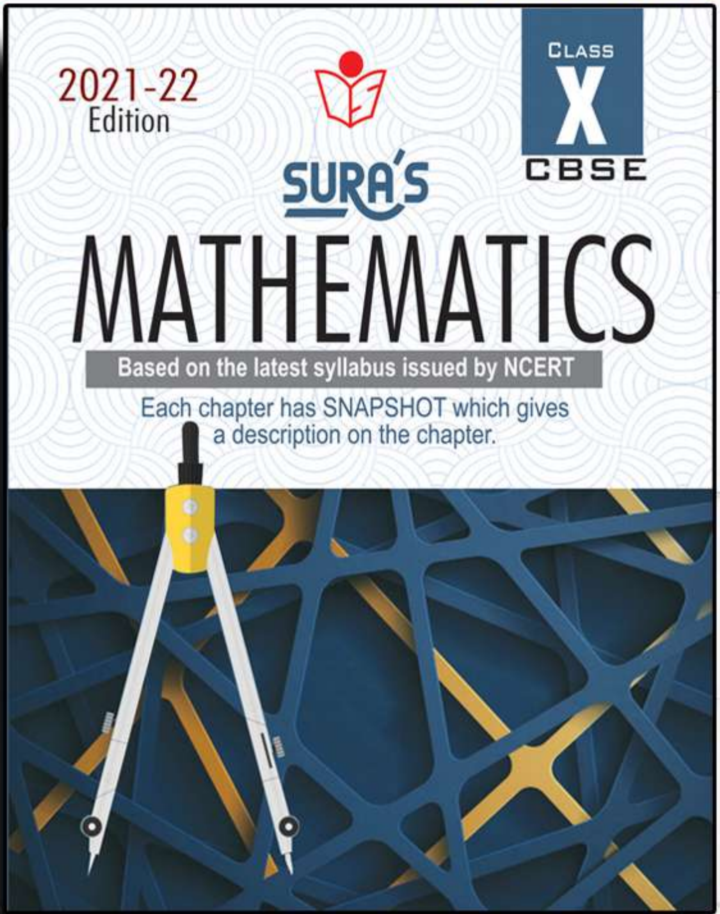
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SURA'S
CLASS
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MATHEMATICS
Based on the latest syllabus issued by NCERT
Each chapter has SNAPSHOT which gives
a description on the chapter.

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MATHEMATICS

Class - X

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Salient Features

- Each chapter has SNAPSHOT which gives a description on the chapter.
- NCERT Examples and Exercises are solved in step by step.
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PREFACE

Respected Principals, Correspondents, Head Masters / Head Mistresses, Teachers,

From the bottom of our heart, we at SURA Publications sincerely thank you for the support and patronage that you have extended to us for more than a decade.

It is in our sincerest effort we take the pride of releasing **SURA's Guide for Mathematics for Class X**, based on the latest syllabus issued by CBSE. This guide has been authored and edited by qualified teachers having teaching experience for over a decade in their respective subject fields. This Guide has been reviewed by reputed Professors who are currently serving as Head of the Department in esteemed Universities and Colleges.

With due respect to Teachers, I would like to mention that this guide will serve as a teaching companion to qualified teachers. Also, this guide will be an excellent learning companion to students with exhaustive exercises including NCERT in-text and bookback questions and Exemplar questions.

In complete cognizance of the dedicated role of Teachers, I completely believe that our students will learn the subject effectively with this guide and prove their excellence in Board Examinations.

I once again sincerely thank the Teachers, Parents and Students for supporting and valuing our efforts.

God Bless all.

Subash Raj, B.E., M.S.
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All the Best.

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DELETED PORTIONS IN MATHS

CHAPTER	TOPICS REMOVED
UNIT I - NUMBER SYSTEMS	
REAL NUMBERS	✦ Euclid's division lemma
UNIT II - ALGEBRA	
POLYNOMIALS	✦ Statement and simple problems on division algorithm for polynomials with real coefficients.
PAIR OF LINEAR EQUATIONS	
IN TWO VARIABLES	✦ Cross multiplication method
	✦ Situational problems based on equations reducible to quadratic equations
ARITHMETIC PROGRESSIONS	✦ Application in solving daily life problems based on sum to *n* terms
UNIT III - COORDINATE GEOMETRY	
COORDINATE GEOMETRY	✦ Area of a triangle.
UNIT IV - GEOMETRY	
TRIANGLES	Proof of the following theorems are deleted
	✦ The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
	✦ In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angle opposite to the first side is a right angle.
CIRCLES	No deletion
CONSTRUCTIONS	✦ Construction of a triangle similar to a given triangle.
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SURFACE AREAS AND VOLUMES	✦ Frustum of a cone.
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	✦ Cumulative Frequency graph
PROBABILITY	No deletion

Unit-wise Mark Distribution

Units	Unit Name	Marks
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II	ALGEBRA	20
III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
	Total	80

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Chapter 1

Real Numbers

Mind Map

REAL NUMBERS

Theorems

1. Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.
2. $\sqrt{2}, \sqrt{3}$ are irrational
3. Let x be a rational number whose x decimal expansion terminates. Then x can be expressed in the form, $\frac{p}{q}$ where p & q are coprime, the prime factorisation of q is of the form $2^n 5^m$ where, are n, m are non-negative integers.
4. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$ where, n, m are non-negative integers. Then, x has decimal expansion which terminates.
5. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$ where, n, m are non-negative integers. Then, x has decimal expansion which is non-terminating repeating.

Euclid's

Division Lemma

Given positive integers a and b, there exist unique integers q and r satisfying $a = bq + r$; $0 \leq r < b$

Division Algorithm

Steps to obtain the HCF of two positive integers, say c and d, with $c > d$.

Step 1: Apply Euclid's Division Lemma, to c & d, $c = dq + r$.

Step 2: If r = zero, d is the HCF of c and d. If $r \neq 0$, apply Euclid's Division lemma to d and r.

Step 3: Continue the process till the remainder is zero.

Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur

Composite Number $x = P_1 \times P_2 \times P_3 ... \times P_n$ where P_1 P_2 ... P_n are prime numbers.

Prime Factorization Method

For any two positive integers, a and b

HCF $(a, b) \times$ LCM $(a, b) = a \times b$

For Example

$f(x) = 3x^2y$

$g(x) = 6xy$

HCF = $3xy$

LCM = $6x^2y^2$

Snapshot

1.1 Euclid's division lemma tells us above divisibility of integers.

- It states that any positive integer can be divided by any other positive integer is in such a way that is leaves a remainder r that is smaller than a.
- It is a usual long division process.
- It provides us a step-wise procedure to compute the HCF of two positive integers.
- This step-wise procedure is known as Euclid's algorithm.
- A lemma is a proven statement used for proving another statement.
- An algorithm is a series of well-defined steps which gives a procedure for solving a type of problem.
- Although, Euclid's division algorithm is stated for any positive integers, it can be extended for all Integers except zero.
- Euclid's division lemma and algorithm has many applications related to finding the properties of numbers.

Example 1 :

Use Euclid's algorithm to find the HCF of 4052 and 12576.

Sol. HCF of 4052 and 12576

Since, 12576 > 4052

By Euclid's division Lemma

(i.e) $a = bq + r$ where $0 \leq r < b$

(i) $a = 12576$ and $b = 4052$

$\Rightarrow$ 12576 = 4052 × 3 + 420 (here, 4052 goes into 12576 thrice and leaves a remainder of 420)

(ii) $r = 420 \neq 0$, apply division lemma to 4052 and 420

Since 4052 > 420

$\Rightarrow$ $a = 4052$ and $b = 420$

Now, 4052 = 420 × 9 + 272 (here 420 goes into 4052 nine times and leaves a remainder of 272)

(iii) $r = 272 \neq 0$, apply division lemma to 272 and 420.

Since 420 > 272, $a = 400$ and $b = 272$

$\Rightarrow$ 420 = 272 ×1 + 148 (here 272 goes into 420 once and leaves a remainder of 148).

(iv) $r = 148 \neq 0$, apply division lemma to 272 and 148.

Since, 272 > 148

$a = 272$ and $b = 148$

$\Rightarrow$ 272 = 148 × 1 + 124 (here 148 goes into 272 once and leaves a remainder of 124).

(v) $r = 124 \neq 0$, apply division lemma to 148 and 124.

Since, 148 > 124,

$a = 148$ and $b = 124$

$\Rightarrow$ 148 = 124 × 1 + 24 (here 124 goes into 148 once and leaves a remainder of 24).

(vi) $r = 24 \neq 0$, apply division lemma to 124 and 24.

Since, 124 > 24,

$a = 124$ and $b = 24$

$\Rightarrow$ $124 = 24 \times 5 + 4$ (here 24 goes into 124 five times and leaves a remainder of 4).

(vii) $r = 4 \neq 0$, apply division lemma to 24 and 4.

Since, $24 > 4$,

$a = 24$ and $b = 4$

$\Rightarrow$ $24 = 4 \times 6 + 0$ (here 4 goes into 24 six times and leaves a remainder of 0).

Since, the remainder has become zero now, the procedure stops.

Thus, the divisor at the last step is 4.

Hence, 4 is the HCF of 12576 and 4052.

Example 2 :

Show that every positive even integer is of the form 2q, and that every positive odd integer is of the form 2q + 1, where q is some integer.

Sol. We know that, Euclid's division lemma is,

$a = bq + r, 0 \leq r < b$

Let a be any positive integer and $b = 2$

By division lemma,

$a = 2q + r, 0 \leq r < 2$ thus r can either b 0 or 1

When $r = 0$

$\Rightarrow$ $a = 2q + 0 = 2q$

$\Rightarrow$ a is positive even integer.

When $r = 1$

$\Rightarrow$ $a = 2q + 1$

$\Rightarrow$ a is positive odd integer.

Example 3 :

Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.

Sol. We know that, Euclid's division lemma is,

$a = bq + r, 0 \leq r < b$

Let 'a' be any positive odd integer and $b = 4$

By Division lemma

$a = 4q + r, 0 \leq r < 4$

thus, here r takes the values 0, 1, 2 and 3.

When $r = 0$ or 2

$\Rightarrow$ $a = 4q$ or $a = 4q + 2$

$\Rightarrow$ $a = 4q$ and $4q + 2$ both are divisible by two.

As a is odd (given), r cannot be 0 or 2.

Hence $a = 4q + 1$ and $a = 4q + 3$ are the forms of every positive odd integer.

Example 4 :

A sweet seller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of that can be placed in each stack for this purpose?

Sol. Maximum number of barfis in each stack = HCF (420, 130)

By Euclid's division lemma (i.e)

$a = bq + r$ and $0 \leq r < b$

(i) Since $420 > 130$,

$a = 420$ and $b = 130$

$\Rightarrow$ $420 = 130 \times 3 + 30$

(here 130 goes into 420 thrice and leaves a remainder of 30)

ii) Since $r = 30 \neq 0$, apply division lemma to 130 and 30.

$a = 130$ and

$b = 30$ $(130 > 30)$

$\Rightarrow$ $130 = 30 \times 4 + 10$

(here 30 goes into 130 four times and leaves a remainder of 10)

iii) $r = 10 \neq 0$, apply division lemma to 30 and 10.

Since $30 > 10$, $a = 30$ and $b = 10$

$\Rightarrow$ $30 = 10 \times 3 + 0$

(here 10 goes into 30 thrice and leaves a remainder of 0)

Since $r = 0 \Rightarrow$ HCF of 420 and 130 is 10

Hence, the number of barfis in each stack would be 10.

Solved NCERT Exercise Questions

Exercise 1.1

1. ***Use Euclid's division algorithm to find the HCF of :***

(i) 135 and 225 *(ii) 196 and 38220*

(iii) 867 and 255

> **Concept:** In this question, we have to find HCF of the given integers using Euclid's division lemma. According to it, the HCF of two positive integers a and b is the largest positive integer that divides both a and b. Now, to obtain the HCF of two integers say a and b with $a > b$, we need to work out the following steps:
>
> **Step 1 :** Apply Euclid's division lemma to a and b to find out the whole numbers q and r so that,
>
> $a = bq + r, 0 \leq r < b$
>
> **Step 2:** If $r = 0$, b is the HCF of a and b. If $r \neq 0$, then apply division lemma to b and r.
>
> **Step 3:** Continue the steps till the remainder is zero. At this step the divisor will be the required HCF.

Sol. **(i)** 135 and 225

Since 225 > 135, we apply the division lemma to 225 and 135 to obtain

$$225 = 135 \times 1 + 90$$

Since remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to obtain

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

$$90 = 2 \times 45 + 0$$

Since the remainder is zero, the process stops.
Since the divisor at this stage is 45,
Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

$$38220 = 196 \times 195 + 0$$

Since the remainder is zero, the process stops.
Since the divisor at this stage is 196,
Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Since 867 > 255, we apply the division lemma to 867 and 255 to obtain

$$867 = 255 \times 3 + 102$$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$$255 = 102 \times 2 + 51$$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$$102 = 51 \times 2 + 0$$

Since the remainder is zero, the process stops.

The divisor at this stage is 51,

Therefore, HCF of 867 and 255 is 51.

2. ***Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.***

> **Concept:** In this question, consider any positive integer a which it is in the form of $6q + r$ where q is some integer. Then, $0 \leq r < 6$ i.e., r = 0 or 1 or 2 or 3 or 4 or 5 as r is smaller than 6. Now, Euclid's division lemma possible values for a can be $6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$.

Sol. Let a be any positive integer and $b = 6$.

Then, by Euclid's algorithm, $a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, Where k_1 is a positive integer

$$6q + 3 = (6q + 2) + 1$$
$$= 2(3q + 1) + 1$$
$$= 2k_2 + 1,$$

where k_2 is an integer

$$6q + 5 = (6q + 4) + 1$$

$= 2(3q + 2) + 1$

$= 2k_3 + 1,$

where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$

3. ***An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?***

> **Concept:** In this question, see to that the army band members and army contingent members have to march in the same number of columns and the number of columns must be the maximum. Now, taking HCF of two numbers is the highest number that divides both the numbers. Hence, we have to find the HCF of the members in the army band and the army contingent.

Sol. HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

4. ***Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.***

> **Concept:** In this question, consider that there is a positive integer a. Using Euclid's lemma, there exist unique integers q and r, such that
>
> $a = bq + r, 0 \leq r < b$
>
> Now, keeping the value of b = 3, then $0 \leq r < 3$ i.e., $r = 0$ or 1 or 2 as r is smaller than 3. Now, Euclid's division lemma possible values for a can be $3q$ or $3q + 1$ or $3q + 2$. Then, find out the square of all the possible values of a. If q is any positive integer then its square say 'm' will also be a positive integer. It should be noted that the square of all the positive integers is either in the form of $3m$ or $3m + 1$ for some integer m.

Sol. Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$ And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$ Or,

$a^2 = (3q)^2$ or $(3q + 1)^2$ or $(3q + 2)^2$

$a^2 = 9q^2$ or $9q^2 + 6q + 1$ or $9q^2 + 12q + 4$

$= 3 \times (3q^2)$ or $3(3q^2 + 2q) + 1$ or $3(3q^2 + 4q + 1) + 1$

$= 3k_1$ or $3k_2 + 1$ or $3k_3 + 1$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

5. ***Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.***

> **Concept:** In this question, consider that there is a positive integer a. Using Euclid's lemma, there exist unique integers q and r, such that
>
> $a = bq + r, 0 \leq r < b$
>
> Now, keeping the value of b = 3, then $0 \leq r < 3$ i.e., $r = 0$ or 1 or 2 as r is smaller than 3. Now, Euclid's division lemma possible values for a can be $3q$ or $3q + 1$ or $3q + 2$. Then, find out the cube of all the possible values of a. If q is any positive integer then its cube say 'm' will also be a positive integer. It should be noted that the cube of all the positive integers is either in the form of $9m$ or $9m + 1$ for some integer m.

Sol. Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$\therefore a = 3q$ or $3q + 1$ or $3q + 2$

Therefore, every number can be represented as these three forms.

There are three cases.

Case I : When $a = 3q$,

$a^3 = (3q)^3 = 27q^3$

$= 9(3q^3) = 9m$

Where m is an integer such that $m = 3q^3$

Case II : When $a = 3q + 1$,

$a^3 = (3q+1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case III : When $a = 3q + 2$,

$a^3 = (3q+2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

Snapshot

1.2 The Fundamental Theorem of Arithmetic.

- The fundamental theorem of Arithmetic states that every composite number can be factorised as a product of primes. It says that any given composite number can be factorised as a product of prime numbers in a unique way.
- The prime factorisation of a natural number is unique except for the order of its factors. This method is called prime factorisation method. The fundamental theorem of arithmetic has many applications.

Example 5 :

Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4n ends with the digit zero.

Sol. Consider 4^n numbers where n is a natural number.

For a number to end with zero, it must be divisible by 5.

For unit's digit to be 0, 4^n should have 2 and 5 as its prime factors.

Since, $4^n = (2 \times 2)^n = (2^2)^n = 2^{(2\times n)} = 2^{2n}$

Thus, it would $4^1, 4^2, 4^3$ .... 4^n (n = 1, 2, 3, ...)

As it does not have 5 as one of its prime factor.

Therefore, 4^n will not end with digit 0

Example 6 :

Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Sol. Prime factorization of 6 and 20,

2	6
3	3
	1

2	20
2	10
5	5
	1

Thus, $6 = 2 \times 3$ and $20 = 2 \times 2 \times 5 = 2^2 \times 5^1$

Since, LCM = Product of each prime factor with highest powers.

$\Rightarrow$ LCM of 6 and 20 $= 2^2 \times 3^1 \times 5^1$

$= 4 \times 3 \times 5 = 60$

HCF = Product of smallest power of each common prime factor.

$\Rightarrow$ HCF of 6 and 20 $= 2^1 = 2$.

Example 7 :

Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

Sol. Prime factorisation of 96 and 404,

2	96
2	48
2	24
2	12
2	6
3	3
	1

2	404
2	202
101	101
	1

Thus, $96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

$404 = 2 \times 2 \times 101$

$= 2^2 \times 101$

$\Rightarrow \text{LCM}(96, 404) = \dfrac{96 \times 404}{\text{HCF}(46, 404)} = \dfrac{96 \times 404}{4}$

$= 9696$

HCF (96, 404) = $2^2 = 4$

Example 8 :

Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

Sol. Prime factorisation of 6, 72 and 120,

2	6
3	3
	1

2	72
2	36
2	18
3	9
3	3
	1

2	120
2	60
2	30
3	15
5	5
	1

Thus, $6 = 2 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

$= 2^3 \times 3^2$

$120 = 2 \times 2 \times 2 \times 3 \times 5$

$= 2^3 \times 3 \times 5$

HCF (6, 72, 120) $= 2^1 \times 3^1 = 2 \times 3 = 6$

LCM (6, 72, 120) $= 2^3 \times 3^2 \times 5^1$

$= 8 \times 9 \times 5 = 360$

Exercise 1.2

1. ***Express each number as a product of its prime factors:***

(i) **140** **(ii)** **156** **(iii)** **3825**
(iv) **5005** **(v)** **7429**

> **Concept:** Using prime factorisation method, find the prime factors of the given numbers. Then, multiply the obtained prime numbers to get the product of the prime numbers.

Sol. **(i)** $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

2	140
2	70
5	35
7	7
	1

(ii) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

2	156
2	78
3	39
13	13
	1

(iii) $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

3	3825
3	1275
5	425
5	85
17	17
	1

(iv) $5005 = 5 \times 7 \times 11 \times 13$

5	5005
7	1001
11	143
	13

(v) $7429 = 17 \times 19 \times 23$

17	5005
19	1001
11	143
23	23
	1

2. ***Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.***

(i) *26 and 91* *(ii)* *510 and 92*
(iii) *336 and 54*

Concept:

- Find the LCM and HCF of the given pairs of the integers.
- Find the prime factors of the given pairs of integers.
- Find the product of smallest power of each common factor in the numbers i.e., HCF..
- Find the product of greatest power of each prime factor in the number i.e., LCM.
- Lastly, find the product of LCM and HCF and also the two given numbers.
- Verify LCM × HCF = product of the two numbers.
- If LHS is equal to the RHS then it will be verified.

Sol. **(i)** 26 and 91

$26 = 2 \times 13$

$91 = 7 \times 13$

2	26
	13

2	91
	13

HCF = 13

LCM = $2 \times 7 \times 13 = 182$

Product of the two numbers = $26 \times 91 = 2366$

HCF × LCM = $13 \times 182 = 2366$

Hence, product of two numbers = HCF × LCM

(ii) 510 and 92

2	510
3	255
5	85
	17

2	92
2	46
	23

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

HCF = 2

LCM = $2 \times 2 \times 3 \times 5 \times 17 \times 23$

= 23460

Product of the two numbers = $510 \times 92 = 46920$

HCF × LCM = $2 \times 23460 = 46920$

Hence, product of two numbers = HCF × LCM

(iii) 336 and 54

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$336 = 2^4 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$54 = 2 \times 3^3$

HCF = $2 \times 3 = 6$

LCM = $2^4 \times 3^3 \times 7$

= 3024

2	336
2	168
2	84
2	42
3	21
	7

2	54
3	27
3	9
	3

Product of the two numbers

= $336 \times 54 = 18144$

HCF × LCM = $6 \times 3024 = 18144$

Hence, product of two numbers = HCF × LCM

3. ***Find the LCM and HCF of the following integers by applying the prime factorisation method.***

(i) **12, 15 and 21** **(ii)** **17, 23 and 29**

(iii) **8, 9 and 25**

Concept:

- Find the prime factors of the given pairs of integers.
- Find the product of smallest power of each common factor in the numbers i.e., HCF.
- Find the product of greatest power of each prime factor in the number i.e., LCM.

Sol. **(i)** 12, 15 and 21

2	12
2	6
	3

3	15
	5

3	21
2	7

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

HCF = 3

LCM = $2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

HCF = 1

LCM = $17 \times 23 \times 29 = 11339$

(iii) 8, 9 and 25

$8 = 2 \times 2 \times 2 \times 1 = 2^3 \times 1$

$9 = 3 \times 3 \times 1 = 3^2 \times 1$

$25 = 5 \times 5 \times 1 = 5^2 \times 1$

HCF = 1

LCM = $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

4. ***Given that HCF (306, 657) = 9, find LCM (306, 657).***

Concept: Use, LCM × HCF = product of the two numbers to find out the unknown LCM or HCF if anyone value is given.

Sol. HCF (306, 657) = 9

We know that, LCM × HCF = Product of two numbers

∴ LCM × HCF = 306×657

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

LCM = 22338

5. ***Check whether 6^n can end with the digit 0 for any natural number n.***

> **Concept:** If any number ends with the digit 0, then it should be divisible by 10, 5 and 2. Try to find the prime factors of 6^n.

Sol. If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as 10 = 2 × 5

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

6. ***Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.***

> **Concept:** Composite numbers are the positive integers which have factors other than 1 and itself while prime numbers are whole numbers whose only factors are 1 and itself.

Sol. Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and itself, only whereas composite numbers have factors other than 1 and itself.

It can be observed that

$$\begin{aligned} 7 \times 11 \times 13 + 13 &= 13 \times (7 \times 11 + 1) \\ &= 13 \times (77 + 1) = 13 \times 78 \\ &= 13 \times 13 \times 6 \end{aligned}$$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$$\begin{aligned} 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 &= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1) \\ &= 5 \times (1008 + 1) \\ &= 5 \times 1009 \end{aligned}$$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

7. ***There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?***

> **Concept:** From the question, it is clear that time taken by Sonia is more than Ravi to complete one round. Since, both are in same direction, we can say that they will meet again at the same time when both complete one round. Now, you have to find time they meet again at the same point. For this, you need to find the number divisible by 18 and 12 by taking LCM.

Sol. It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$$\begin{aligned} 18 &= 2 \times 3 \times 3 = 2 \times 3^2 \\ \text{and, } 12 &= 2 \times 2 \times 3 = 2^2 \times 3 \\ \text{LCM of 12 and 18} &= 2 \times 2 \times 3 \times 3 = 36 \end{aligned}$$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

Snapshot

1.3 Irrational Numbers

A number 's' is called irrational if it cannot be written in the form $\frac{p}{q}$; where p and q are integers and $q \neq 0$. In this section, we will prove that square root numbers are irrational. The proof is based on the method called 'Proof by Contradiction'.

Example 9 :

Prove that $\sqrt{3}$ is irrational.

Sol. Let us assume that $\sqrt{3}$ is a rational number.

$\Rightarrow$ $\sqrt{3}$ can be written in the form of $\frac{p}{q}$ where p and q ($q \neq 0$) are co-prime (no common factor other than 1).

$$\Rightarrow \quad \sqrt{3} = \frac{p}{q}$$

$$\sqrt{3}q = p$$

Squaring on both sides,

$$3q^2 = p^2$$

$$q^2 = \frac{p^2}{3}$$

Hence, 3 divides p^2

Since, if p divides a^2 then p divides a, {(Theorem 1.3)} where a is a positive number.

$\Rightarrow$ 3 divides p

Now, let $\frac{p}{3} = r$ where r is an integer.

$$\Rightarrow \quad p = 3r$$

$$\sqrt{3}\,.\,q = 3r \text{ (since } p = \sqrt{3}q)$$

Squaring on both sides

$$3q^2 = 9r^2$$

$$r^2 = \frac{q^2}{3}$$

Hence, 3 divides q^2

By theorem:

If p is a prime number, and p divides a^2, then p divides a, where a is a positive number.

$\Rightarrow$ 3 divides q

Since 3 divides both p and q

$\Rightarrow$ 3 is a factor of p and q which contradicts our assumption.

Thus, $\sqrt{3}$ is irrational.

Example 10 :

Show that $5 - \sqrt{3}$ is irrational.

Sol. Let us assume $5 - \sqrt{3}$ to be rational.

$\Rightarrow$ $(5 - \sqrt{3})$ can be written in the form of $\frac{p}{q}$ where p and q ($q \neq 0$) are co-prime.

$$\text{Thus,} \quad 5 - \sqrt{3} = \frac{p}{q}$$

$$\sqrt{3} = 5 - \frac{p}{q}$$

$$\sqrt{3} = \frac{5q - p}{q}$$

Here, $\sqrt{3}$ is irrational and $\frac{5q-p}{q}$ is rational.

Since irrational $\neq$ rational which contradicts our assumption.

Hence, $(5 - \sqrt{3})$ is irrational.

Example 11 :

Show that $3\sqrt{2}$ is irrational.

Sol. Let us assume that $3\sqrt{2}$ is rational.

$\therefore$ We can find co-prime p and q ($q \neq 0$) such that $3\sqrt{2} = \frac{p}{q} \Rightarrow \sqrt{2} = \frac{p}{3q}$.

Since 3, q and p are integers, $\frac{p}{3q}$ is rational, where as $\sqrt{2}$ is irrational.

Since irrational $\neq$ rational, which contradicts our assumption.

Hence $3\sqrt{2}$ is irrational.

Exercise 1.3

1. ***Prove that $\sqrt{5}$ is irrational.***

Concept: This question can be solved with the help of contradiction method. If $\sqrt{5}$ is rational, then it can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Here, p and q have common factors. But when we cancel them, you will get $\frac{a}{b}$ where a and b are co-primes which will have no common factor other than 1.

Squaring both the sides, if a^2 is divisible by 5, then a is also divisible by 5. Therefore $a = 5c$

By squaring again, we will get the value of a^2. Substituting the value of a^2 in the above equation, we will get $\frac{b^2}{5} = c^2$, this means b^2 is divisible by 5 and so b is also divisible by 5. Therefore, a and b have at least 5 as a common factor.

However, this contradicts with the fact that a and b are coprime because of incorrect assumption that $\sqrt{5}$ is a rational number. So, we conclude that $\sqrt{5}$ is an irrational number.

Sol. Let us assume that $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$. Let a and b have a common factor other than 1. Then we can divide them by the common factor and assume that a and b are co-prime.

$$a = \sqrt{5}b$$

$$\Rightarrow \text{Squaring, } a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that a is divisible by 5.

$$\text{Let } a = 5k, \text{ where } k \text{ is an integer}$$

$$(5k)^2 = 5b^2$$

$$\Rightarrow 5k^2 = b^2$$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$. So that $\sqrt{5}$ is irrational.

2. ***Prove that $3 + 2\sqrt{5}$ is irrational.***

Sol. Let us assume $3 + 2\sqrt{5}$ is rational.

Therefore, we can find two co-prime integers a, b ($b \neq 0$) such that

$$3 + 2\sqrt{5} = \frac{a}{b}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$$

Since a and b are integers, $\frac{1}{2}\left(\frac{a}{b} - 3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3 + 2\sqrt{5}$ is rational is false. Therefore, $3 + 2\sqrt{5}$ is irrational.

3. ***Prove that the following are irrationals:***

(i) $\frac{1}{\sqrt{2}}$ *(ii)* $7\sqrt{5}$ *(iii)* $6 + \sqrt{2}$

Sol. **(i)** $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ be rational.

Therefore, we can find two co-prime integers a, b ($b \neq 0$) such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b} \text{ Or } \sqrt{2} = \frac{b}{a}$$

$\frac{b}{a}$ is rational as a and b are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ be rational.

Therefore, we can find two co-prime integers a, b ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b} \text{ for some integers } a \text{ and } b$$

$$\therefore \sqrt{5} = \frac{a}{7b}$$

$\frac{a}{7b}$ is rational as a and b are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii) $6 + \sqrt{2}$

Let $6 + \sqrt{2}$ be rational.

Therefore, we can find two co-prime integers a, b ($b \neq 0$) such that

$$6 + \sqrt{2} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{a}{b} - 6$$

Since a and b are integers, $\frac{a}{b} - 6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, our assumption is false and hence, $6 + \sqrt{2}$ is irrational.

Snapshot

1.4 Rational Number and their Decimal Expansions

In this session, we are going to consider a rational number and explore when the decimal expansion of $\frac{p}{q}$ is terminating and when it is non-terminating repeating or recurring.

Theorem 1.5 :

- Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$ where p and q are coprime and the prime factorisation of q is of the form $\alpha^n s^m$ where n, m are non-negative integers

Theorem 1.6 :

- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form 2^n5^m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

Theorem 1.7:

- Let $x = \frac{p}{q}$, where p and q are coprimes, be a rational number, such that the prime factorisation of q is not of the form 2^n5^m, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

Exercise 1.4

1. ***Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:***

(i) $\frac{13}{3125}$ *(ii)* $\frac{17}{8}$ *(iii)* $\frac{64}{455}$

(iv) $\frac{15}{1600}$ *(v)* $\frac{29}{343}$ *(vi)* $\frac{23}{2^5 5^2}$

(vii) $\frac{129}{2^2 5^7 7^5}$ *(viii)* $\frac{6}{15}$ *(ix)* $\frac{35}{50}$

(x) $\frac{77}{210}$

Concept: Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n \times 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates

Sol. (i) $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$455 = 5 \times 7 \times 13$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$1600 = 2^6 \times 5^2$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$343 = 7^3$

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 5^2}$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 5^2}$ is terminating.

(vii) $\frac{129}{2^2 5^7 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$

The denominator is of the form 5^n.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50} = \frac{7 \times 5}{10 \times 5} = \frac{7}{10}$

$10 = 2 \times 5$

The denominator is of the form $2^m \times 5^n$,

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210} = \frac{11 \times 7}{30 \times 7} = \frac{11}{30}$

$30 = 2 \times 3 \times 5$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 3 as its factors, the decimal expansion of $\frac{77}{210}$ is non-terminating repeating.

2. ***Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.***

(i)	$\frac{13}{3125}$	*(ii)*	$\frac{17}{8}$	*(iii)*	$\frac{64}{455}$
(iv)	$\frac{15}{1600}$	*(v)*	$\frac{29}{343}$	*(vi)*	$\frac{23}{2^3 5^2}$
(vii)	$\frac{129}{2^2 5^7 7^5}$	*(viii)*	$\frac{6}{15}$	*(ix)*	$\frac{35}{50}$
(x)	$\frac{77}{210}$				

Concept: Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n \times 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates

Sol. **(i)** $\frac{13}{3125} = 0.00416$

$$3125\overline{)13.00000}\quad 0.00416$$

0
130
0
1300
0
13000
12500
5000
3125
18750
18750
0

(ii) $\frac{17}{8} = 2.125$

$$8\overline{)17}\quad 2.125$$

16
10
8
20
16
40
40
0

(iii) $\frac{64}{455}$

Factorizing the denominator, we get

$455 = 5 \times 7 \times 13$

There is 7 and 13 in the denominator, which is not in the form of $2^n \times 5^m$.

Hence, $\frac{64}{455}$ is non-terminating repeating decimal expansion.

(iv) $\frac{15}{1600} = 0.009375$

$$1600\overline{)15.000000}\quad 0.009375$$

0
150
0
1500
0
15000
14400
6000
4800
12000
11000
8000
8000
0

(v) $\frac{29}{343}$

Factorizing the denominator, we get

$343 = 7 \times 7 \times 7 = 7^3$

The denominator is not in the form of $2^n \times 5^m$.

Hence $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 \times 5^2} = \frac{23}{200} = 0.115$

$$200\overline{)23.000}\quad 0.115$$

0
230
200
300
200
1000
1000
0

(vii) $\frac{129}{2^2 5^7 7^5}$

Here, the denominator has 7 which is not in the form of $2^n \times 5^m$.

Hence $\frac{129}{2^2 5^7 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5} = 0.4$

$$5\overline{)2.0}\quad 0.4$$

0
20
20
0

(ix) $\frac{35}{50} = 0.7$

$$50\overline{)35.0}\quad 0.7$$

0
350
350
0

(x) $\frac{77}{210}$

Simplifying the above fraction, we get $\frac{11}{30}$.

Factorize the denominator 30 = 2 × 3 ×5

The denominator 3 is not in the form of $2^n \times 5^m$.

Hence $\frac{77}{210}$ is non-terminating reapeating.

4. ***The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of q?***

(i) ***43.123456789***

(ii) ***0.120120012000120000...***

(iii) ***43.123456789***

> **Concept:** Let x be a rational number whose decimal expansion terminates.
>
> Then x can be expressed in the form $\frac{p}{q}$, where p and q are coprime, and the prime factorisation of q is of the form $2^n \times 5^m$, where n, m are non-negative integers.

Sol. **(i)** 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$ i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.120120012000120000 …

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not in the form $2^m \times 5^n$ i.e., the prime factors of q will also have a factor other than 2 or 5.

Additional Question and Answers

Multiple choice questions :

Choose the correct answer from the given four options in the following questions.

Q1. ***For some integer m, every even integer is of the form*** ***(Exemplar)***

(A) m (B) m + 1 (C) 2m (D) 2m + 1

Ans : (C) 2*m*

Sol. We know that, even integers are 2,4,6, …….

Where,

m is an integer

[Since, here integer is represented by *m*]

m = ..., –1, 0, 1, 2, 3......

$\therefore 2m$ = ..., –2, 0, 2, 4, 6,......

Q2. ***If HCF (a, b) = 12 and a × b = 1800 then LCM (a, b) is***

(A) 1200 (B) 510 (C) 150 (D) 900

Ans : (C) 150

Q3. ***For some integer q, every odd integer is of the form*** ***(Exemplar)***

(A) q (B) q+1 (C) 2q (D) 2q+1

Ans : (D) 2q + 1

Sol. We know that, odd integers are 1, 3, 5, ….

So, it can be written in the form of 2q +1

Where,

q is an integer

or q = ...,–1, 0, 1, 2, 3,.....

$\therefore 2q + 1$ = ..., –3, –1, 1, 3, 5......

Q4. ***$n^2 - 1$ is divisible by 8, if n is***

(A) an integer (B) a natural number
(C) an odd integer (D) an even integer

Ans : (C) an odd integer

Sol. Let $a = n^2 - 1$

Here n can be even or odd

Case I :

n = Even i.e., $n = 2k$, where k is an integer.

$\Rightarrow \quad a = (2k)^2 - 1$

$\Rightarrow \quad a = 4k^2 - 1$

At $k = -1$,

$a = 4(-1)^2 - 1 = 4 - 1 = 3$,

Which is not divisible by 8.

At $k = 0$,

$a = 4(0)^2 - 1 = 0 - 1 = -1$,

Which is not divisible by 8.

Case II :

n = odd i.e., $n = 2k + 1$,

Where k is an integer

$\Rightarrow \quad a = (2k + 1)^2 - 1$

$\Rightarrow \quad a = 4k^2 + 4k + 1 - 1$

$\Rightarrow \quad a = 4k^2 + 4k$

$\Rightarrow \quad a = 4k\ (k + 1)$

At $k = -1$,

$a = 4(-1)\ (-1 + 1) = 0$

Which is divisible by 8

At $k = 0$,

$a = 4(0)\ (0 + 1) = 0$

Which is divisible by 8

At $k = 1$,

$a = 4\ (1)\ (1 + 1) = 8$ Which is divisible by 8

Hence, we can conclude from above two cases, if n is odd, then $n^2 - 1$ is divisible by 8.

Q5. ***If $m^n = 32$, where m and n are positive integers, then the value of (n)mn is***

(A) 9755625 (B) 9785625
(C) 9765625 (D) 9865265

Ans : (C) 9765625

Q6. ***For any positive integer a and b, there exist unique integers q and r such that a = 3q + r, where r must satisfy.***

(A) $0 < r \leq 3$ (B) $0 \leq r < 3$
(C) $1 < r < 3$ (D) $0 < r < 3$

Ans : (B) $0 \leq r < 3$

Q7. ***If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of m is (Exemplar)***

(A) 4 (B) 2 (C) 1 (D) 3

Ans : (B) 2

Sol. Using Euclid's division algorithm,

$b = aq + r,\ 0 \leq r < a$ [∵ Dividend = Divisor × Quotient + Remainder]

$\Rightarrow \quad 117 = 65 \times 1 + 52$

$\Rightarrow \quad 65 = 52 \times 1 + 13$

$\Rightarrow \quad 52 = 13 \times 4 + 0$

∴ HCF (65, 117) = 13 ...(i)

Also, given that, HCF (65, 117) = $65m - 117$ ...(ii)

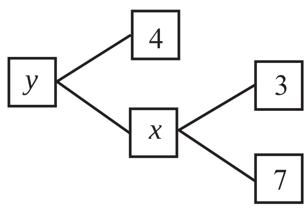
From Equations (i) and (ii), we get

$65m - 117 = 13$

$\Rightarrow \quad 65m = 130$

$\Rightarrow \quad m = 2$

Q8. ***The values of x and y is the given figure are***



(A) $x + 10, y = 15$ (B) $x = 10, y = 40$
(C) $x = 21, y = 25$ (D) $x = 21, y = 84$

Ans : (C) $x = 21, y = 25$

Q9. ***The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is (Exemplar)***

(A) 13 (B) 65 (C) 875 (D) 1750

Ans : (A) 13

Sol. Since, 5 and 8 are the remainders of 70 and 125, respectively.

After subtracting these remainders from the numbers, we have the numbers 65 = (70 – 5), 117 = (125 – 8), which is divisible by the required number.

Now, required number = HCF of 65, 117

[Since we need the largest number]

For this, $117 = 65 \times 1 + 52$

[$\because$ Dividend = Divisor × Quotient + Remainder]

$\Rightarrow \quad 65 = 52 \times 1 + 13$

$\Rightarrow \quad 52 = 13 \times 4 + 0$

$\therefore \quad$ HCF $= 13$

Hence, 13 is the largest number which divides 70 and 125, leaving remainders 5 and 8.

Q10. ***If two positive integers a and b are written and $a = x^3y^2$ and $b = xy^3$, where x, y are the prime numbers, then HCF (a, b) is*** ***(Exemplar)***

(A) xy (B) xy^2 (C) x^3y^3 (D) x^2y^2

Ans : (B) xy^2

Sol. Given that,

$a = x^3 y^2 = x \times x \times x \times y \times y$ and

$b = xy^3 = x \times y \times y \times y$

$\therefore$ HCF of a and b

$= \text{HCF}(x^3 y^2, xy^3)$

$= x \times y \times y = xy^2$

[Since, HCF is the product of the smallest power of each common prime factor involved in the number]

Q11. ***Given that LCM of (91, 26) = 182 then HCF (91, 26) is***

(A) 21 (B) 31 (C) 7 (D) 13

Ans : (D) 13

Q12. ***If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM(p, q) is*** ***(Exemplar)***

(A) ab (B) a^2b^2 (C) a^3b^2 (D) a^3b^3

Ans : (C) a^3b^2

Sol. Given that,

$p = ab^2 = a \times b \times b$ and

$q = a^3b = a \times a \times a \times b$

$\therefore$ LCM of p and q $= \text{LCM}(ab^2, a^3b)$

$= a \times b \times b \times a \times a = a^3b^2$

[Since, LCM is the product of the greatest power of each prime factor involved in the numbers]

Q13. ***If HCF of 55 and 99 is expressible in the form 55 m – 99, then the value of m:***

(A) 6 (B) 2 (C) 5 (D) 7

Ans : (B) 2

Q14. ***The product of a non-zero rational and an irrational number is*** ***(Exemplar)***

(A) always irrational (B) always rational

(C) rational or irrational (D) one

Ans : (A) always irrational

Sol. Product of a non-zero rational and an irrational number is always irrational.

For example :

$\frac{3}{4}$ is a rational and $\sqrt{2}$ is irrational number but their product is an irrational number.

$$\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$$

Q15. ***If the LCM of 12 and 42 is 10 m + 4 then the value of m is***

(A) 8 (B) 32 (C) 1 (D) 5

Ans : (A) 8

Q16. ***The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is*** ***(Exemplar)***

(A) 10 (B) 1000 (C) 504 (D) 2520

Ans : (D) 2520

Sol. Factors of 1 to 10 numbers are as follows :

1	= 1	6	= 1 × 2 × 3
2	= 1 × 2	7	= 1 × 7
3	= 1 × 3	8	= 1 × 2 × 2 × 2
4	= 1 × 2 × 2	9	= 1 × 3 × 3
5	= 1 × 5	10	= 1 × 2 × 5

$\therefore$ LCM of number 1 to 10

$= \text{LCM}(1, 2, 3, 4, 5, 6, 7, 8, 9, 10)$

$= 1 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2520$

Q17. ***If A = 2n + 13, B = n + 7 where n is a natural number then HCF of A and B.***

(A) 4 (B) 3

(C) 1 (D) None of the above

Ans : (C) 1

Q18. ***The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after :*** ***(Exemplar)***

(A) one decimal place (B) two decimal places

(C) three decimal places (D) four decimal places

Ans : (D) four decimal places

Sol. Given rational number

$$= \frac{14587}{1250} = \frac{14587}{2^1 \times 5^4}$$
$$= \frac{14587}{10 \times 5^3} \times \frac{(2)^3}{(2)^3} = \frac{14587 \times 8}{10 \times 1000}$$
$$= \frac{116696}{10000} = 11.6696$$

Hence, given rational number will terminate after four decimal places.

Q19. ***If n is a natural number, then exactly one of numbers n, n + 2 and n + 1 must be a multiple of***

(A) 4 (B) 3 (C) 1 (D) 8

Ans : (B) 3

Q20. ***The LCM of 2.5, 0.5 and 0.175 is***

(A) 0.875 (B) 5.5
(C) 7 (D) None of the above

Ans : (A) 0.875

Q21. ***The rational number between 72 and 73 is***

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $\frac{6}{5}$ (D) $\frac{2}{5}$

Ans: (A) $\frac{3}{2}$

Fill in the Blank

1. If the HCF and LCM of two positive integers a and b are x and y respectively, then $\frac{x^2 y^2}{a^2 b^2}$ ________.

Ans : 1

2. The LCM of the smallest prime number and the smallest composite number is ________.

Ans : 4

3. If the prime factorization of a natural number n is $2^4 \times 3^4 \times 5^3 \times 7$, then the number of consecutive zeros in n, is ________.

Ans : 3

4. The LCM of $2a$, $5a$ and $7a$ is _________, where a is a positive integer.

Ans : $70a$

5. The decimal expansion of $\frac{17}{8}$ will terminate after ________ places of decimal.

Ans : 3

6. The values of the remainder r, when a positive integers is always divisible by ________.

Ans : 0, 1 and 2

7. Two numbers are in the ratio 21 : 17. If their HCF is 5, the numbers are __________ and _________.

Ans : 85, 105

8. If a and b are two positive co-prime integers such that $a = 12b$, then HCF (a, b) = _________

Ans : 12

9. A positive integer m when divided by 11 gives remainder 6. If 4m + 5 is divided by 11, the remainder is ________.

Ans : 7

10. The sum of the exponents of prime factors in the prime factorization of 250 is ________.

Ans : 4

11. If two positive integers m and n are expressible in the form $m = a^2 b^3$ and $n = a^3 b^2$, where a, b are prime numbers, then HCF (m, n) = _________ and LCM (a, b) = ______.

Ans : a^2b^2, a^3b^3

12. The ratio between the HCF and LCM of 5, 15 and 20 is _______.

Ans : 1 : 12

13. If the least prime factors of two positive integers a and b are 5 and 13 respectively, then the least prime factor of $a + b$, is _______.

14. 6^n cannot end with digit 5 for __________ value of n.

Ans : any

15. If $2^3 \times 3^a \times b \times 7$ is the prime factorization of 2520, then $5a + 2b$ = ________.

Ans : 20

Very short answer questions :

Q1. ***Write whether every positive integer can be of the form 4q + 2, where q is an integer. Justify your answer. (Exemplar)***

Sol. No, by Euclid's Lemma, $b = aq + r$, $0 \le r < a$

[∵ Dividend = Divisor × Quotient + Remainder]

Here, b is any positive integer and $a = 4$, $b = 4q + r$ for $0 \le r < 4$

i.e., $r = 0, 1, 2, 3$

So, this must be in the form $4q$, $4q + 1$, $4q + 2$ or $4q + 3$.

Q2. ***"The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons?*** ***(Exemplar)***

Sol. Yes, it is true.

Two consecutive integers can be n, $(n + 1)$. So, one number of these two must be divisible by 2. Hence, product of numbers is divisible by 2.

For example :

4×5 is divisible by 2,

14×15 is divisible by 2,

40×41 is divisible by 2 and so on.

Q3. ***'The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.*** ***(Exemplar)***

Sol. Yes, it is true.

Three consecutive integers can be n, $(n + 1)$ and $(n + 2)$

So, one number of these three must be divisible by 2 another one must be divisible by 3.

Hence, product of numbers is divisible by 6.

For example :

$2 \times 3 \times 4$ is divisible by 6,

$12 \times 13 \times 14$ is divisible by 6,

$82 \times 83 \times 84$ is divisible by 6 and so on.

Q4. ***Write whether the square of any positive integer can be of the form 3m + 2, where m is a natural number. Justify your answer.*** ***(Exemplar)***

Sol. No,

By Euclid's lemma, $b = aq + r$, $0 \le r \ge a$

Here, b is any positive integer and $a = 3$, $b = 3q + r$ for $0 \le r \ge 2$.

So, any positive integer is of the form $3k$, $3k + 1$ or $3k + 2$.

$$\text{Now, } (3k)^2 = 9k^2 = 3m \quad [\text{Where, } m = 3k^2]$$

$$\text{and } (3k + 1)^2 = 9k^2 + 6k + 1 = 3(3k^2 + 2k) + 1 = 3m + 1 \quad [\text{Where, } m = 3k^2 + 2k]$$

$$\text{Also, } (3k + 2)^2 = 9k^2 + 12k + 4 \quad [\because (a + b)^2 = a^2 + 2ab + b^2]$$

$$= 9k^2 + 12k + 3 + 1 = 3(3k^2 + 4k + 1) + 1 = 3m + 1 \quad [\text{Where, } m = 3k^2 + 4k + 1]$$

Which is in the form of $3m$ and $3m + 1$.

Hence, square of any positive number cannot be of the form $3m + 2$.

Q5. ***A positive integer is of the form 3q + 1, q being a natural number. Can you write its square in any form other than 3m + 1, i.e., 3m or 3m + 2 for some integer m? Justify you answer.*** ***(Exemplar)***

Sol. No,

By, Euclid's Lemma,

$$b = aq + r, \quad 0 \le r \ge a$$

Here, b is any positive integer and

$$a = 3, b = 3q + r \text{ for } 0 \le r \ge 3$$

So, this must be in the form $3q$, $3q + 1$ or $3q + 2$.

$$\text{Now, } (3q)^2 = 9q^2 = 3m \quad [\text{Where, } m = 3q^2]$$

$$\text{and } (3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1$$

$$\text{Also, } (3q + 2)^2 = 9q^2 + 12q + 4 = 9q^2 + 12q + 3 + 1 = 3(3q^2 + 4q + 1) + 1 = 3m + 1$$

$$[\text{Where, } m = 3q^2 + 4q + 1]$$

Hence, square of a positive integer is of the form $3q + 1$ is always in the form $3m + 1$ for some integer m.

Q6. ***The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify your answer.*** ***(Exemplar)***

Sol. Since, the HCF (525 and 3000) = 75

By Euclid's Lemma, $3000 = 525 \times 5 + 375$

[$\because$ Dividend = Divisor × Quotient + Remainder]

$$525 = 375 \times 1 + 150$$

$$375 = 150 \times 2 + 75$$

$$150 = 75 \times 2 + 0$$

The number 3, 5, 15, 25 and 75 divides the numbers 525 and 3000. It mean these terms are common in both 525 and 3000.

So, the highest common factor among these is 75.

Q7. ***Explain why 3 × 5 × 7 + 7 is a composite number.*** ***(Exemplar)***

Sol. We have,

$$a = 3 \times 5 \times 7 + 7 = 7(3 \times 5 + 1) = 7 \times 16$$

Since, a has more than two factors namely 1, 7 and 16

So, it is a composite number.

> ***Note :*** Prime numbers has only two factors 1 and numbers itself.

Q8. ***Can two numbers have 18 as their HCF and 380 as their LCM ? Given reasons.*** ***(Exemplar)***

Sol. No.

Because HCF is always a factor of LCM but here 18 is not a factor of 380.

Q9. ***Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.*** ***(Exemplar)***

Sol. Yes, it is terminating decimal expansion.

Simplified denominator has factor in the form of $2^m . 5^n$. So, this is terminating decimal.

Now,

$$\frac{987}{10500} = \frac{47}{500} = \frac{47}{5^3.2^2} \times \frac{2}{2}$$

$$= \frac{94}{5^3 \times 2^3} = \frac{94}{1000} = 0.094$$

Q10. ***A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form $\frac{p}{q}$? Given reasons.*** ***(Exemplar)***

Sol. 327.7081 is terminating decimal number. So, it represents a rational number and also its denominator must have the form $2^m \times 5^n$.

Thus,

$$327.7081 = \frac{3277081}{10000} = \frac{p}{q}$$

$$\therefore q = 10^4 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$$

$$= 2^4 \times 5^4 = (2 \times 5)^4$$

Hence, the prime factorization of q contains only factors of 2 and 5.

Short answer questions :

Q1. ***Show that the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.*** ***(Exemplar)***

Sol. Let a be an arbitrary positive integer. Then, by Euclid's division algorithm, corresponding to the positive integers a and 4, there exist non-negative integers m and r, such that

$$a = 4m + r, \text{ where } 0 \leq r < 4$$

$$\Rightarrow a^2 = (4m + r)^2$$

$$= 16m^2 + r^2 + 8mr \quad ...(i)$$

Where, $0 \leq r < 4$

Case I :

When $r = 0$, then putting $r = 0$ in equation (i), we get

$$a^2 = 16m^2 = 4(4m^2) = 4q$$

Where, $q = 4m^2$ is an integer

Case II :

When $r = 1$, then putting $r = 1$ in Equation (i), we get

$$a^2 = 16m^2 + 1 + 8m$$

$$= 4(4m^2 + 2m) + 1 = 4q + 1$$

Where, $q = 4(4m^2 + 2m)$ is an integer

Case III :

When $r = 2$, then putting $r = 2$ in Equation(i), we get

$$a^2 = 16m^2 + 4 + 16m$$

$$= 4(4m^2 + 4m + 1) = 4q$$

Where, $q = (4m^2 + 4m + 1)$ is an integer.

Case IV :

When $r = 3$, then putting $r = 3$ in Equation (i), we get

$$a^2 = 16m^2 + 24m + 9$$

$$= 16m^2 + 24m + 8 + 1$$

$$= 4(4m^2 + 6m + 2) + 1$$

$$= 4q + 1$$

Where, $q = (4m^2 + 6m + 2)$ is an integer

Hence, the square of any positive integer is either of the form $4q$ or $4q + 1$ from some integer q.

Q2. ***Show that cube of any positive integer is of the form, 4m, 4m + 1 or 4m + 3, for some integer m.*** ***(Exemplar)***

Sol. Let a be an arbitrary positive integer. Then, by Euclid's division algorithm, corresponding to the positive integers a and 4, there exist non-negative integers q and r such that

$$a = 4q + r, \text{ where } 0 \leq r \geq 4$$

$$\Rightarrow a^3 = (4q + r)^3$$

$$= 64q^3 + r^3 + 12r^2 + 48q^2r$$

$$\Rightarrow a^3 = (64q^2 + 48q^2 + 12q^2) + r^3 \quad ...(i)$$

Where, $0 \leq r \geq 4$

Case I :

When $r = 0$,

Putting $r = 0$ in equation (i), we get

$$a^3 = 64q^3 = 4(16q^3) = 4m$$

Where $m = 16q^3$ is an integer.

Case II :

When $r = 1$

Putting $r = 1$ in equation (i), we get

$a^3 = 64q^3 + 48q^2 + 12q + 1$

$= 4(16q^3 + 12q^2 + 3q) + 1$

$= 4m + 1$

Where $m = (16q^3 + 12q^2 + 3q)$

is an integer.

Case III :

When $r = 2$,

$a^3 = 64q^3 + 8 + 96q^2 + 48q$

$= 4(16q^3 + 24q + 12q + 2) = 4m$

Where $m = 16q^3 + 24q^2 + 12q + 2$

is an integer.

Case IV:

When $r = 3$,

Putting $r = 2$ in equation (i), we get

$a^3 = 64q^3 + 144q^2 + 108q + 27$

$= 64q^3 + 144q^2 + 108q + 24 + 3$

$= 4(16q^3 + 36q^2 + 27q + 6) + 3$

$= 4m + 3$

Where $m = (16q^3 + 36q^2 + 27q)$

is an integer.

Hence, the cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$ for some integer m.

Q3. ***Show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.*** ***(Exemplar)***

Sol. Let a be an arbitrary positive integer.

Then, by Euclid's divisions algorithm, corresponding to the positive integers a and 5, there exist non-negative integers m and r such that

$a = 5m + r$, Where $0 \le r < 5$

$\Rightarrow a^2 = (5m + r)^2 = 25m^2 + r^2 + 10mr$

$\Rightarrow a^2 = 5(5m^2 + 2mr) + r^2$ ...(i)

Where, $0 \le r < 5$

Case I :

When $r = 0$,

Putting $r = 0$ in equation (i) we get

$a^2 = 5(5m^2) = 5q$

Where, $q = 5m^2$ is an integer.

Case II :

When $r = 1$,

Putting $r = 1$ in equation (i) we get

$a^2 = 5(5m^2 + 2m) + 1$

$\Rightarrow q = 5q + 1$

Where, $q = (5m^2 + 2m)$ is an integer.

Case III :

When $r = 2$,

Putting $r = 2$ in equation (i) we get

$a^2 = 5(5m^2 + 4m) + 4$

Where, $q = (5m^2 + 4m)$ is an integer.

Case IV :

When $r = 3$,

Putting $r = 3$ in equation (i) we get

$a^2 = 5(5m^2 + 6m) + 9$

$= 5(5m^2 + 6m) + 5 + 4$

$= 5(5m^2 + 6m + 1) + 4$

$= 5q + 4$

$\Rightarrow a = 5q + 4$

Where, $q = (5m^2 + 6m + 1)$ is an integer.

Case V :

When $r = 4$,

Putting $r = 4$ in equation (i) we get

$a^2 = 5(5m^2 + 8m) + 16$

$= 5(5m^2 + 8m) + 15 + 1$

$a^2 = 5(5m^2 + 8m + 3) + 1$

$= 5q + 1$

$\Rightarrow a = 5q + 1$

Where, $q = (5m^2 + 8m + 3)$ is an integer.

Hence, the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q.

Q4. ***Show that the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.*** ***(Exemplar)***

Sol. Let a be an arbitrary positive integer, then by Euclid's division algorithm, corresponding to the positive integers a and 6, there exist non-negative integers q and r such that

$a = 6q + r$, where $0 \le r < 6$

$\Rightarrow a^2 = (6q + r)^2 = 36q^2 + r^2 + 12qr$

$\Rightarrow a^2 = 6(6q^2 + 2qr) + r^2$ ...(i)

Where, $0 \le r < 6$

Case I :

When $r = 0$,

Putting $r = 0$ in equation (i) we get

$a^2 = 6(6q)^2 = 6m$

Where, $m = 6q^2$ is an integer.

Case II :

When $r = 1$,

Putting $r = 1$ in equation (i), we get

$a^2 = 6(6q^2 + 2q) + 1$

$= 6m + 1$

$\Rightarrow a^2 = 6m + 1$

Where, $m = (6q^2 + 2q)$ is an integer.

Case III :

When $r = 2$,

Putting $r = 2$ in equation (i) we get,

$a^2 = 6(6q^2 + 4q) + 4$

$= 6m + 4$

$\Rightarrow a^2 = 6m + 4$

Where, $m = (6q^2 + 4q)$ is an integer.

Case IV :

When $r = 3$,

Putting $r = 3$ in equation (i), we get

$a^2 = 6(6q^2 + 6q) + 9$

$= 6(6q^2 + 6q) + 6 + 3$

$= 6(6q^2 + 6q + 1) + 3$

$= 6m + 3$

$\Rightarrow a^2 = 6m + 3$

Where, $m = (6q^2 + 6q + 1)$ is an integer.

Case V :

When $r = 4$,

Putting $r = 4$ in equation (i), We get

$a^2 = 6(6q^2 + 8q) + 16$

$= 6(6q^2 + 8q) + 12 + 4$

$= 6(6q^2 + 8q + 2) + 4$

$= 6m + 4$

$\Rightarrow a^2 = 6m + 4$

Where, $m = (6q^2 + 8q + 2)$ is an integer

Case VI :

When $r = 5$,

Putting $r = 5$ in equation (i) we get

$a^2 = 6(6q^2 + 10q) + 25$

$= 6(6q^2 + 10q) + 24 + 1$

$= 6(6q^2 + 10q + 4) + 1$

$= 6m + 1$

$\Rightarrow a^2 = 6m + 1$

Where, $m = (6q^2 + 10q + 4)$ is an integer.

Hence, the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m.

Q5. ***Show that the square of any odd integer is of the form 4q + 1, for some integer q.*** ***(Exemplar)***

Sol. By Euclid's division algorithm,

We have $a = bq + r$, where $0 \leq r < 4$

Putting $b = 4$, we get

$a = 4q + r$, where $0 \leq r < 4$

i.e. $r = 0, 1, 2, 3.$

If $r = 0$

$a = 4q$, which is divisible by 2

$\Rightarrow 4q$ is even

If $r = 1$

$a = 4q + 1$,

which is not divisible by 2

If $r = 2$

$a = 4q + 2 = 2(2q + 1)$,

which is divisible by 2

$\Rightarrow 2(2q + 1)$ is even

If $r = 3$

$a = 4q + 3$

which is not divisible by 2

So, for any positive integer q, $4q + 1$ and $4q + 3$ are odd integers.

Now,

$a^2 = (4q + 1)^2$

$= 16q^2 + 1 + 8q$

$= 4(4q^2 + 2q) + 1$

$[\because (a + b)^2 = a^2 + 2ab + b^2]$

Which is of the form $4q + 1$, where

$q = (4q^2 + 2q)$ is an integer.

Now, $a^2 = (4q + 3)^2 = 16q^2 + 9 + 24q$

$= 4(4q^2 + 6q + 2) + 1$

Which is of the form, where $q = (4q^2 + 6q + 2)$ is an integer.

Hence, for some integer q, the square of any odd integer is of the form $4q + 1$.

So, when we apply division algorithm remainder should be zero.

$\therefore\ (7k + 21) = 0$ and $(2k^2 + 8k + 6) = 0$

$\Rightarrow\ 7k + 21 = 0$ and $2k^2 + 8k + 6 = 0$

$\Rightarrow\ k = -3$ and $k^2 + 4k + 3 = 0$

$\Rightarrow\ k = -3$ and $k^2 + 3k + k + 3 = 0$

$\Rightarrow\ k = -3$ and $k(k + 3) + 1(k + 3) = 0$

$\Rightarrow\ k = -3$ and $k = -1$ or -3

Here, only $k = -3$ satisfy the required condition.

Thus, the required value of k is -3.

Now, Dividend = Divisor × Quotient + Remainder

$\Rightarrow 2x^4 + x^3 - 14x^2 + 5x + 6 = (x^2 + 2x - 3)(2x^2 - 3x - 2) + 0$

Using factorization method,

$(x^2 + 3x - x - 3)(2x^2 - 4x + x - 2)$

$= \{x(x + 3) - 1(x + 3)\}\ \{2x(x - 2) + 1(x - 2)\}$

$= (x - 1)(x + 3)(x - 2)(2x + 1)$

Hence, the zeroes of $x^2 + 2x - 3$ are $1, -3$ and the zeroes of $2x^4 + x^3 - 14x^2 + 5x + 6$ are $1, -3, 2, -\frac{1}{2}$.

Q8. ***Find all the zeroes of the polynomial $4x^4 - 20x^3 + 23x^2 + 5x - 6$ if two of its zeros are 2 and 3.***

Sol. Let $P(x) = 4x^4 - 20x^3 + 23x^2 + 5x - 6$

Given 2 and 3 are the zeroes of the Polynomial $P(x)$

So, $(x - 2)(x - 3)$ are factors of the Polynomial $P(x)$

Thus, $(x^2 - 5x + 6)$ is a factor of the Polynomial $P(x)$

We have,

$$\begin{array}{r|l} & 4x^2 - 1 \\ \hline x^2 - 5x + 6 & 4x^4 - 20x^3 + 23x^3 + 5x - 6 \\ & 4x^4 - 20x^3 + 24x^2 \\ & (-)\quad (+)\quad (-) \\ \hline & -x^2 + 5x - 6 \\ & -x^2 + 5x - 6 \\ & (+)\quad (-)\quad (+) \\ \hline & 0 \end{array}$$

$\therefore\ 4x^4 - 20x^3 + 23x^2 + 5x - 6 = (x^2 - 5x + 6)(4x^2 - 1)$

$= (x - 2)(x - 3)(2x - 1)(2x + 1)$

Therefore, $2, 3, \frac{1}{2}, \frac{-1}{2}$ are the zeroes of $P(x)$.

Q9. ***Given that $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.*** ***(Exemplar)***

Sol. Let $f(x) = x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$ and given that, $(x - \sqrt{5})$ is one of the factor of $f(x)$.

Now, using division algorithm,

$$\begin{array}{r|l} & x^2 - 2\sqrt{5}x + 3 \\ \hline x - \sqrt{5} & x^3 - 3\sqrt{5}x^3 + 13x + 3\sqrt{5} \\ & x^3 - \sqrt{5}\,x^2 \\ & -\quad + \\ \hline & +2\sqrt{5}x^2 + 13x - 3\sqrt{5} \\ & +2\sqrt{5}\,x^2 + 10x \\ & (-)\qquad - \\ \hline & 3x - 3\sqrt{5} \\ & 3x - 3\sqrt{5} \\ & -\quad + \\ \hline & 0 \end{array}$$

$\therefore\ x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5} = (x^2 - 2\sqrt{5}x + 3) \times (x - \sqrt{5}) + 0$

[$\therefore$ Dividend = Divisor × Quotient + Remainder]

$= (x - \sqrt{5})\ [x^2 - \{(\sqrt{5} + \sqrt{2}) + (\sqrt{5} - \sqrt{2}\}x + 3]$

$= (x - \sqrt{5})\ [x^2 - (\sqrt{5} - \sqrt{2})x - (\sqrt{5} - \sqrt{2})x + (\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})]$

[$\because 3 = (\sqrt{5} + \sqrt{2})\ (\sqrt{5} - \sqrt{2})$]

$= (x - \sqrt{5})[x\{x - (\sqrt{5} + \sqrt{2})\} - (\sqrt{5} - \sqrt{2})\{x - (\sqrt{5} + \sqrt{2})\}]$

$= (x - \sqrt{5})\{x - (\sqrt{5} + \sqrt{2})\}\{x - (\sqrt{5} - \sqrt{2})\}$

Hence, all the zeroes of polynomial are $\sqrt{5}, (\sqrt{5} + \sqrt{2})$ and $(\sqrt{5} - \sqrt{2})$.

Q10. ***For which values of a and b, are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? which zeroes of $p(x)$ are not the zeroes of $q(x)$?*** ***(Exemplar)***

Sol. Given that the zeroes of $q(x) = x^3 + 2x^2 + a$ are also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$ i.e., $q(x)$ is a factor of $p(x)$. Using division algorithm.

$$
\begin{array}{l|l}
 & x^2 - 3x + 2 \\
\hline
x^3 + 2x^2 + a & x^5 - x^4 - 4x^3 + 3x^2 + 3x + b \\
 & x^5 + 2x^4 \qquad + ax^2 \\
 & - \quad - \qquad\quad - \\
\hline
 & -3x^4 - 4x^3 - (a-3)x^2 + 3x + b \\
 & -3x^4 - 6x^3 \qquad\qquad -3ax \\
 & + \quad + \qquad\qquad\qquad + \\
\hline
 & 2x^3 - (a-3)x^2 + (3+3a)x + b \\
 & 2x^3 + \quad 4x^2 \qquad\qquad + 2a \\
 & - \quad - \qquad\qquad\qquad - \\
\hline
 & -(a+1)x^2 + (3+3a)x + b - 2a
\end{array}
$$

If $(x^3 + 2x^2 + a)$ is a factor of $x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$, then remainder should be zero.

i.e., $-(1 + a)x^2 + (3 + 3a)x + (b - 2a) = 0.\ x^2 + 0.x + 0$

On comparing the coefficient of x^2 and constant term, we get

$$a + 1 = 0$$

$\Rightarrow$ $a = -1$ and $b - 2a = 0$

$\Rightarrow$ $b = 2a$

$\Rightarrow$ $b = 2(-1) = -2$ $[\because a = -1]$

For $a = -1$ and $b = -2$, the zeroes of $q(x)$ are also the zeroes of the polynomial $p(x)$.

$\therefore q(x) = x^3 + 2x^2 - 1$

And $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x - 2$

Now, Dividend = divisor × quotient + remainder

$p(x) = (x^3 + 2x^2 - 1)(x^2 - 3x + 2) + 0$

$= (x^3 + 2x^2 - 1)\{x^2 - 2x - 2 + x\}$

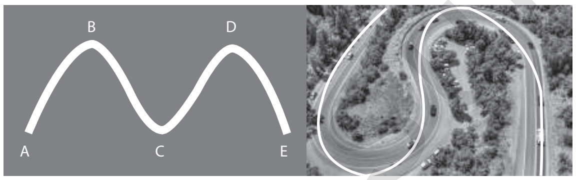
$= (x^3 + 2x^2 - 1)(x - 2)(x - 1)$

Hence, the zeroes of $p(x)$ are 1 and 2 which are not the zeroes of $q(x)$.

Case Study Question and Answers

1. ***Rani travelled in a car to visit Ooty in a summer. The path travelled by the car in some places describes parabola, which is expressed in the form of a polynomial as***

$P(x) = a_0x_n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots$



Sol. **(i)** What is the shape of the curve CDE?

(A) Para bola (B) Straight line
(C) Circle (D) Ellipse

Ans : (A) Para bola

(ii) If the shape of the curve ABC is represented by $x^2 - yx + 12$, then its zeroes are

(A) (2, –3) (B) (3, 4)
(C) (4, –5) (D) (3, –5)

Ans : (B) (3, 4)

(iii) The path traced by the car, whose zeroes are 2 and –4 is

(A) $x^2 - 4x - 8$ (B) $x^2 + 2x - 8$
(C) $x^2 + 2x + 8$ (D) $x^2 - 2x + 8$

Ans : (B) (3, 4)

(iv) The given path is shown on the co-ordinate axis, which is shown below:

Find the number of zeroes of he given curve.

(A) 3 (B) 2 (C) 4 (D)

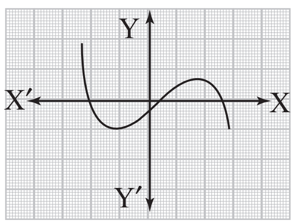
Ans : (C) 4

(v) If the path ABC traced by the car represented by $x^2 + 8x + 15$, then the distance between G and F is

(A) 2 (B) 3
(C) 1 (D) Cannot be determined

Ans : (A) 2

2. ***Malu dropped a ball from a building of height 10 m. The ball touched the balcony of the first floor and bounced which traced the following path. It is expressed as a polynomial as y = P(x).***



Sol. **(i)** The graph $y = P(x)$ is shown in the figure. How many zeroes does $P(x)$ have?

(A) 1 (B) 2 (C) 3 (D) 4

Ans : (C) 3

(ii) If the path is given by $P(x) = x^2 - 3x - 4$ what is the value of $P(x)$ at $x = 2$.

(A) 3 (B) –3 (C) 6 (D) –6

Ans : (D) –6

(iii) What is the value of $P(x) = x^2 - 3x - 4$ at $x = -1$?

(A) 0 (B) 1 (C) –1 (D) 2

Ans : (A) 0

(iv) If the co-efficient of x and the constant term in a linear polynomial are 5 and –3 respectively. Find its zero.

(A) 0 (B) $\frac{3}{5}$ (C) $\frac{5}{3}$ (D) 1

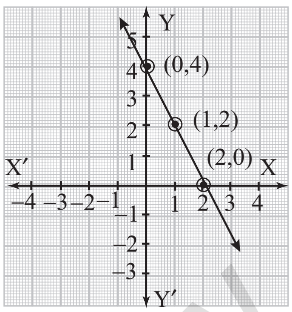
Ans : (B) $\frac{3}{5}$

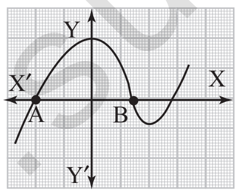
(v) Find the sum of the zeroes of the polynomial $2x^2 - 8x + 6$.

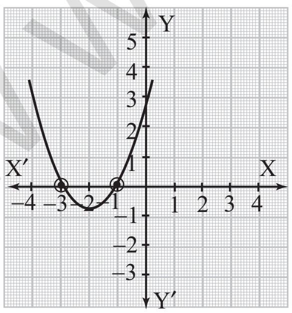
(A) 1 (B) 2 (C) 3 (D) 4

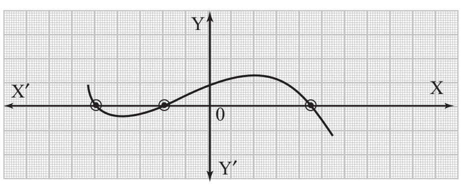
Ans : (D) 4

3. ***In a class the teacher asks four students to write the equations of the path travelled by them to school from their home. They draw the following figures for the polynomial P(x) = ax² + bx + c. Observe the graph and answer the following questions.***









Sol. **(i)** In the second student's path write the number of zeroes of the polynomial.

(A) 0 (B) 1 (C) 2 (D) 3

Ans : (C) 2

(ii) The path of the Student I is a ....

(A) Parabola (B) Ellipse

(C) Straight line (D) Circle

Ans : (C) Straight line

(iii) In third students path the zeroes of the polynomial is ....

(A) (–1, –3) (B) (–1, 3)

(C) (1, –3) (D) (1, 3)

Ans : (A) (–1, –3)

(iv) Write the number of zeroes in fourth students path.

(A) 0 (B) 1 (C) 2 (D) 3

Ans : (D) 3

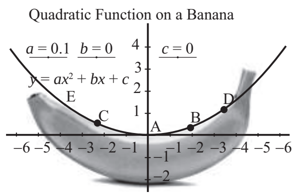
(v) Write a quadratic polynomial, the sum and product of whose zeroes are 3 and –2 respectively.

(A) $x^2 - 3x + 2$ (B) $x^2 - 3x - 2$

(C) $x^2 + 3x + 2$ (D) $x^2 + 3x - 2$

Ans : (B) $x^2 - 3x - 2$

4. ***The shape of a banana that Sonu give to his child is in the shape of a parabola. Since it is in the shape of a parabola it must have a quadratic function which have its standard form as f(x) = ax² + bx + c. From the figure using a=0.1, b=0 and c=0. We can get the equation as f(x) = 0.1 x².***



Sol. **(i)** Name the shape of the banana curve from the given figure.

(A) Ellipse (B) Semi circle

(C) Parabola (D) Straight line

Ans : (C) Parabola

(ii) Find the number of zeroes of the polynomial for the shape of the banana.

(A) 0 (B) 1 (C) 2 (D) 3

Ans : (B) 1

(iii) If the curve of the banana is represented by $f(x) = x^2 - x - 12$, find its zeroes.

(A) (–4, 3) (B) (–4, –3)

(C) (4, –3) (D) (4, 3)

Ans : (C) (4, –3)

(iv) If the representation of banana curve whose one zero is 4 and the sum of the zeroes is zero, then find the quadratic polynomial.

(A) $x^2 - 16$ (B) $x^2 + 16$

(C) $x^2 + 4$ (D) $x^2 - 4$

Ans : (A) $x^2 - 16$

(v) General form of the quadratic equation formed by the banana curve is ....

(A) $f(x) = ax^2 + bx + c$

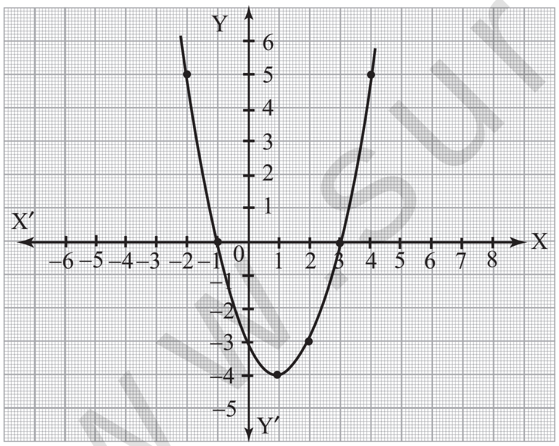
(B) $f(x) = bx^2 + bx + c$

(C) $f(x) = bx + c$

(D) $f(x) = a + b$

Ans : (A) $f(x) = ax^2 + bx + c$

5. ***David found one rainy day that an electric wire got bent as shown in the figure. It followed a mathematical shape. Answer the following questions.***



Sol. (i) Name the shape in which the where is bent.

(A) spiral (B) ellipse

(C) linear (D) parabola

Ans : (D) parabola

(ii) How many zeroes are there for the polynomial shape wire?

(A) 2 (B) 3 (C) 1 (D) 0

Ans : (A) 2

(iii) The zeroes of the polynomial are

(A) (–1, 5) (B) (–1, 3)

(C) (3, 5) (D) (–4, 2)

Ans : (B) (–1, 3)

(iv) What will be the expression of the polynomial?

(A) $x^2 + 2x - 3$ (B) $x^2 - 2x + 3$

(C) $x^2 - 2x - 3$ (D) $x^2 + 2x + 3$

Ans : (C) $x^2 - 2x - 3$

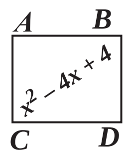
(v) What is the value of the polynomial if $x = -1$?

(A) 6 (B) –18 (C) 18 (D) 0

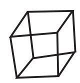
Ans : (D) 0

6. ***In a geometry box there are some shapes with the following dimensions.***

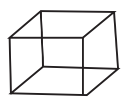
(a) A square of area $x^2 - 4x + 4$



(b) A cube with the equation $-4x^3 + 3x^2 + 9x - 7$



(c) A cuboid of volume $x^3 - 6x^2 + 11x - 6$



Sol. (i) Degree of the polynomial $P(x) = (2^2 - 1)(x^3 + 1)$ is

(A) 3 (B) 4 (C) 5 (D) 6

Ans : (C) 5

(ii) If the area of a square is $x^2 - 4x + 4$, the length of its diagonal is

(A) $\sqrt{2}(x - \sqrt{2})$ (B) $\sqrt{2}x - 2\sqrt{2}$

(C) $2(x-2)^2$ (D) $x - 2$

Ans : (B) $\sqrt{2}x - 2\sqrt{2}$

(iii) Find the sum of the zeroes of the polynomial $P(x) = 3x^2 - 4x^3 + 9x - 7$

(A) $\frac{3}{4}$ (B) $\frac{-3}{4}$ (C) $\frac{4}{3}$ (D) $\frac{-4}{3}$

Ans : (A) $\frac{3}{4}$

(iv) One fourth of the co-efficient of x in the expression of $(x-4)^3$ is

(A) 48 (B) –12 (C) 16 (D) 12

Ans : (D) 12

(v) If the volume of a cuboid is $x^3 - 6x^2 + 11x - 6$, then

I. $(x - 3)$ is one of the possible dimensions

II. $(x - 3)$ is a factor of $P(x)$

(A) I and II are true, II is the correct reason for I

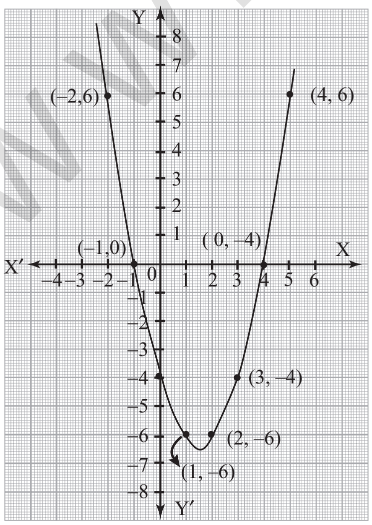
(B) I and II are true, II is not the correct reason for I

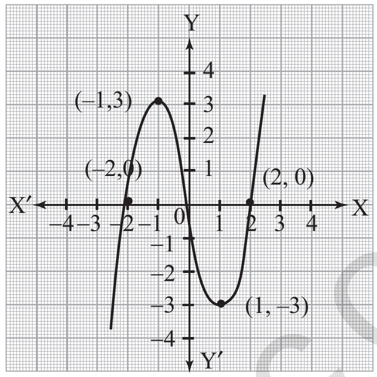
(C) I is true, II is false

(D) I is false, II is also false

Ans : (A) I and II are true, II is the correct reason for I

7. ***For any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding equation. $y = ax^2 + bx + c$ has one of the two shapes either open upwards like ∪ or open downward like ∩ depending on whether $a > 0$ or $a < 0$. Observe the following graphs and answer the following questions.***





Sol. **(i)** The sum of degrees of polynomials represented in figure A and figure B is

(A) 2 (B) 3 (C) 4 (D) 5

Ans : (D) 5

(ii) From the graph in figure A, we can conclude that

(A) The leading coefficient is negative

(B) Leading coefficient is positive

(C) The graph represents a bi-quadratic polynomial

(D) None

Ans : (B) Leading coefficient is positive

(iii) Identify the figure depicted in figure B.

(A) $x^3 + 2x^2 - 2x + 4$ (B) $x^3 - 4x$

(C) $x^3 - 2x^2 - 2x - 4$ (D) $x^3 + 4x$

Ans : (B) $x^3 - 4x$

(iv) Obtain a polynomial whose zeroes are 1 and –3.

(A) $x^2 + 2x - 3$ (B) $x^2 + 3$

(C) $x^2 - 3$ (D) x

Ans : (A) $x^2 + 2x - 3$

(v) Find the number of zeroes from the given graph B.

(A) 4 (B) 5 (C) 2 (D) 3

Ans : (D) 3

Unit Test

(30 Marks)

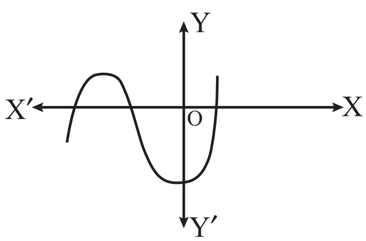
SECTION - A

Multiple choice questions : (4 × 1 = 4)

1. The maximum number of zeroes that a polynomial of degree 4 can have is

(A) Four (B) Seven
(C) Three (D) Five

2. In figure given below, the number of zeroes of the polynomial $f(x)$ is



(A) 2 (B) 3
(C) 1 (D) None

3. A polynomial of degree 3 is called

(A) a quadratic polynomial
(B) a cubic polynomial
(C) a biquadratic polynomial
(D) a linear polynomial

4. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then

(A) $a = -7, b = -1$ (B) $a = 5, b = -1$
(C) $a = 2, b = -6$ (D) $a = 0, b = -6$

SECTION - B

Very short questions : (2 × 2 = 4)

1. Divide $2x^2 + 3x + 1$ by $x + 2$.

2. Find a quadratic polynomial in the given number as the sum and product of its zeroes respectively $-\frac{1}{4}, \frac{1}{4}$.

SECTION - C

Short questions : (4 × 3 = 12)

1. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

2. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in the following:
$p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

3. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

4. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in case: $x^3 - 4x^2 + 5x - 2$; 2, 1, 1.

SECTION - D

Long questions (2 × 5 = 10)

1. For which values of a and b, are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? which zeroes of $p(x)$ are not the zeroes of $q(x)$?

2. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also, find all the zeroes of the two polynomials.

Chapter 3 Pair of Linear Equations in Two Variables

Mind Map

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

General Form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$a_1, b_1, c_1, a_2, b_2, c_2$, – Real numbers

Solution Graphically

Each solution (x, y), corresponds to a point on the line representing the equation and vice-versa

Algebraic Methods

By Substitution

Solve : $7x - 15y = 2$ ... (i) ; $x + 2y = 3$ ... (ii)

Solution : From equation (ii),

$x = 3 - 2y$ substitute value of x in equation (i)

$$7(3 - 2y) - 15y = 2$$

$$-29y = -19 \Leftrightarrow y = \frac{19}{29}$$

Now, from $x = 3 - 2y$

$$x = 3 - 2\left(\frac{19}{20}\right) = \left(\frac{49}{29}\right)$$

By Elimination

Solve : $x + 3y = 6$ ... (i)

$2x - 3y = 12$ ... (ii)

Now, Adding equation (i) and (ii)

$3 = 18$ or $x = 6$

Again, from (i) × 2 – (ii)

$3y = 0$ or, $y = 0$

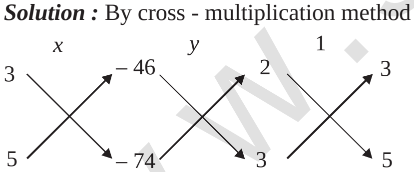
Hence, $x = 6, y = 0$

By Cross-Multiplication

Solve : $2x + 3y - 46 = 0$ ... (i)

$3x + 5y - 74 = 0$ ... (ii)

Solution : By cross - multiplication method



Then, $\frac{x}{3(-74) - 5(-46)} = \frac{y}{(-46)(3) - (-74)(2)} = \frac{1}{2(5) - 3(3)}$

$$\frac{x}{-222 + 230} = \frac{y}{-138 + 148} = \frac{1}{10 - 9} = \frac{x}{8} = \frac{y}{10} = \frac{1}{1}$$

$\Leftrightarrow \frac{x}{8} = \frac{1}{1}$ and $\frac{y}{10} = \frac{1}{1}$ i.e. $x = 8$ and $y = 10$

Graphical Representation

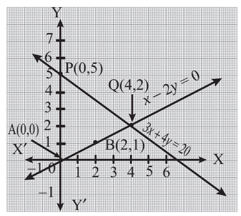
Pair of Lines $x - 2y = 0$

$3x + 4y - 20 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{4}, \frac{c_1}{c_2} = \frac{0}{-20}$

Compare the Ratios = $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Graphical Representation



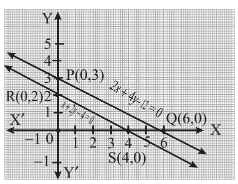
Intersecting Lines

Pair of Lines = $x + 2y - 4 = 0$

$2x + 4y - 12 = 0$

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4}, \frac{c_1}{c_2} = \frac{-4}{-12}$

Compare the Ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$



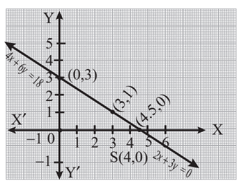
Parallel Lines

Pair of Lines = $2x + 3y - 9 = 0$

$4x + 6y - 18 = 0$

$\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{3}{6}, \frac{c_1}{c_2} = \frac{-3}{-18}$

Compare the Ratios $\frac{a}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$



Coincident Lines

Snapshot

3.1 In this chapter, we shall discuss about a linear equation in two variables, solution if a system of linear equations in two variables and graphics and algebraic methods of solving a solving a system of linear equations in two variables. We shall also be discussing about some applications of linear equations in two variables in different areas.

3.2 Simultaneous linear equations in two variables. A pair of linear equations in two variables is said to form of a system of simultaneous linear equations. The general forms of a pair of linear equations in two variables x and y is

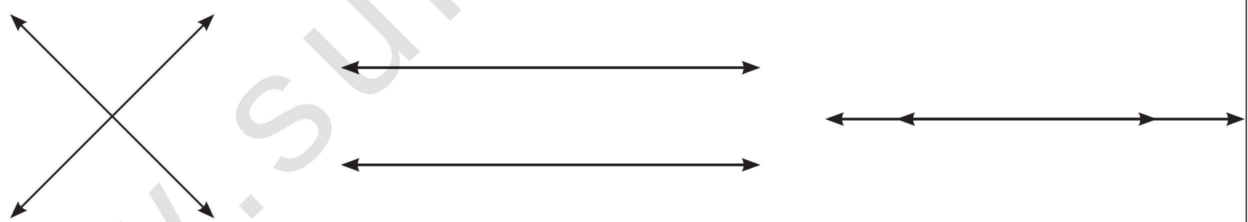
$$a_1x + b_1y + c_1 = 0 \text{ and}$$
$$a_2x + b_2y + c_2 = 0$$

Where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers and $a_1^2 + b_1^2 \neq 0$, $a_2^2 + b_2^2 \neq 0$. This is known as the algebraic representation of a system of simultaneous linear equations in two variables. A pair of values if the variables x and y satisfying each one of the equations in a given system of two simultaneous linear equations in x and y is called a solution of the system.

A system of simultaneous linear equations is said to be consistent, if it has atleast one solution. On the other hand, a system of simultaneous linear equations is said to be in-consistent, if it has no solution.

3.3 Graphical representation of linear equations in two variables is represented as a straight line. Both to be considered together. However, given two lines in a plane, only one of the following three possibilities can takes place:

(a) The two lines will intersect at one point.
(b) The two lines will not intersect, i.e., they are parallel
(c) The two lines will be coincident.



(a) (b) (c)

Solved NCERT Exercise Questions

Example 1 :

Let us take the example given in Section 3.1. Akhila goes to a fair with ₹ 20 and wants to have rides on the Giant Wheel and play Hoopla. Represent this situation algebraically and graphically (geometrically).

Sol. Let x be the number of rides she had on Giant Wheel and y be the number of times she played Hoopla.

$\Rightarrow \quad y = \frac{1}{2}x$

$2y = x \Rightarrow 2y - x = 0$ ...(1)

Since, each ride costs ₹3 and game of Hoopla cost ₹4

$\Rightarrow \quad 3x + 4y = 20$ ...(2)

For equation (1)

$\Rightarrow \quad 2y - x = 0$

When $x = 0$

$\Rightarrow \quad 2y - 0 = 0$

$\Rightarrow \quad y = 0$

[0, 0] is a solution.

When $x = 2$

$\Rightarrow \quad 2y - 2 = 0$

$\Rightarrow \quad 2y = 2$

$\Rightarrow \quad y = 1$

[2, 1] is a solution.

For equation (2)

$\Rightarrow \quad 3x + 4y = 20$

When $x = 0$

$\Rightarrow \quad 3(0) + 4y = 20$

$\Rightarrow \quad 4y = 20$

$\Rightarrow \quad y = 5$

[0, 5] is a solution.

When $x = 4$

$\Rightarrow \quad 3(4) + 4y = 20$

$\Rightarrow \quad 12 + 4y = 20$

$\Rightarrow \quad 4y = 8$

$\Rightarrow \quad y = 2$

[4, 2] is a solution.

Equation (1)

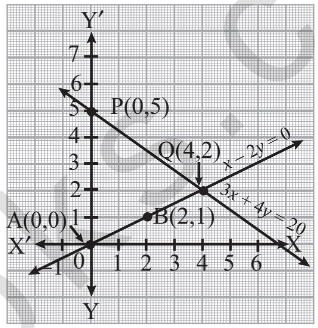
$\Rightarrow \quad 2y - x = 0$

x	0	2
y	0	1

Equation (2)

$\Rightarrow \quad 3x + 4y = 20$

x	0	4
y	5	2



Example 2 :

Romila went to a stationery shop and purchased 2 pencils and 3 erasers for ₹9. Her friend Sonali saw the new variety of pencils and erasers with Romila, and she also bought 4 pencils and 6 erasers of the same kind for ₹18. Represent this situation algebraically and graphically.

Sol. Let x be the cost of pencil and y be the cost of eraser.

According to conditions

$\Rightarrow \quad 2x + 3y = 9$ ...(1)

$4x + 6y = 18$ ...(2)

For equation (1)

$\Rightarrow \quad 2x + 3y = 9$

When $x = 0$

$\Rightarrow \quad 2(0) + 3y = 9$

$\Rightarrow \quad 3y = 9$

$\Rightarrow \quad y = 3$

[0, 3] is a solution.

When $x = 3$

$\Rightarrow \quad 2(3) + 3y = 9$

$\Rightarrow \quad 6 + 3y = 9$

$\Rightarrow \quad 3y = 3$

$\Rightarrow \quad y = 1$

[3, 1] is a solution.

For equation (2)

$\Rightarrow \quad 4x + 6y = 18$

Equation (2) ÷ 2

$\Rightarrow \quad 2x + 3y = 9$

Thus, (0,3) and (3, 1) is solution. Hence,

For Equation (1)

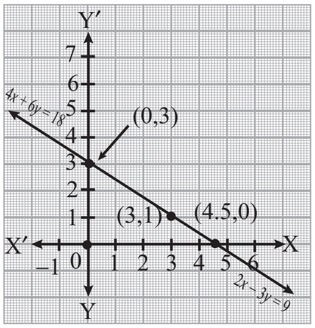
$\Rightarrow \quad 2x + 3y = 9$

x	0	3
y	3	1

For Equation (2)

$\Rightarrow \quad 4x + 6y = 18$

x	0	3
y	3	1



Example 3 :

Two rails are represented by the equations x + 2y – 4 = 0 and 2x + 4y – 12 = 0. Represent this situation geometrically.

Sol. Given equations are,

$x + 2y = 4$ ...(1)

$2x + 4y = 12$ ...(2)

For equation (1)

$\Rightarrow \quad x + 2y = 4$

When $\quad x = 0$

$\Rightarrow \quad 0 + 2y = 4$

$\Rightarrow \quad y = 2$

[0, 2] is a solution.

When $\quad x = 2$

$\Rightarrow \quad 2 + 2y = 4$

$\Rightarrow \quad 2y = 2$

$\Rightarrow \quad y = 1$

[2, 1] is a solution.

For equation (2)

$\Rightarrow \quad 2x + 4y = 12$

Equation (2) ÷ 2

$\Rightarrow \quad x + 2y = 6$

When $\quad x = 0$

$\Rightarrow \quad 0 + 2y = 6$

$\Rightarrow \quad 2y = 6$

$\Rightarrow \quad y = 3$

When $\quad x = 2$

$\Rightarrow \quad 2 + 2y = 6$

$\Rightarrow \quad 2y = 4$

$\Rightarrow \quad y = 2$

Thus, (0,3) and (2, 2) are solutions. Hence,

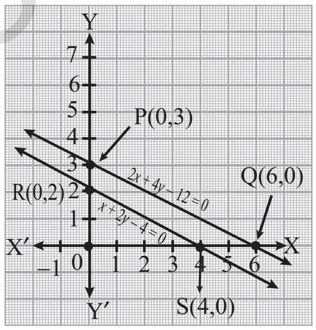
For Equation (1)

$\Rightarrow \quad x + 2y = 4$

x	0	2
y	2	1

For Equation (2) $\Rightarrow 2x + 4y = 12$

x	0	2
y	3	2



Exercise 3.1

1. ***Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.***

> **Concept:** In this question, assume the present ages as x and y, then find their age 7 years from now and 3 years ago in terms of x and y to obtain two linear equations. Using algebraic equation and truth table represent it graphically.

Sol. Let the present age of Aftab be x.

And, present age of his daughter = y

Seven years ago,

Age of Aftab = $x - 7$

Age of his daughter = $y - 7$

According to the condition,

$$\text{Aftab age} = 7 \text{ (his daughter's age)}$$
$$(x-7) = 7(y-7)$$
$$x-7 = 7y-49$$
$$x-7y = -42 \quad \text{...(i)}$$

After three years,

$$\text{Age of Aftab} = x+3$$
$$\text{Age of his daughter} = y+3$$

According to the question,

$$(x+3) = 3(y+3)$$
$$(x+3) = 3y+9$$
$$x-3y = 6 \quad \text{...(ii)}$$

Therefore, the algebraic representation of equation (i)

$$x-7y = -42$$
$$x-3y = 6$$
$$\text{For } x-7y = -42,$$
$$x = -42+7y$$

The solution table is

x	–7	0	7
y	5	6	7

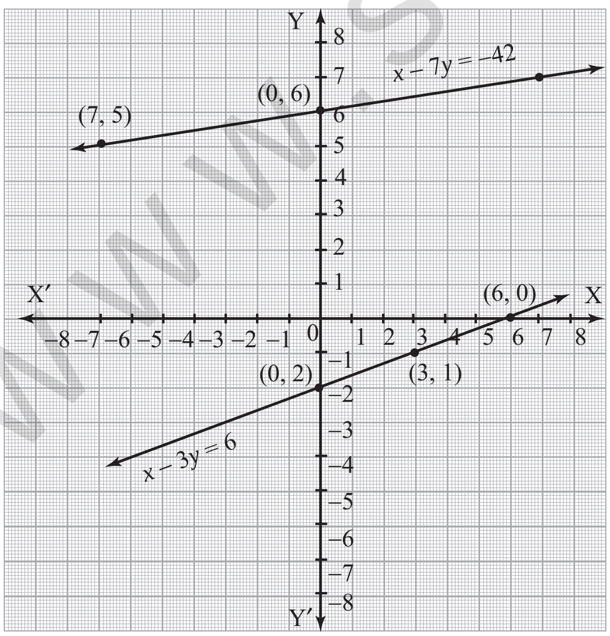
Therefore the algebraic representation of equation (i)

$$\text{For } x-3y = 6.$$
$$x = 6+3y$$

The solution table is

x	6	3	0
y	0	–1	–2

The graphical representation is as follows.



2. ***The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, she buys another bat and 3 more balls of the same kind for ₹1300. Represent this situation algebraically and geometrically.***

> **Concept:** In this question, assume the cost of 1 bat as ₹ x and the cost of 1 ball as ₹ y to obtain two linear equations.

Sol. Let the cost of a bat be ₹ x.

And, cost of a ball = ₹ y

Cost of 3 bats = ₹ $3x$

According to the question, the algebraic representation is

$$3x+6y = 3900$$
$$\Rightarrow \quad x+2y = 1300 \quad \text{... (1)}$$
$$\text{Also cost of 1 bat} = 1x$$
$$\text{Cost of 3 balls} = 3y$$
$$x+3y = 1300 \quad \text{... (2)}$$

Thus, (1) and (2) are the algebraic representation of the given situation.

Geometrical representation :

We have, for equation (1) :

$$x+2y = 1300$$
$$\Rightarrow \quad y = \frac{1300-x}{2}$$

The solution table is

x	300	100	–100
y	500	600	700

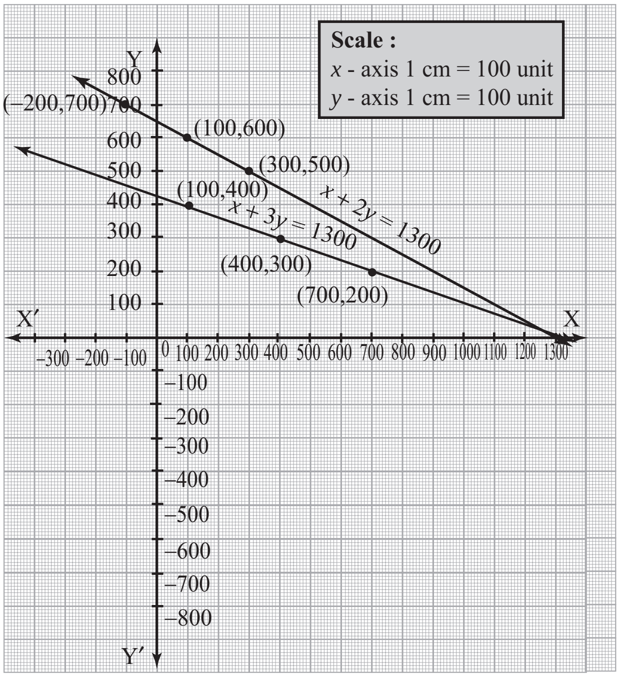
and for equation (2)

$$x+3y = 1300,$$
$$y = \frac{1300-x}{3}$$

The solution table is

x	400	700	100
y	300	200	400

The graphical representation is as follows.



3. ***The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹ 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹ 300. Represent the situation algebraically and geometrically.***

> **Concept:** In this question, assume the cost of 1kg of apples as ₹ x and the cost of 1kg of grapes as ₹ y to obtain two linear equations.

Sol. Let the cost of 1 kg of apples be ₹ x.

And, cost of 1 kg of grapes = ₹ y

According to the question, the algebraic representation is

$$2x + y = 160$$
$$4x + 2y = 300$$

For $2x + y = 160$,
$$y = 160 - 2x$$

The solution table is

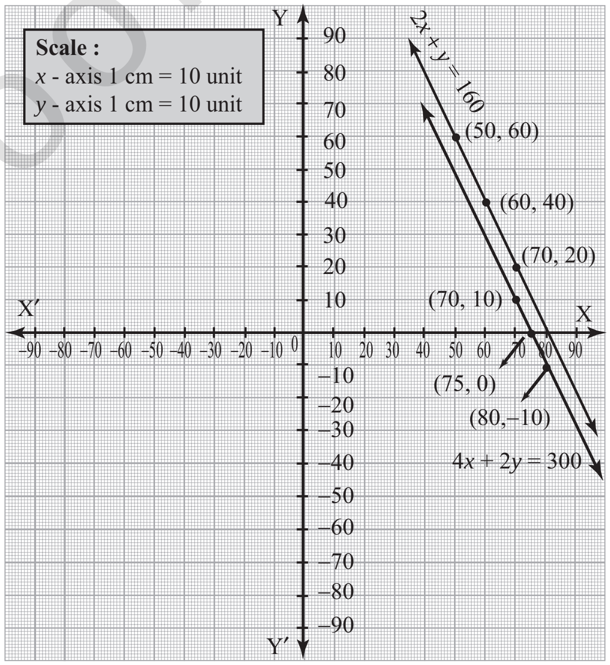
x	50	60	70
y	60	40	20

For $4x + 2y = 300$,
$$y = \frac{300 - 4x}{2}$$

The solution table is

x	70	80	75
y	10	–10	0

The graphical representation is as follows.



Snapshot

3.3.1 Graphical method of solution of a pair of linear equations

In this section, we will use the knowledge of construction of graphs of linear equations in sowing systems if simultaneous linear equations in two variables. If there is a system of simultaneous linear equations in two variables such that the lines representing the equations intersect at a point $p(\alpha, \beta)$. The point p lies on both the lines, so that its coordinates will satisfy both the equations in the system. Therefore, $x = \alpha$, $y = \beta$ is the solution of the given system of equations. If the lines represented by the two equations are coincident, they have infinitely many common points. On the other hand, if the lines represented by the two equations are parallel, they do not have the common point and so the system has no solution, i.e., it is in-consistent.

The procedure for solving a system of simultaneous linear equations in two variables by drawing their graphs is known as the graphical method. The algerithm used to solve the linear equation is as follows:

Step 1 : Obtain the given system of simultaneous linear equations in x and y.

Step 2 : Draw the graphs of the equations representing the lines, l_1 and l_2.

Step 3 : If the lines l_1 and l_2 intersect at a point and $(\alpha, \beta)$are the co-ordinates, then the given system has a unique. Solution given by $x = \alpha$, $y = \beta$ otherwise go to Step 1,

Step 4 : If the lines l_1 and l_2 are coincident, then the system is consistent and has infinitely many solutions. In this case, every solution of one of the equations is a solution of one system, otherwise go to Step 5.

Step 5 : if the lines l_1 and l_2 are parallel, then the given system of equations is in-consistent and hence, it has no solution.

Example 4 :

Check graphically whether the pair of equations

$$x + 3y = 6 \quad ...(1)$$
$$\text{and } 2x - 3y = 12 \quad ...(2)$$

Sol. Given equations are,

$$x + 3y = 6 \quad ...(1)$$
$$2x - 3y = 12 \quad ...(2)$$

For equation (1)

$\Rightarrow$ $x + 3y = 6$

When $x = 0$

$\Rightarrow$ $0 + 3y = 6$

$\Rightarrow$ $3y = 6$

$\Rightarrow$ $y = 2$

When $x = 6$

$\Rightarrow$ $6 + 3y = 6$

$\Rightarrow$ $3y = 0$

$\Rightarrow$ $y = 0$

Thus, [0, 2] and [6, 0] are solutions.

For equation (2)

$\Rightarrow$ $2x - 3y = 12$

When $x = 0$

$\Rightarrow$ $2(0) - 3y = 12$

$\Rightarrow$ $-3y = 12$

$\Rightarrow$ $y = -4$

When $x = 6$

$\Rightarrow$ $2(6) - 3y = 12$

$\Rightarrow$ $-3y = 0$

$\Rightarrow$ $y = 0$

Thus, (0, – 4) and (6, 0) are solutions.

Hence,

For Equation (1)

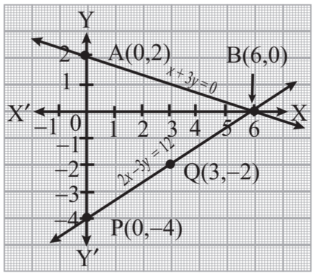
$\Rightarrow$ $x + 3y = 6$

x	0	6
y	2	0

For Equation (2)

$\Rightarrow$ $2x - 3y = 12$

x	0	6
y	– 4	0



Since the lines intersect at (6, 0), the given equations are consistent.

Thus, the solution is (6, 0)

Example 5 :

Graphically, find whether the following pair of equations has no solution,unique solution or infinitely many solutions:

$$5x - 8y + 1 = 0$$

$$3x - \frac{24}{5}y + \frac{3}{5} = 0$$

Sol. Given equations are,

$$5x - 8y + 1 = 0$$

$$\Rightarrow \quad 5x - 8y = -1 \quad ...(1)$$

$$3x - \frac{24}{5}y + \frac{3}{5} = 0$$

Multiplying by 5 we get,

$$\Rightarrow \quad 15x - 24y + 3 = 0 \quad ...(2)$$

For equation (1)

When $x = 0$, $-8y + 1 = 0 \Rightarrow y = \frac{1}{8} = 0.125$

When $y = 0$, $5x + 1 = 0 \Rightarrow x = -\frac{1}{5} = -0.2$

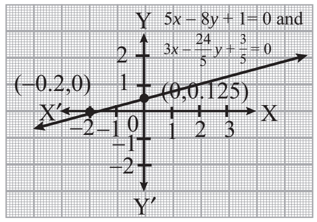
x	0	– 0.2
y	0.125	0

For equation (2)

When $x = 0$, $y = +\frac{1}{8} = +0.125$

When $y = 0$, $x = -\frac{1}{5} = -0.2$

x	0	– 0.2
y	0.125	0



Since the given lines coincide, they have infinitely many solution.

Example 6 :

Champa went to a 'Sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased". Help her friends to find how many pants and skirts Champa bought.

Sol. Let x be the number of pants and y be the number of skirts purchased by Champa.

According to the conditions,

$$y = (2x) - 2$$

$$\Rightarrow \quad 2x - y = 2 \quad ...(1)$$

$$y = 4x - 4$$

$$4x - y = 4 \quad ...(2)$$

For equation (1)

$\Rightarrow \quad 2x - y = 2$

When $x = 0$

$\Rightarrow \quad 2(0) - y = 2$

$\Rightarrow \quad y = -2$

When $x = 1$

$\Rightarrow \quad 2(1) - y = 2$

$\Rightarrow \quad y = 0$

Thus, [0, –2] and [1, 0] are solutions.

For equation (2)

$\Rightarrow \quad 4x - y = 4$

When $x = 0$

$\Rightarrow \quad 4(0) - y = 4$

$\Rightarrow \quad y = -4$

When $x = 1$

$\Rightarrow \quad 4(1) - y = 4$

$\Rightarrow \quad y = 0$

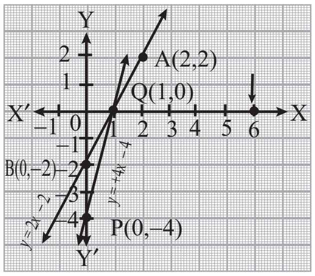
Thus, (0, –4) and (1, 0) are solutions. Hence, Equation (1)

$\Rightarrow \quad 2x - y = 2$

x	0	1
y	–2	0

Equation (2) $\Rightarrow 4x - y = 4$

x	0	1
y	–1	0



Thus, solution of equations given is (1, 0)

Therefore,

Number of pants and skirts bought by Champa are 1 and 0 respectively.

Exercise 3.2

1. ***Form the pair of linear equations in the following problems, and find their solutions graphically.***

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

Concept:

(i) In this question, assume the number of boys as x and the number of girls as y to obtain two linear equations.

(ii) In this question, assume the cost of 1 pencil as ₹ x and the cost of 1 pen as ₹ y to obtain two linear equations.

Sol. (i) Let the number of girls be x and the number of boys be y.

According to the question, the algebraic representation is

$x + y = 10$ ... (1)

$x - y = 4$ [$\because y = 4 + x$] ... (2)

For $x + y = 10$, [From equation (1)]

$x = 10 - y$

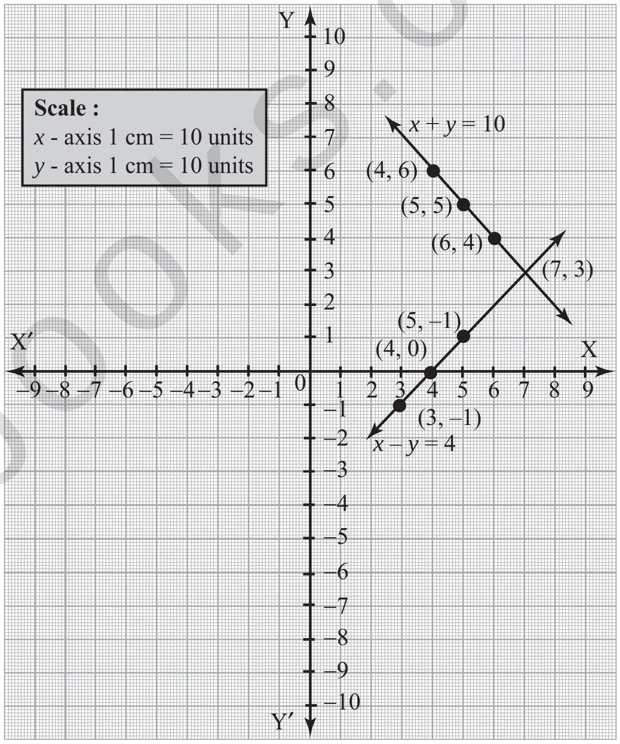
x	5	4	6
y	5	6	4

For $x - y = 4$, [From equation (2)]

$x = 4 + y$

x	5	4	3
y	1	0	–1

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (7, 3).

Therefore, the number of girls and boys in the class are 7 and 3 respectively.

Sol. (ii) Let the cost of 1 pencil be ₹ x and the cost of 1 pen be ₹ y.

According to the question, the algebraic representation is

$5x + 7y = 50$ ...(1)

$7x + 5y = 46$ ... (2)

For $5x + 7y = 50$, [From equation (1)]

$$x = \frac{50 - 7y}{5}$$

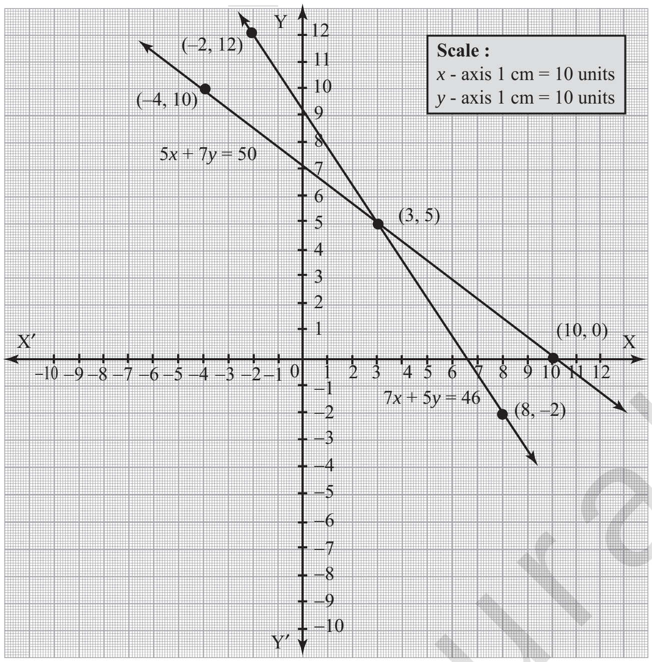
x	3	10	–4
y	5	0	10

$$7x + 5y = 46 \quad \text{[From equation (2)]}$$

$$x = \frac{46 - 5y}{7}$$

x	8	3	–2
y	–2	5	12

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (3, 5).

Therefore, the cost of a pencil and a pen are ₹ 3 and ₹ 5 respectively.

2. ***On comparing the ratios*** $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ ***and*** $\frac{c_1}{c_2}$***, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:***

(i) $5x - 4y + 8 = 0$, $7x + 6y - 9 = 0$

(ii) $9x + 3y + 12 = 0$, $18x + 6y + 24 = 0$

(iii) $6x - 3y + 10 = 0$, $2x - y + 9 = 0$

Concept: We know that, For any pair of linear equation

$a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$

(a) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (Intersecting Lines)

(b) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (Coincident Lines)

(c) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (Parallel Lines)

Sol. **(i)** $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Comparing these equations with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we obtain

$a_1 = 5, b_1 = -4, c_1 = 8$

$a_2 = 7, b_2 = 6, c_2 = -9$

$$\frac{a_1}{a_2} = \frac{5}{7}$$

$$\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the lines representing the given pair of equations have a unique solution and the pair of lines intersects at exactly one point.

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing these equations with $a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$, we obtain

$a_1 = 9, \quad b_1 = 3, \quad c_1 = 12$

$a_2 = 18, \quad b_2 = 6, \quad c_2 = 24$

$$\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$