APTITUDE & MENTAL ABILITY TEST

MATHEMATICS

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Simple Arithmetic Computations

The following formulas will be very useful.

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^2-b^2 = (a+b)(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^4 - b^4 = (a-b)(a+b)(a^2+b^2)$

$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

$(a+b)^2 - (a-b)^2 = 4ab.$

H.C.F.

Factors which divide two or more numbers without a remainder are called their Common Factor. Out of the common factors, the greatest is called the **Highest Common Factor** (H.C.F.) of those numbers.

(e.g) The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
The factors of 30 are 1, 2, 3, 5, 6, 10, 15, 30
Their common factors are 1, 2, 3, 6
The greatest of them is 6
Hence H.C.F. of 24 and 30 is 6.

L.C.M.

The lowest common multiple of two or more expressions or numbers is called the Lowest or Least Common Multiple (L.C.M.) of the two or more expressions.

(e.g) L.C.M. of 6, 8 and 10.
$6 = 2 \times 3$; $8 = 2\times2\times2$; $10 = 2\times5$
L.C.M. $= 2^3 \times 3 \times 5 = 120$.

Fractions and to simplify the expressions

In questions on fraction, signs +, –, ×, ÷ (of signifies multiplication) and brackets are often involved. In simplifying these questions, the following order must be followed.

1. Begin with brackets, solve the part found within brackets. Also evaluate the brackets in the order ( ), { }, [ ].
2. Then, come to division.
3. Then, do multiplication.
4. Finally add and subtract.

The rule is BODMAS

'B' stands for brackets; 'O' for of; 'D' for division; 'M' for multiplication;

'A' for addition and 'S' for subtraction.

(e.g) $(90 \times 35+90 \times 15) - (90 \times 10)$
$= 90\ (35+15) - (90\times10) = 90 \times 50 - 90 \times 10$
$= 90\times50 - 900 = 4500 - 900 = 3600$

Decimal fraction

A fraction involving decimal point is called Decimal Fraction. For converting decimal fraction into vulgar fraction, write down the given number in the numerator omitting the decimal point and for the denominator write 1 followed by as many zeros as there are figures on the right of the decimal point.

(e.g) $46.78 = \frac{4678}{100}$; $188.0834 = \frac{1880834}{10000}$

Square Root

To find the square root, divide the number into periods of two digits each, by placing dot over every second figure beginning with that in the units place and proceeding towards the left. Find the greatest number whose square is contained in the first period. This will be the first figure of the required square root, subtract its square from the first period, and to the remainder bring down the next period.

(e.g) Find the square root of 980100

```
        990
    9 | 98,01,00
      | 81
  189 | 1701
      | 1701
    0 | 000
```

∴ Square root = 990.

Example Problems

1. Find the value of
$12 \times 12 + 2 \times 12 \times 15 + 15 \times 15$.

Solution : $12 \times 12 + 2 \times 12 \times 15 + 15 \times 15$
$= (12 + 15)^2 = (27)^2$
$= 729$ [$\because (a+b)^2 = a^2 + 2ab + b^2$]

2. Find the value of $(99)^2$

Solution : $(99)^2 = (100 - 1)^2$ use $(a - b)^2$
$= (100)^2 - 2 \times 100 \times 1 + (1)^2$
$= 10000 - 200 + 1 = 9801$

3. Find the value of $(999)^2$

Solution : $(999)^2 = (1000 - 1)^2$
$= (1000)^2 - 2 \times 100 \times 1 + (1)^2$
$= 10,00,000 - 2000 + 1 = 9,98,001$

4. Find the value of $(65^2 - 15^2)$ use $a^2 - b^2 = (a+b)(a-b)$
$(65^2-15^2) = (65 - 15)(65 + 15)$

Solution : $= (50)(80) = 4000$

5. Find the value of $(12.122)^2 - (12.022)^2$

$(12.122)^2 - (12.022)^2$

Solution : $[a^2 - b^2] = (a+b)(a-b)$

$= (12.122 - 12.022)(12.122 + 12.022)$

$= (0.1)(24.144)$

$= 2.4144$

6. Find the value of $\dfrac{(25\times25)-(15\times15)}{(25+15)}$

Solution : $\left[\dfrac{a^2-b^2}{a+b} = \dfrac{(a+b)(a-b)}{a+b} = a-b\right]$

Here, a = 25 ; b = 15

$\therefore \dfrac{(25\times25)-(15\times15)}{(25+15)} = \dfrac{(25+15)\times(25-15)}{(25+15)}$

$= 25 - 15 = 10$

7. Find the value of $\dfrac{15^3-4^3}{15^2+15\times4+4^2}$

Solution : $\left[\dfrac{a^3-b^3}{a^2+ab+b^2} = a-b\right]$ Here a = 15 ; b = 4

$\therefore \dfrac{15^3-4^3}{15^2+15\times4+4^2} = 15 - 4 = 11$

8. Find the value of $\dfrac{1.7\times1.7\times1.7-1.5\times1.5\times1.5}{1.7\times1.7+1.7\times1.5+1.5\times1.5}$

Solution :

The required value $= \dfrac{(1.7)^3-(1.5)^3}{(1.7)^2+(1.7)(1.5)+(1.5)^2}$

$= 1.7 - 1.5 = 0.20$

9. Simplify $\dfrac{(.57)^3-(0.1)^3}{(.57)^2+(0.57)(0.1)+(0.1)^2} + \dfrac{0.67}{1+\frac{24}{43}}$

Solution : Solution : The first term can be simplified by using

$= \dfrac{a^3-b^3}{a^2+ab+b^2} = a-b$

Here a = 0.57 and b = 0.1

$\therefore$ The required value $= 0.57 - 0.1 + \dfrac{0.67}{1+\frac{24}{43}}$

$= 0.47 + 0.67 \times \dfrac{43}{67}$

$= 0.47 + \dfrac{43}{67} = 0.47 + 0.43 = 0.90$

10. Find the value of $15^4 - 5^4$

Solution :

$(a^4 - b^4) = (a+b)(a-b)(a^2+b^2)$

Here a $= 15$; b $= 5$

$15^4 - 5^4 = (15 + 5)(15 - 5)(15^2 + 5^2)$

$= 20 \times 10 \times 250 = 50{,}000$

11. Find the value of 6 + [5 – {4 + (3 – (2 + 1)}]

Solution : {The rule to be followed for this type of problems is BODMAS and brackets in the order (), {}, []}

$\therefore 6 + [5 - \{4 + (3 - 2 + 1)\}]$

$= 6 + [5 - \{4 + (3 - 3)\}]$

$= 6 + [5 - \{4 + 0\}] = 6 + [5 - 4] = 7$

12. Determine the value of

1 + [2 – {3 + (4 – 5 + 6)}]

Solution : The given expression

$= 1 + [2 - \{3 + (4 - 5 + 6)\}]$

$= 1 + [2 - \{3 + (4 + 1)\}]$

$= 1 + [2 - \{3 + 5\}]$

$= 1 + [2 - 8] = 1 + [-6] = 1 - 6 = -5$

13. Simplify $\dfrac{2\frac{4}{9}\div3\frac{2}{3}\text{ of }\frac{2}{5}\times\frac{3}{5}+1\frac{1}{9}}{1\frac{1}{9}\times\frac{3}{4}\text{ of }1\frac{2}{5}\div\frac{21}{38}-\frac{1}{3}} - \dfrac{5\frac{1}{2}-\frac{3}{4}}{2\frac{1}{5}\times1\frac{9}{11}}$

Solution :

$= \dfrac{\frac{22}{9}\div\frac{11}{3}\,of\,\frac{2}{5}\times\frac{3}{5}+\frac{10}{9}}{\frac{10}{9}\times\frac{3}{4}\,of\,\frac{7}{5}\div\frac{21}{38}-\frac{1}{3}} - \dfrac{\frac{11}{2}-\frac{3}{4}}{\frac{11}{5}\times\frac{20}{11}}$

$= \dfrac{\frac{22}{9}\div\frac{22}{15}\times\frac{3}{5}+\frac{10}{9}}{\frac{10}{9}\times\frac{21}{20}\div\frac{21}{38}-\frac{1}{3}} \quad \dfrac{\frac{22}{4}\ \ 3}{}$

$= \dfrac{\left(\frac{22}{9}\times\frac{15}{22}\right)\times\frac{3}{5}+\frac{10}{9}}{\frac{10}{9}\times\left(\frac{21}{20}\times\frac{38}{21}\right)-\frac{1}{3}} - \dfrac{19}{4}\times\dfrac{1}{4}$

$= \dfrac{\frac{5}{3}\times\frac{3}{5}+\frac{10}{9}}{\frac{10}{9}\times\frac{19}{10}-\frac{1}{3}} - \dfrac{19}{16} = \dfrac{1+\frac{10}{9}}{\frac{19}{9}-\frac{1}{3}} - \dfrac{19}{16}$

$\dfrac{19}{16}\times\dfrac{9}{16} - \dfrac{19}{16} = \dfrac{19}{16} - \dfrac{19}{16} = 0$

14. Simplify $7\frac{1}{2} - \frac{1}{9}\left[3\frac{3}{4}\div\left\{\frac{5}{6}\text{ of }\frac{2}{3}\left(\frac{1}{3}-\left(\frac{1}{4}-\frac{1}{6}\right)\right)\right\}\right]$

Solution $= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4}\div\left\{\frac{5}{6}\,of\,\frac{2}{3}\left(\frac{1}{3}-\left(\frac{3-2}{12}\right)\right)\right\}\right]$

$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4}\div\left\{\frac{5}{6}\,of\,\frac{2}{3}\left(\frac{1}{3}-\frac{1}{12}\right)\right\}\right]$

$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{5}{6} of \frac{2}{3}\left(\frac{3}{12}\right)\right\}\right]$

$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \left\{\frac{10}{18}\left(\frac{3}{12}\right)\right\}\right]$

$= 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \div \frac{5}{36}\right] = 7\frac{1}{2} - \frac{1}{9}\left[\frac{15}{4} \times \frac{36}{5}\right]$

$= 7\frac{1}{2} - \frac{1}{9} \times [3 \times 9] = 7\frac{1}{2} - \frac{1}{9} \times 27$

$= 7\frac{1}{2} - 3 = 4\frac{1}{2}$

15. Simplify $\frac{0.1 \times 0.1 \times 0.1 + 0.01 + 0.01 \times 0.01}{0.2 \times 0.2 \times 0.2 + 0.08 + 0.04 \times 0.02}$

Solution $= \frac{\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} + \frac{1}{100} + \frac{1}{100} \times \frac{1}{100}}{\frac{2}{10} \times \frac{2}{10} \times \frac{2}{10} + \frac{8}{100} + \frac{4}{100} \times \frac{2}{100}}$

$= \frac{\frac{1}{1000} + \frac{1}{100} + \frac{1}{10000}}{\frac{8}{1000} + \frac{8}{100} + \frac{8}{10000}}$

$= \frac{\left(\frac{1}{1000} + \frac{1}{100} + \frac{1}{10000}\right)}{8\left(\frac{1}{1000} + \frac{1}{100} + \frac{1}{10000}\right)} = \frac{1}{8}$

16. Simplify 15 of

$$\frac{\frac{2}{3} of \frac{4}{9} - 3 \times \frac{3}{5} of \frac{4}{9} + 3 \times \frac{9}{25} of \frac{2}{3} - 0.6 of 0.36}{\frac{4}{9} - 2 \times 0.6 of \frac{2}{3} + 0.36}$$

Solution :

$= 15 \text{ of } \frac{\frac{2}{3} \times \frac{4}{9} - 3 \times \frac{3}{5} \times \frac{4}{9} + 3 \times \frac{9}{25} \times \frac{2}{3} - \frac{6}{10} \times \frac{36}{100}}{\frac{4}{9} - 2 \times \frac{6}{10} \times \frac{2}{3} + \frac{36}{100}}$

$= 15 \text{ of } \frac{\frac{8}{27} - \frac{4}{5} + \frac{18}{25} - \frac{216}{1000}}{\frac{4}{9} - \frac{4}{5} + \frac{36}{100}}$

$= 15 \text{ of } \frac{8000 - 21600 + 19440 - 5832}{\frac{27000}{400 - 720 + 324}}$

$= 15 \text{ of } \frac{\frac{8}{27000}}{\frac{4}{900}} = 15 of \frac{8}{27000} \times \frac{900}{4}$

$= 15 \times \frac{1}{15} = 1$

17. Simplify : $\frac{500 \times 0.00125}{6.25}$

$= \frac{500 \times \frac{125}{100000}}{\frac{625}{100}} = 500 \times \frac{125}{100000} \times \frac{100}{625}$

$= \frac{1}{10} = 0.1$

18. Find the H.C.F. of 630, 1050, 1260

630 = 2.3.3.5.7
1050 = 2.3.5.5.7
1260 = 2.2.3.3.5.7
∴ H.C.F. is 2.3.5.7 = 210

19. What is the highest number of four digits which will leave a remainder of 1 when divided by any of the numbers 6,9,12,15 or 18 ?

L.C.M. of 6, 9, 12, 15, 18 = 180
Greatest no. of 4 digits = 9999
Greatest no. of 4 digits divisible by 180 is
= 9999 – 99 = 9900

```
        55
   ------------
180 | 9999
      900
   ------------
      999
      900
   ------------
       99
```

∴ Required Number = 9900 + 1 = 9901

20. Find the square root of 2480625.

```
        1575
   ------------
1    | 2,48,06,25
       1
   ------------
25   | 148
       125
   ------------
307  | 2306
       2149
   ------------
3145 | 15725
       15725
   ------------
          0
```

∴ Square root of 2480625 is 1575.

21. Find the value of $\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$ correct to three decimal places.

$$\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}}$$

(Rationalising the denominator)

$$= \sqrt{\frac{(2+\sqrt{3})^2}{4-3}} = 2 + \sqrt{3}$$

$$= 2 + 1.732 = 3.732$$

22. Simplify : $\frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$ correct to three decimal places.

$$\frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{5-\sqrt{5}.\sqrt{3}}{5-3}$$

Square root of 15 = 3.872

$$\therefore = \frac{5-3.872}{2} = \frac{1.128}{2} = 0.564$$

23. Find the smallest fraction which, when added to the expression

$$\left(2\frac{1}{2}-1\frac{5}{8}\right)\left(2\frac{1}{4}+1\frac{7}{8}\right) \div 1\frac{3}{4}$$

will give a whole number.

$$2\frac{1}{2}-1\frac{5}{8} = \frac{5}{2}-\frac{13}{8}\ ;\ 2\frac{1}{4}+1\frac{7}{8} = \frac{9}{4}+\frac{15}{8}$$

$$\frac{20-13}{8} \times \frac{18+15}{8} \div \frac{7}{4} = \frac{33}{8}$$

$\therefore$ The given expression $= \left(\frac{7}{8}\right)\left(\frac{33}{8}\right) \div \frac{7}{4}$

$$\left(\frac{7}{8}\right)\left(\frac{33}{8}\right)\left(\frac{4}{7}\right) = \frac{7}{8} \times \frac{33}{8} \times \frac{4}{7} = \frac{33}{16} = 2\frac{1}{16}$$

$\therefore$ To make this a whole number, the smallest fraction required is $\frac{15}{16}$

[Note: Here the immediate higher whole number is 3]

Simple and Compound Interest

Introduction :

When a person is in need of money, he borrows it from a bank, a financial institution or from another person. He has to pay a charge on the sum of money he borrows. This charge is called the Interest. The sum that is borrowed is the principal. In this Chapter, we shall discuss the concepts of Simple Interest and Compound Interest and the methods of calculating them.

Notations used :

P - Principal
A - Amount
N - No. of years
R - Rate of interest
SI - Simple interest
CI - Compound interest

Important Formulae :

1. $SI = \frac{PNR}{100}$
2. $P = \frac{100 \times SI}{N \times R}$
3. $N = \frac{100 \times SI}{P \times R}$
4. $R = \frac{100 \times SI}{P \times N}$
5. $A = P + SI$
6. $CI = P\left(1+\frac{\quad}{100}\right)$ P

Where $P\left(1+\frac{R}{100}\right)^N$ is the amount on CI, where interest is compounded annually.

Note : a) When interest is compounded half-yearly Amount $= P\left(1+\frac{(R/2)}{100}\right)^{2n}$

b) When interest is compounded quarterly Amount $= P\left(1+\frac{(R/4)}{100}\right)^{4n}$

Example Problems

1. A construction company borrowed ₹ 8,50,000 for 5 years at interest rate $1\frac{1}{2}$ p.a. How much annual interest, the company had to pay?

A) ₹ 63,750 **B)** ₹ 1,27,500
C) ₹ 12,750 **D)** ₹ 25,000

Solution : **Ans : A**

$$\text{S.I.} = \frac{8,50,000 \times 3 \times 5}{2 \times 100} = 4250 \times 15 = ₹\ 63,750$$

2. A sum of ₹ 20,000 is to be divided between two brothers aged 15 years and 13 years, so that at the age of 18 they get equal amount, while the compound rate of interest is 5% per annum. What is the amount received by the younger brother?

A) ₹ 8,125.50 **B)** ₹ 1,213.50
C) ₹ 9,512.50 **D)** ₹ 1,000.50