
Maximize Z = $6x_1 + 5x_2$ Subject to the constraints $3x_1 + 5x_2 \le 15$, $5x_1 + 2x_2 \le 10$ and $x_1, x_2 \ge 0$

Solution :

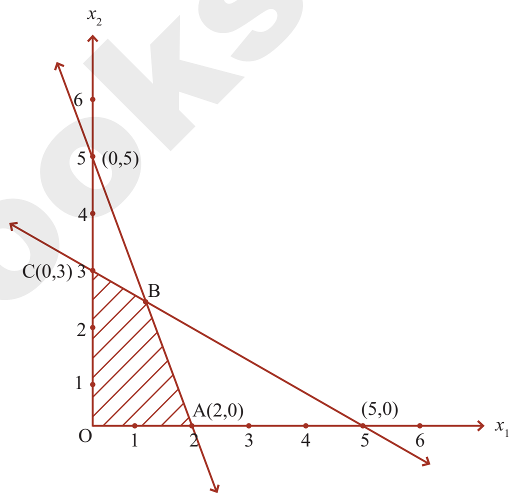
Since the decision variables x_1, x_2 are non-negative, the solution lies in the I quadrant. Consider the equations

$3x_1 + 5x_2 = 15$

x_1	0	5
x_2	3	0

$5x_1 + 2x_2 = 10$

x_1	0	2
x_2	5	0



The feasible region is OABC and its co-ordinates are O(0, 0) A(2, 0) C(0, 3) and B is the point of intersection of the lines

$$3x_1 + 5x_2 = 15 \quad \text{... (1)}$$

and

$$5x_1 + 2x_2 = 10 \quad \text{... (2)}$$

Verification of B:

$(1) \times 5 \Rightarrow \quad 15x_1 + 25x_2 = 75$

$\quad (-) \quad (-) \quad (-)$

$(2) \times 3 \Rightarrow \quad 15x_1 + 6x_2 = 30$

$$19x_2 = 45$$

$$\Rightarrow \quad x_2 = \frac{45}{19}$$

$$\text{From (1),} \quad 3x_1 + 5\left(\frac{45}{19}\right) = 15$$

$$\Rightarrow \quad 3x_1 = 15 - \frac{225}{19}$$

$$\Rightarrow \quad x_1 = \frac{20}{19}$$

$$\therefore \text{ B is } \left(\frac{20}{19}, \frac{45}{19}\right)$$

On 23.02.2019, Model Question Paper is released by the Govt.

11th STD.

GOVT. MODEL QUESTION PAPER - 2019
Business Mathematics & Statistics

Time : 3.00 Hours | Maximum Marks : 90

Section - I

Note : (i) Answer ***all*** the questions. **[20 × 1 = 20]**

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. The value of $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix}$ is

(a) 0 (b) 1
(c) $(a-b)(b-c)(c-a)$ (d) 2

2. The value of n, when $np_2 = 20$ is

(a) 3 (b) 6 (c) 5 (d) 4

3. The total number of 9 digit number which have all different digit is

(a) 10! (b) 9!
(c) 9 × 9! (d) 10 × 10!

4. Sum of the binomial co-efficients is

(a) 2^n (b) n^2 (c) $2n$ (d) $n+17$

5. If the lines $2x-3y-5=0$ and $3x-4y-7=0$ are the diameters of a circle, then its centre is

(a) (–1, 1) (b) (1,1)
(c) (1, –1) (d) (–1, –1)

6. The distance between directrix and focus of a parabola $y^2 = 4ax$ is

(a) a (b) $2a$ (c) $4a$ (d) $3a$

7. The value of sin 15º cos 15º is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{4}$

8. If tan A = $\frac{1}{2}$ and tan B = $\frac{1}{3}$ then tan (2A + B) is equal to

(a) 1 (b) 2 (c) 3 (d) 4

9. $\lim_{x\to 0} \frac{\log(1+x)^4}{x} =$

(a) 4 (b) 0 (c) 1 (d) –4

10. The graph of $y = e^x$ intersect the y - axis at

(a) (0,0) (b) (1,0)
(c) (0,1) (d) (1,1)

11. If $y = e^{2x}$ then $\frac{d^2y}{dx^2}$ at $x = 0$ is

(a) 4 (b) 9 (c) 2 (d) 0

12. Marginal revenue of the demand function $p = 20-3x$ is

(a) $20-6x$ (b) $20-3x$
(c) $20+6x$ (d) $20+3x$

13. If $q = 1000 + 8p_1 - p_2$ then, $\frac{\partial q}{\partial p_1}$ is

(a) –1 (b) 8
(c) 1000 (d) $1000-p_2$

14. The annual income on 500 shares of face value 100 at 15% is

(a) ₹7,500 (b) ₹5,000 (c) ₹8,000 (d) ₹8,500

15. When calculating the average growth of economy, the correct mean to use is?

(a) Weighted mean (b) Arithmetic mean
(c) Geometric mean (d) Harmonic mean

16. The median of 10, 14, 11, 9, 8, 12, 6 is

(a) 10 (b) 12 (c) 14 (d) 9

17. The correlation coefficient from the following data $\bar{x} = \bar{y} = 0, \Sigma x_i y_i = 1.2, \sigma_x = 3, \sigma_y = 2$ and $n = 10$

(a) 0.4 (b) 0.3 (c) 0.2 (d) 0.1

11th STD.

Register Number

COMMON QUARTERLY EXAMINATION - 2019

Business Mathematics & Statistics

Time Allowed : 2.30 Hours | Maximum Marks : 90

Instructions:

i. Check the question paper for fairness of printing. It there is any lack of fairness, inform the Hall Supervisor immediately.

ii. Use **Blue** or **Black** ink to write and underline and **pencil** to draw diagrams.

Part – I

Note : (i) Answer ***all*** the questions.

[20 × 1 = 20]

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. The inverse matrix of $\begin{pmatrix} \frac{4}{5} & \frac{-5}{12} \\ \frac{-2}{5} & \frac{1}{2} \end{pmatrix}$ is

(a) $\frac{30}{7}\begin{pmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$ (b) $\frac{7}{30}\begin{pmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{2}{5} & \frac{4}{5} \end{pmatrix}$

(c) $\frac{7}{30}\begin{pmatrix} \frac{1}{2} & \frac{5}{12} \\ \frac{-2}{5} & \frac{1}{5} \end{pmatrix}$ (d) $\frac{30}{7}\begin{pmatrix} \frac{1}{2} & \frac{-5}{12} \\ \frac{-2}{5} & \frac{4}{5} \end{pmatrix}$

2. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then $|2A|$ is equal to

(a) $4\cos 2\theta$ (b) 2 (c) 4 (d) 1

3. If n is the order of the matrix A, then $|\text{adj}A| =$

(a) $|A|$ (b) $|A|^{n-1}$ (c) $|A|^{n+1}$ (d) $|A|^n$

4. The inventor of input-output analysis is

(a) Sir Francis Galton (b) Fisher
(c) Arthur Caylay
(d) Prof.Wassily W.Leontief.

5. For all positive integer n, $nc_1 + nc_2 + nc_3 + \ldots\ldots + nc_n =$

(a) 2^n (b) n^2 (c) $2^n - 1$ (d) $n^2 - 1$

6. If $\frac{kx}{(x+4)(2x-1)} = \frac{4}{x+4} + \frac{1}{2x-1}$ then k is equal to

(a) 9 (b) 11 (c) 5 (d) 7

7. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of there parallel lines is

(a) 12 (b) 9 (c) 6 (d) 8

8. The number of diagonals that can be drawn by joining the angular points of octagon is

(a) 22 (b) 20 (c) 24 (d) 26

9. If m_1 and m_2 are the slopes of the pair of lines given by $ax^2 + 2hxy + by^2 = 0$, then the value of $m_1 + m_2$ is

(a) $\frac{2h}{b}$ (b) $-\frac{2h}{a}$ (c) $\frac{-2h}{b}$ (d) $\frac{2h}{a}$

10. The equation of the circle with centre on the x axis and passing through the origin is

(a) $x^2 - 2ax + y^2 = 0$
(b) $y^2 - 2ay + x^2 = 0$
(c) $x^2 + y^2 = a^2$
(d) $x^2 - 2ay + y^2 = 0$

11. The double ordinate passing through the focus is

(a) latus retum (b) focal chord
(c) directrix (d) axis

12. Who is the father of Analytical geometry?

(a) Seki kowa (b) G.W.Leibnitz
(c) Rene Descartes (d) Sir Issac Newton

13. The value of sin 15° is

(a) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

11th STD.

Register Number

COMMON HALF-YEARLY EXAMINATION - 2019

BUSINESS MATHEMATICS & STATISTICS

TIME ALLOWED : 3.00 Hours MAXIMUM MARKS : 90

Instructions:

i. Check the question paper for fairness of printing. It there is any lack of fairness, inform the Hall Supervisor immediately.

ii. Use **Blue** or **Black** ink to write and underline and **pencil** to draw diagrams.

PART – I

Note : (i) Answer ***all*** the questions. **[20 × 1 = 20]**

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $ad - bc \neq 0$ then A^{-1} is

(a) $\frac{1}{ad-bc}\begin{pmatrix} d & b \\ -c & a \end{pmatrix}$ (b) $\frac{1}{ad-bc}\begin{pmatrix} d & b \\ c & a \end{pmatrix}$

(c) $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ (d) $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ c & a \end{pmatrix}$

2. If n is the order of the matrix A, then $|\text{adj}A|$ =

(a) $|A|$ (b) $|A|^{n-1}$ (c) $|A|^{n+1}$ (d) $|A|^n$

3. Sum of the binomial co-efficients is

(a) $2n$ (b) $n2$

(c) $2n$ (d) $n + 17$

4. The no. of permutations of English words A,E,I,O,U taking two at a time:

(a) 20 (b) 120 (c) 5 (d) 2

5. The double ordinate passing through the focus is

(a) focal chord b) latus rectum

(c) directrix (d) axis

6. In a pair of straight lines, if the lines are perpendicular, then:

(a) $ab = 0$ b) $a - b = 0$

(c) $a + b = 0$ (d) none of these

7. The value of sin (– 420°) is

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

8. The domain of $\sin^{-1}x$:

(a) [–1, 1] (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(c) (–∞, ∞) (d) [0, π]

9. The graph of $y = e^x$ intersect the y - axis at

(a) (0,0) (b) (1,0) (c) (0,1) (d) (1,1)

10. If $f(x) = x^n$, $f'(1) = 5$. Then the value of n is:

(a) 5 (b) 10 (c) 25 (d) 1

11. For the cost function C = $\frac{1}{25}e^{5x}$, the marginal cost is

(a) $\frac{1}{25}$ (b) $\frac{1}{5}e^{5x}$

(c) $\frac{1}{125}e^{5x}$ (d) $25e^{5x}$

12. The graph of demand function lies in:

(a) I Quadrant (b) II Quadrant

(c) III Quadrant (d) IV Quadrant

13. An annuity in which payments are made at the beginning of each payment period is called

(a) Annuity due

(b) An immediate annuity

(c) perpetual annuity (d) none of these

14. When a stock is purchased, brokerage is _____ to the cost price.

(a) added (b) subtracted

(c) multiplied (d) dividend

15. Harmonic mean is better than other means if the data are for

(a) Speed or rates.

(b) Heights or lengths.

(c) Binary values like 0 and 1.

(d) Ratios or proportions.