SURA'S

MATHEMATICS

10th Standard

Strictly as per the Reduced (Prioritised) Syllabus released on 13th August, 2021 (G.O.(Ms).No126)

Salient Features

- This guide is specially prepared as per the reduced syllabus for the year 2021 - 22
- Model Question Papers 1 to 6 (PTA) : Questions are incorporated in the appropriate sections.
- Govt. Model Question Paper-2019, Quarterly Exam - 2019, Half yearly Exam - 2019, Supplementary Exam - 2020 are incorporated in the appropriate sections.
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PREFACE

The woods are lovely, dark and deep.
But I have promises to keep, and
miles to go before I sleep - Robert Frost

Respected Principals, Correspondents, Head Masters / Head Mistresses, Teachers, and dear Students.

From the bottom of our heart, we at SURA Publications sincerely thank you for the support and patronage that you have extended to us for more than a decade.

It is in our sincerest effort we take the pride of releasing **SURA'S Mathematics** for 10 Standard . This guide has been authored and edited by qualified teachers having teaching experience for over a decade in their respective subject fields. This Guide has been reviewed by reputed Professors who are currently serving as Head of the Department in esteemed Universities and Colleges.

With due respect to Teachers, I would like to mention that this guide will serve as a teaching companion to qualified teachers. Also, this guide will be an excellent learning companion to students with exhaustive exercises, additional problems and 1 marks as per new model in addition to precise answers for exercise problems.

In complete cognizance of the dedicated role of Teachers, I completely believe that our students will learn the subject effectively with this guide and prove their excellence in Board Examinations.

I once again sincerely thank the Teachers, Parents and Students for supporting and valuing our efforts.

God Bless all.

Subash Raj, B.E., M.S.
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All the Best

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Chapter 1

RELATIONS AND FUNCTIONS

EXERCISE 1.1

1. Find A× B, A×A and B ×A
(i) A = {2,−2,3} and B ={1,−4} (ii) A= B ={*p*,*q*}
(iii) A= {*m*,*n*} ; B = ϕ [PTA - 1]

Sol. (i) A = {2, –2, 3}, B = {1, –4}

A × B = {(2, 1), (2, –4), (–2, 1), (–2, –4), (3, 1), (3, –4)}

A × A = {(2, 2), (2, –2), (2, 3), (–2, 2), (–2, –2), (–2, 3), (3, 2), (3, –2), (3, 3)}

B × A = {(1, 2), (1, –2), (1, 3), (–4, 2), (–4, –2), (–4, 3)}

(ii) A = B = {(*p*,*q*)

A × B = {(*p*, *p*), (*p*, *q*), (*q*, *p*), (*q*, *q*)}

A × A = {(*p*, *p*), (*p*, *q*), (*q*, *p*), (*q*, *q*)}

B × A = {(*p*, *p*), (*p*, *q*), (*q*, *p*), (*q*, *q*)}

(iii) A = {*m*,*n*} , B = ϕ

A × B = { }

A × A = {(*m*,*m*), (*m*,*n*), (*n*, *m*), (*n*, *n*)}

B × A = { }

2. Let A = {1, 2, 3} and B = {*x* | *x* is a prime number less than 10}. Find A× B and B × A.

Sol. A = {1, 2, 3}, B = {2, 3, 5, 7}

A × B = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

3. If B ×A={(−2, 3),(−2, 4),(0, 3),(0, 4),(3, 3),(3, 4)} find A and B. [Qy - 2019]

Sol. Given B × A = {(–2, 3), (–2, 4), (0, 3), (0, 4), (3, 3), (3, 4)}

Here B = { –2, 0, 3}
[All the first elements of the order pair]

and A = {3, 4}
[All the second elements of the order pair]

4. If A ={5, 6}, B = {4, 5, 6} , C ={5, 6, 7}, Show that A×A = (B × B)∩(C ×C).

Sol. A = {5, 6}, B = {4, 5, 6}, C = {5, 6, 7}

A × A = {(5, 5), (5, 6), (6, 5), (6, 6)} ...(1)

B × B = {(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)} ...(2)

C × C = {(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)} ...(3)

(B × B) ∩ (C × C) = {(5, 5), (5, 6), (6, 5), (6, 6)} ...(4)

(1) = (4)

A × A = (B × B) ∩ (C × C). It is proved.

5. Given A ={1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if(A∩C) × (B∩D) = (A×B)∩(C × D) is true? [Qy - 2019]

Sol. LHS = {(A ∩ C) × (B ∩ D)

A ∩ C = {3}

B ∩ D = {3, 5}

(A ∩ C) × (B ∩ D) = {(3, 3) , (3, 5)} ...(1)

RHS = (A × B) ∩ (C × D)

A × B = {(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5)}

C × D = {(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)}

(A × B) ∩ (C × D) = {(3, 3), (3, 5)} ...(2)

∴ (1) = (2) ∴ It is true.

6. Let $A=\{x \in \mathbb{W} \mid x < 2\}$, $B = \{x \in \mathbb{N} \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ **[PTA - 2]**

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ **[PTA - 5]**

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Sol.

$A = \{x \in \mathbb{W} \mid x < 2\} = \{0, 1\}$

[Whole numbers less than 2]

$B = \{x \in \mathbb{N} \mid 1 < x \leq 4\} = \{2, 3, 4\}$

$C = \{3, 5\}$

[Natural numbers from 2 to 4]

LHS $= A \times (B \cup C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\} = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(1)

RHS $= (A \times B) \cup (A \times C)$

$(A \times B) = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1,5)\}$

$(A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(2)

(1) = (2), LHS = RHS Hence it is proved.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

LHS $= A \times (B \cap C)$

$(B \cap C) = \{3\}$

$A \times (B \cap C) = \{(0, 3), (1, 3)\}$ ...(1)

RHS $= (A \times B) \cap (A \times C)$

$(A \times B) = \{(0, 2), \underline{(0, 3)}, (0, 4), (1, 2), \underline{(1, 3)}, (1, 4)\}$

$(A \times C) = \{\underline{(0, 3)}, (0, 5), \underline{(1, 3)}, (1,5)\}$

$(A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ...(2)

$(1) = (2) \Rightarrow$ LHS = RHS.

Hence it is verified.

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

LHS $= (A \cup B) \times C$

$A \cup B = \{0, 1, 2, 3, 4\}$

$(A \cup B) \times C = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$ ...(1)

RHS $= (A \times C) \cup (B \times C)$

$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$(B \times C) = \{(2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$

$(A \times C) \cup (B \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$...(2)

(1) = (2)

$\therefore$ LHS = RHS. Hence it is verified.

7. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ **[Sep. - 2020]**

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ **[PTA - 1]**

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$

$C = \{2\}$

[$\because$ 2 is the only even prime number]

Sol. (i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

LHS $= (A \cap B) \times C$

$A \cap B = \{2, 3, 5, 7\}$

$(A \cap B) \times C = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ...(1)

RHS $= (A \times C) \cap (B \times C)$

$(A \times C) = \{(1, 2), \underline{(2, 2)}, \underline{(3, 2)}, (4, 2), \underline{(5, 2)}, (6, 2), \underline{(7, 2)}\}$

$(B \times C) = \{\underline{(2, 2)}, \underline{(3, 2)}, \underline{(5, 2)}, \underline{(7, 2)}\}$

$(A \times C) \cap (B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ...(2)

(1) = (2)

$\therefore$ LHS = RHS. Hence it is verified.

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

LHS $= A \times (B - C)$

$(B - C) = \{3, 5, 7\}$

$A \times (B - C) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\}$ ...(1)

RHS $= (A \times B) - (A \times C)$

$(A \times B) = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7), (7, 2), (7, 3), (7, 5), (7,7)\}$

$(A \times C) = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$

$(A \times B) - (A \times C) = (1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\}$ ...(2)

(1) = (2) ⇒ LHS = RHS. Hence it is verified.

EXERCISE 1.2

1. Let A = {1,2,3,7} and B = {3,0,–1,7}, which of the following are relation from A to B ?

(i) $\mathbb{R}_1 = \{(2,1), (7,1)\}$

(ii) $\mathbb{R}_2 = \{(-1,1)\}$

(iii) $\mathbb{R}_3 = \{(2,-1), (7,7), (1,3)\}$

(iv) $\mathbb{R}_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

Sol. Given A = {1, 2, 3, 7} and B = {3, 0, –1, 7}

(i) $R_1 = \{(2, 1), (7, 1)\}$

2 and 7 cannot be related to 1 since $1 \notin B$

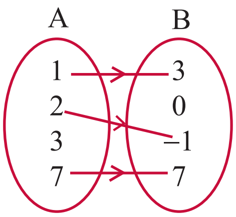
∴ R_1 is not a relation.

(ii) $R_2 = \{(-1, 1)\}$

–1 cannot be related to 1 since $-1 \notin A$ and $1 \notin B$

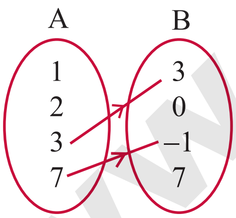
∴ R_2 is not a relation.

(iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$



R_3 is a relation since 2 is related to –1, 7 is related to 7 and 1 is related to 3.

(iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$



7 is related to –1
3 is related to 3
Since $0 \notin A$, 0 cannot be related to 3 and 7.
∴ R_4 is not a relation.

2. Let A={1, 2, 3, 4,...,45} and R be the relation defined as "is square of " on A. Write $\mathbb{R}$ as a subset of A × A. Also, find the domain and range of $\mathbb{R}$.

Sol. Given A ={1, 2, 3, 4, . . . 45}

∴ A × A = {(1, 1) (1, 2) (1,3) ... (1, 45) (2, 1) (2, 2) ... (2, 45) (45, 1) (45, 2) (45, 3) ... (45, 45)} ... (1)

R is defined as "is square of "

∴ R = {(1,1) (2,4) (3,9) (4,16) (5, 25) (6,36)}... (2)

[∵ 1 is the square of 1, 2 is the square of 4 and so on]

From (1) and (2), R is the subset of A × A

∴ R ⊂ A × A

Domain of R = {1, 2, 3, 4, 5, 6}

[All the first elements of the order pair in (2)]

Range of R = {1, 4, 9, 16, 25, 36}

[All the second elements of the order pair in (2)]

3. A Relation $\mathbb{R}$ is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range. **[PTA - 5]**

Sol. Given $\mathbb{R} = \{(x, y) / y = x + 3\}$ and $x \in \{0, 1, 2, 3, 4, 5\}$

When $x = 0$, $y = 0 + 3 = 3$ [∵ $y = x + 3$]

When $x = 1$, $y = 1 + 3 = 4$

When $x = 2$, $y = 2 + 3 = 5$

When $x = 3$, $y = 3 + 3 = 6$

When $x = 4$, $y = 4 + 3 = 7$

When $x = 5$, $y = 5 + 3 = 8$

∴ $\mathbb{R}$ = {(0, 3),(1, 4),(2, 5),(3, 6),(4, 7), (5, 8)}

∴ Domain of $\mathbb{R}$ = {0, 1, 2, 3, 4, 5}

[All the first element in $\mathbb{R}$]

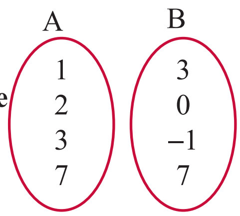
Range of $\mathbb{R}$ = {3, 4, 5, 6, 7, 8}

[All the second element in $\mathbb{R}$]

4. Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y)|x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$

(ii) $\{(x, y)|y = x + 3, x, y$ are natural numbers < 10}



(i) $R = \{(x, y)| x = 2y, x \in \{2, 3, 4, 5\}$ and $y \in \{1, 2, 3, 4\}\}$

When $x = 2$, $y = \frac{x}{2} = \frac{2}{2} = 1$

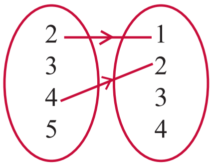
[∵ $x = 2y \Rightarrow y = \frac{x}{2}$]

When $x = 3$, $y = \frac{3}{2}$

When $x = 4$, $y = \frac{4}{2} = 2$

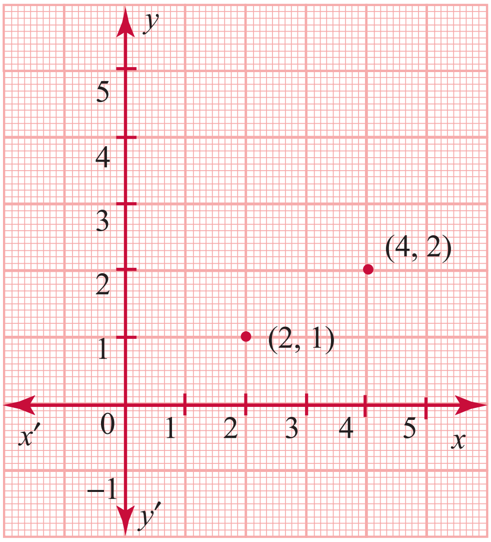
When $x = 5$, $y = \frac{5}{2}$

(a) an arrow diagram



3 cannot be related to $\frac{3}{2}$ and 5 cannot be related to $\frac{5}{2}$.

(b) a graph



(c) Roster form : R = {(2, 1), (4, 2)}

(ii) R = $\{(x, y) | y = x + 3$, x and y are natural numbers <10}

x = {1, 2, 3, 4, 5, 6, 7, 8, 9

y = {1, 2, 3, 4, 5, 6, 7, 8, 9}

[$\because$ x and y are natural numbers less than 10]

Given $y = x + 3$

When $x = 1$, $y = 1 + 3 = 4$

When $x = 2$, $y = 2 + 3 = 5$

When $x = 3$, $y = 3 + 3 = 6$

When $x = 4$, $y = 4 + 3 = 7$

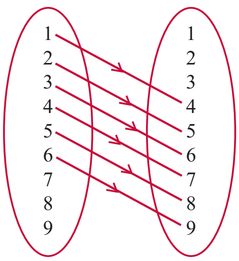
When $x = 5$, $y = 5 + 3 = 8$

When $x = 6$, $y = 6 + 3 = 9$

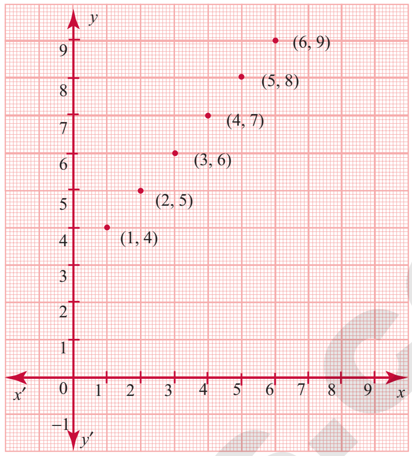
$\left.\begin{array}{ll} \text{When } x = 7, & y = 7 + 3 = 10 \\ \text{When } x = 8, & y = 8 + 3 = 11 \\ \text{When } x = 9, & y = 9 + 3 = 12 \end{array}\right\} [10, 11, 12 \notin y]$

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

(a) an arrow diagram



(b) a graph



(c) Roster form :

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation $\mathbb{R}$ is defined by $x\mathbb{R}y$, where x is the salary given to person y, express the relation $\mathbb{R}$ through an ordered pair and an arrow diagram.

Sol.

A – Assistants $\rightarrow$ A_1, A_2, A_3, A_4, A_5

C – Clerks $\rightarrow$ C_1, C_2, C_3, C_4

M – Managers $\rightarrow$ M_1, M_2, M_3

E – Executive officer $\rightarrow$ E_1, E_2

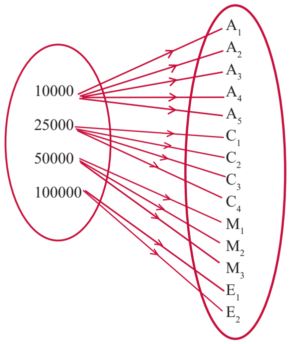
xRy is defined as x is the salary for assistants is ₹10,000, clerks is ₹25,000, Manger is ₹50,000 and for the executing officer ₹1,00,000.

(a) $\therefore$ R = {(10,000, A_1), (10,000, A_2), (10,000, A_3), (10,000, A_4), (10,000, A_5),

(25,000, C_1), (25,000, C_2), (25,000, C_3), (25,000, C_4)

(50,000, M_1), (50,000, M_2), (50,000, M_3), (1,00,000, E_1), (1,00,000, E_2)}

(b)



EXERCISE 1.6

Multiple choice questions.

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is

(A) 1 (B) 2 (C) 3 (D) 6

[Ans. (C) 3]

Hint:

$$\text{If } n(A \times B) = 6$$
$$A = \{1, 1\},\ n(A) = 2$$
$$n(B) = 3$$

2. $A=\{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ is [PTA - 3]

(A) 8 (B) 20 (C) 12 (D) 16

[Ans. (C) 12]

Hint:

$$A = \{a, b, p\},\ B = \{2, 3\},$$
$$C = \{p, q, r, s\}$$
$$n\,(A \cup C) \times B$$
$$A \cup C = \{a, b, p, q, r, s\}$$
$$(A \cup C) \times B = \{(a,2), (a,3), (b,2), (b,3), (p,2), (p,3), (q,2), (q,3), (r,2), (r,3), (s, 2), (s, 3\}$$
$$n\,[(A \cup C) \times B] = 12$$

3. If $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$ then state which of the following statement is true. [Sep.- 2020]

(A) $(A \times C) \subset (B \times D)$

(B) $(B \times D) \subset (A \times C)$

(C) $(A \times B) \subset (A \times D)$

(D) $(D \times A) \subset (B \times A)$

[Ans. (A) $(A \times C) \subset (B \times D)$]

Hint:

$$A = \{1, 2\},\ B = \{1, 2, 3, 4\},$$
$$C = \{5, 6\},\ D = \{5, 6, 7, 8\}$$
$$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$
$$B \times D = \{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8)\}$$
$\therefore (A \times C) \subset B \times D$ It is true.

4. If there are 1024 relations from a set $A = \{1, 2, 3, 4, 5\}$ to a set B, then the number of elements in B is [PTA - 2]

(A) 3 (B) 2 (C) 4 (D) 8

[Ans. (B) 2]

Hint:

$$n(A) = 5$$
$$n(B) = x$$
$$n(A \times B) = 1024 = 2^{10}$$
$$2^{5x} = 2^{10} \quad \Rightarrow 5x = 10$$
$$\Rightarrow \quad x = 2$$

5. The range of the relation $R= \{(x, x^2) \mid x$ is a prime number less than 13$\}$ is [PTA - 4; Hy - 2019]

(A) $\{2,3,5,7\}$ (B) $\{2,3,5,7,11\}$

(C) $\{4,9,25,49,121\}$ (D) $\{1,4,9,25,49,121\}$

[Ans. (C) $\{4, 9, 25, 49, 121\}$]

Hint: $R = \{(x, x^2)/x$ is a prime number $<13\}$

The squares of 2, 3, 5, 7, 11 are $\{4, 9, 25, 49, 121\}$

6. If the ordered pairs $(a + 2, 4)$ and $(5, 2a + b)$ are equal then (a, b) is

(A) $(2, -2)$ (B) $(5,1)$

(C) $(2,3)$ (D) $(3, -2)$

[Ans. (D) $(3, -2)$]

Hint:

$$(a + 2, 4), (5, 2a + b) \Rightarrow a + 2 = 5$$
$$a = 3 \Rightarrow 2a + b = 4$$
$$6 + b = 4 \Rightarrow b = -2$$

7. Let $n(A) = m$ and $n(B) = n$ then the total number of non-empty relations that can be defined from A to B is

(A) m^n (B) n^m (C) $2^{mn} - 1$ (D) 2^{mn}

[Ans. (D) 2^{mn}]

Hint:

$$n(A) = m,\ n(B) = n$$
$$n(A \times B) = 2^{mn}$$

8. If $\{(a, 8), (6, b)\}$ represents an identity function, then the value of a and b are respectively [PTA - 1]

(A) (8,6) (B) (8,8) (C) (6,8) (D) (6,6)

[Ans. (A) (8,6)]

Hint: $\{(a, 8), (6, b)\} \Rightarrow a = 8 \Rightarrow b = 6$

9. Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10\}$. A function $f : A \to B$ given by $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$ is a [PTA - 4]

(A) Many-one function

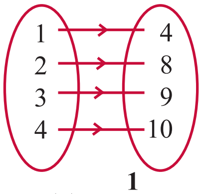
(B) Identity function

(C) One-to-one function

(D) Into function

[Ans. (C) One-to one function]

Hint: A = {1, 2, 3, 4), B = {4, 8, 9, 10}



10. **If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, Then fog is** **[Hy - 2019]**

(A) $\frac{3}{2x^2}$ (B) $\frac{2}{3x^2}$ (C) $\frac{2}{9x^2}$ (D) $\frac{1}{6x^2}$

[Ans. (C) $\frac{2}{9x^2}$]

Hint: $f(x) = 2x^2 \Rightarrow g(x) = \frac{1}{3x}$

$$fog = f(g(x)) = f\left(\frac{1}{3x}\right) = 2\left(\frac{1}{3x}\right)^2$$

$$= 2 \times \frac{1}{9x^2} = \frac{2}{9x^2}$$

11. **If f : A $\rightarrow$ B is a bijective function and if n(B) = 7 , then n(A) is equal to** **[PTA - 2]**

(A) 7 (B) 49 (C) 1 (D) 14

[Ans. (A) 7]

Hint: In a bijective function, $n(A) = n(B) \Rightarrow n(A) = 7$

12. **Let f and g be two functions given by f = {(0, 1), (2, 0), (3, −4), (4, 2), (5, 7)} g = {(0, 2),(1, 0), (2, 4), (−4, 2), (7,0)} then the range of fog is**

(A) {0,2,3,4,5} (B) {–4,1,0,2,7}

(C) {1,2,3,4,5} (D) {0,1,2}

[Ans. (D) {0, 1, 2}]

Hint:

$gof = g(f(x))$

$fog = f(g(x))$

$= \{(0, 2),(1, 0),(2, 4),(-4, 2),(7, 0)\}$

Range of fog = {0, 1, 2}

13. **Let $f(x) = \sqrt{1+x^2}$ then**

(A) $f(xy) = f(x).f(y)$ (B) $f(xy) \geq f(x).f(y)$

(C) $f(xy) \leq f(x).f(y)$ (D) None of these

[Ans. (C) $f(xy) \leq f(x).f(y)$]

Hint: $\sqrt{1+x^2y^2} \leq \sqrt{(1+x^2)}\sqrt{(1+y^2)}$

$\Rightarrow f(xy) \leq f(x) . f(y)$

14. **If g = {(1, 1),(2, 3),(3, 5),(4, 7)} is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are** **[PTA - 6]**

(A) (−1, 2) (B) (2, −1)

(C) (−1, −2) (D) (1,2) **[Ans.(B) (2, –1)]**

Hint:

$g(x) = \alpha x + \beta$

$\alpha = 2$

$\beta = -1$

$g(x) = 2x - 1$

$g(1) = 2(1) - 1 = 1$

$g(2) = 2(2) - 1 = 3$

$g(3) = 2(3) - 1 = 5$

$g(4) = 2(4) - 1 = 7$

15. **$f(x) = (x + 1)^3 - (x - 1)^3$ represents a function which is** **[PTA - 5; Qy - 2019]**

(A) linear (B) cubic

(C) reciprocal (D) quadratic

[Ans. (D) quadratic]

Hint: $f(x) = (x+1)^3 - (x-1)^3$

$= x^3 + 3x^2 + 3x + 1 - [x^3 - 3x^2 + 3x - 1]$

$= \cancel{x^3} + 3x^2 + \cancel{3x} + 1 - \cancel{x^3} + 3x^2 - \cancel{3x} + 1 = 6x^2 + 2$

It is a quadratic function.

Unit Exercise - 1

1. **If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and (–2,5) are equal, then find x and y.**

Sol.

$(x^2 - 3x, y^2 + 4y) = (-2, 5)$

$x^2 - 3x = -2$

$x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 2, 1$

$y^2 + 4y = 5$

$y^2 + 4y - 5 = 0$

$(y + 5)(y - 1) = 0$

$y = -5, 1$

2: –2, –1

–5: +5, –1

2. **The cartesian product A × A has 9 elements among which (–1, 0) and (0,1) are found. Find the set A and the remaining elements of A × A.**

Sol.

A = {–1, 0, 1}, B = {1, 0, –1}

A × B = {(–1, 1), (–1, 0), (–1, –1), (0, 1), (0, 0), (0, –1), (1, 1), (1, 0), (1, –1)}

(ii) $p(x) = \frac{-5}{4x^2+1}$

The domain is R.

(iii) $g(x) = \sqrt{x-2}$

The domain = [2, ∝]

(iv) $h(x) = x + 6$

The domain is R.

3. Let A={9,10,11,12,13,14,15,16,17} and let f : A→ N be defined by $f(n)$= the highest prime factor of n∈ A. Write f as a set of ordered pairs and find the range of f.

Sol.
$A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$
$f : A \to \mathbb{N}$
$f(n)$ = the highest prime factor of $n \in A$
$f = \{(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)\}$
Range $= \{3, 5, 11, 13, 7, 2, 17\} = \{2, 3, 5, 7, 11, 13, 17\}$

4. Let A = {1, 2} and B = {1, 2, 3, 4} , C = {5, 6} and D = {5, 6, 7, 8} . Verify whether A×C is a subset of B×D?

Sol.
$A = \{1, 2), B = \{1, 2, 3, 4\}$
$C = \{5, 6\}, D = \{5, 6, 7, 8\}$
$A \times C = \{(\underline{1, 5}), (\underline{1, 6}), (\underline{2, 5}), (\underline{2, 6})\}$
$B \times D = \{(\underline{1, 5}), (\underline{1, 6}), (1, 7), (1, 8), (\underline{2, 5}), (\underline{2, 6}), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$
$(A \times C) \subset (B \times D)$ It is proved.

PTA EXAM QUESTION & ANSWERS

1 MARK

1. If n(A) = p, n (B) = q then the total number of relations that exist between A and B is [PTA -1]

(A) 2^p (B) 2^q (C) 2^{p+q} (D) 2^{pq}

[Ans. (D) 2^{pq}]

2 MARKS

1. A relation R is given by the set $\{(x, y)/y = x^2 + 3, x \in \{0,1,2,3,4,5\}\}$ Determine its domain and range. [PTA - 2]

Sol.
Domain $= \{0, 1, 2, 3, 4, 5\}$
$x = 0, y = 0^2 + 3 = 3$
$x = 1, y = 1^2 + 3 = 4$
$x = 2, y = 2^2 + 3 = 7$
$x = 3, y = 3^2 + 3 = 12$
$x = 4, y = 4^2 + 3 = 19$
$x = 5, y = 5^2 + 3 = 28$
Range $= \{3, 4, 7, 12, 19, 28\}$

2. Let A = {1, 2, 3, ..., 100} and R be the relation defined as "is cube of" on A. Find the domain and range of R. [PTA - 4]

Sol.
$R = \{(1,1) (2,8), (3,27), (4, 64)\}$
Domain $= \{1, 2, 3, 4\}$
Range $= \{1, 8, 27, 64\}$

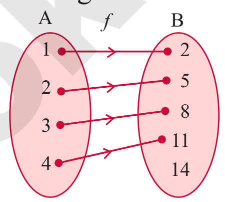
5 MARKS

1. Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let f : A → B be a function given by $f(x) = 3x – 1$ Represent this function. [PTA - 3]
(i) by arrow diagram [Sep.-2020]
(ii) in a table form
(iii) as a set of ordered pairs
(iv) in a graphical form

Sol. Let A = {1, 2, 3, 4} ; B = {2, 5, 8, 11, 14};
$f(x) = 3x - 1$
$f(1) = 3(1) - 1 = 3 - 1 = 2; f(2) = 3(2) - 1 = 6 - 1 = 5$
$f(3) = 3(3) - 1 = 9 - 1 = 8; f(4) = 4(3) - 1 = 12 - 1 = 11$

(i) Arrow diagram

Let us represent the function $f : A \to B$ by an arrow diagram



(ii) Table form

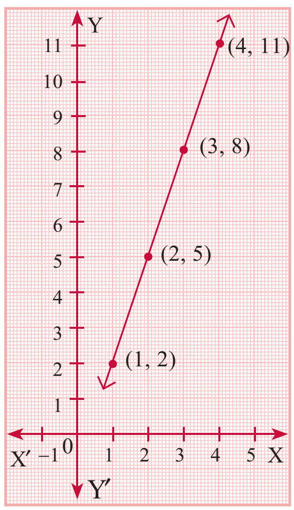
The given function f can be represented in a tabular form as given below

x	1	2	3	4
$f(x)$	2	5	8	11

(iii) Set of ordered pairs

The function f can be represented as a set of ordered pairs as
$f = (1, 2),(2, 5),(3, 8),(4, 11)$

(iv) Graphical form



In the adjacent xy -plane the points (1,2), (2,5), (3,8), (4,11) are plotted

2. Let A = $\{x \in \mathbb{W} / 0 < x < 5\}$, B = $\{x \in \mathbb{W} / 0 \le x \le 2\}$, C = $\{x \in \mathbb{W} / x < 3\}$ then verify that A × (B ∩ C) = (A × B) ∩ (A× C) **[PTA - 3]**

Sol.

$A = \{1, 2, 3, 4\}$

$B = \{0, 1, 2\}$

$C = \{0, 1, 2\}$

$B \cap C = \{0, 1, 2\} \cap \{0, 1, 2\} = \{0, 1, 2\}$

$A \times (B \cap C) = \{1, 2, 3, 4\} \times \{0, 1, 2\}$

$= \{(1,0), (1,1), (1, 2), (2,0), (2,1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4,2)\}$ ... (1)

$A \times B = \{1, 2, 3, 4\} \times \{0, 1, 2\}$

$= \{(1,0), (1,1), (1,2), (2,0),(2,1),(2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2)$

$A \times C = \{1, 2, 3, 4\} \times \{0, 1, 2\}$

$= \{(1,0), (1,1), (1,2), (2,0),(2,1),(2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2)$

$(A \times C) \cap (A \times C) = \{1, 2, 3, 4\} \times \{0, 1, 2\}$

$= \{(1,0), (1,1), (1,2), (2,0),(2,1),(2,2), (3,0), (3,1), (3,2), (4,0), (4,1), (4,2)$ ... (2)

$(1) = (2)$

Hence it is proved.

GOVT. EXAM QUESTION & ANSWERS

1 MARK

Multiple choice questions.

1. If $n(A)= p$ and $n(B)= q$ then $n(A\times B)=$ ______ **[Qy - 2019]**

(A) $p + q$ (B) $p - q$ (C) $p \times q$ (D) $\frac{p}{q}$

[Ans. (C) $p \times q$]

Hint: $n(A \times B) = n(A) \times n(B) = p \times q$

2 MARKS

1. Let A = {1, 2, 3, 4, 5}, B = W and f : A→B is defined by $f(x) = x^2 - 1$ find the range of f. **[Qy - 2019]**

Sol. $f(1) = 0; f(2) = 3; f(3) = 8; f(4) = 15; f(5) = 24$

Range of $f = \{0, 3, 8, 15, 24\}$

5 MARKS

1. Let A=$\{x \in \mathbb{N} / 1 < x < 4\}$, B=$\{x \in \mathbb{W}/0 \le x < 2\}$ and C = $\{x \in \mathbb{N} / x < 3\}$.Then verify that A × (B ∩ C) = (A × B) ∩ (A × C). **[Hy - 2019]**

Sol.

$A = \{x \in \mathbb{N}/ 1 < x < 4\} = \{2, 3\}$

$B = \{x \in \mathbb{W}/0 \le x < 2\} = \{0, 1\}$

$C = \{x \in \mathbb{N}/x < 3\} = \{1, 2\}$

$A\times(B \cap C) = (A\times B) \cap (A\times C)$

$B \cap C = \{0,1\} \cap \{1,2\} = \{1\}$

$A \times (B \cap C) = \{2,3\}\times\{1\} = \{(2,1),(3,1)\}$

$A \times B = \{2,3\}\times\{0,1\}$

$= \{(2,0),(2,1),(3,0),(3,1)\}$

$A \times C = \{2,3\}\times\{1,2\}$

$= \{(2,1),(2,2),(3,1),(3,2)\}$

$(A\times B) \cap (A\times C) = \{(2,0),(2,1),(3,0),(3,1)\} \cap \{(2,1),(2,2),(3,1),(3,2)\}$

$= \{(2,1),(3,1)\}$

From (3) and (4), $A\times(B \cap C)=(A\times B) \cap(A\times C)$ is verified.

❁❁❁❁❁

Chapter 2

NUMBERS AND SEQUENCES

EXERCISE 2.1

1. Find all positive integers, when divided by 3 leaves remainder 2.

Sol. The positive integers when divided by 3 leaves remainder 2.

By Euclid's division lemma

$a = bq + r, 0 \leq r < b.$

Here $a = 3q + 2$, where $0 \leq q < 3$.

When $q = 0$, $a = 3(0) + 2 = 2$

When $q = 1$, $a = 3(1) + 2 = 5$

When $q = 2$, $a = 3(2) + 2 = 8$

When $q = 3$, $a = 3(3) + 2 = 11$

The required positive numbers are 2, 5, 8, 11,...

2. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over. [PTA - 1]

Sol. By Euclid's division algorithm,

$a = bq + r, \quad 0 \leq r < b.$

Here $532 = 21q + r$ ... (1)

$\Rightarrow$ $532 = 21(25) + 7$ ... (2)

$\therefore q = 25$, and $r = 7$

[Comparing (1) and (2)]

$$\begin{array}{r|l} & 25 \\ \hline 21 & 532 \\ & 42 \\ \hline & 112 \\ & 105 \\ \hline & 7 \end{array}$$

$\therefore$ Number of completed rows = 25 and the leftover flower pots = 7.

3. Prove that the product of two consecutive positive integers is divisible by 2.

Sol. Let $n - 1$ and n be two consecutive positive integers. Then their product is $(n-1)n$.

$(n-1)(n) = n^2 - n.$

We know that any positive integer is of the form $2q$ or $2q + 1$ for some integer q. So, following cases arise.

Case I. When $n = 2q$. In this case, we have

$n^2 - n = (2q)^2 - 2q = 4q^2 - 2q = 2q(2q-1)$

$\Rightarrow n^2 - n = 2r$, where $r = q(2q-1)$

$\Rightarrow n^2 - n$ is divisible by 2.

Case II : When $n = 2q + 1$. In this case, we have

$n^2 - n = (2q+1)^2 - (2q+1)$

$= (2q+1)(2q+1-1) = 2q(2q+1)$

$\Rightarrow n^2 - n = 2r$, where $r = q(2q+1)$.

$\Rightarrow n^2 - n$ is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n.

Hence it is proved.

4. When the positive integers a, b and c are divided by 13, the respective remainders are 9, 7 and 10. Show that $a + b + c$ is divisible by 13.

Sol. When a is divided by 13, the remainder is 9.

By Euclid's lemma, $a = bq + r, 0 \leq r < b$

$\Rightarrow$ $a = 13q + 9$ ... (1)

Similarly when the positive integers b and c are divided by 13, the remainders are 7 and 10.

$\therefore b = 13q + 7$ ... (2)

and $c = 13q + 10$ ... (3)

Adding (1), (2) and (3) we get,

$a + b + c = 13q + 9 + 13q + 7 + 13q + 10$

$= 39q + 26 = 13(3q + 2)$

Which is divisible by 13.

$\therefore a + b + c$ is divisible by 13.

5. Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.

Sol. Let x be any integer.

The square of x is x^2.

Case (i) : Let x be an even integer.

$\Rightarrow$ $x = 2q$ [$\because x$ is even]

Where q is some integer

$\Rightarrow$ $x^2 = (2q)^2 = 4q^2$

$\Rightarrow$ $x^2 = 4(q^2)$

$\Rightarrow x^2$ is divisible by 4.

$\therefore$ When x is an even integer, x^2 is divisible by 4.

$\Rightarrow x^2$ leaves the remainder 0 when divided by 4.

Case (ii) : Let x be an odd integer.

$$\therefore x = 2k+1 \text{ for some integer } k.$$
$$\therefore x^2 = (2k+1)^2 = 4k^2+4k+1$$
$$= 4k(k+1)+1$$
$$= 4q+1 \text{ where } q = k(k+1)$$

$\Rightarrow$ $x^2 = 4q+1$ leaves the remainder 1 when divided by 4.

From Case (i) and Case (ii), the square of any integer leaves the remainder either 0 or 1 when divided by 4.

6. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of
(i) 340 and 412 (ii) 867 and 255
(iii) 10224 and 9648 (iv) 84, 90 and 120

Sol. (i) To find the HCF of 340 and 412. Using Euclid's division algorithm.

We get $412 = 340 \times 1 + 72$

The remainder $72 \neq 0$

Again applying Euclid's division algorithm

$$340 = 72 \times 4 + 52$$

The remainder $52 \neq 0$.

Again applying Euclid's division algorithm

$$72 = 52 \times 1 + 20$$

The remainder $20 \neq 0$.

Again applying Euclid's division algorithm,

$$52 = 20 \times 2 + 12$$

The remainder $12 \neq 0$.

Again applying Euclid's division algorithm.

$$20 = 12 \times 1 + 8$$

The remainder $8 \neq 0$.

Again applying Euclid's division algorithm

$$12 = 8 \times 1 + 4$$

The remainder $4 \neq 0$.

Again applying Euclid's division algorithm

$$8 = \boxed{4} \times 2 + 0$$

The remainder is zero.

Therefore HCF of 340 and 412 is 4.

(ii) To find the HCF of 867 and 255, using Euclid's division algorithm.

$$867 = 255 \times 3 + 102$$

The remainder $102 \neq 0$.

Again using Euclid's division algorithm

$$255 = 102 \times 2 + 51$$

The remainder $51 \neq 0$.

Again using Euclid's division algorithm

$$102 = 51 \times 2 + 0$$

The remainder is zero.

Therefore the HCF of 867 and 255 is 51.

(iii) To find HCF 10224 and 9648. Using Euclid's division algorithm.

$$10224 = 9648 \times 1 + 576$$

The remainder $576 \neq 0$.

Again using Euclid's division algorithm

$$9648 = 576 \times 16 + 432$$

Remainder $432 \neq 0$.

Again applying Euclid's division algorithm

$$576 = 432 \times 1 + 144$$

Remainder $144 \neq 0$.

Again using Euclid's division algorithm

$$432 = 144 \times 3 + 0$$

The remainder is zero.

There HCF of 10224 and 9648 is 144.

(iv) To find HCF of 84, 90 and 120.

Using Euclid's division algorithm

$$90 = 84 \times 1 + 6$$

The remainder $6 \neq 0$.

Again using Euclid's division algorithm

$$84 = 6 \times 14 + 0$$

The remainder is zero.

$\therefore$ The HCF of 84 and 90 is 6. To find the HCF of 6 and 120 using Euclid's division algorithm.

$$120 = 6 \times 20 + 0$$

The remainder is zero.

Therefore HCF of 120 and 6 is 6

$\therefore$ HCF of 84, 90 and 120 is 6.

7. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

Sol. The required number is the HCF of the numbers.

$$1230 - 12 = 1218,$$
$$1926 - 12 = 1914$$

First we find the HCF of 1218 & 1914 by Euclid's division algorithm.

$$1914 = 1218 \times 1 + 696$$

The remainder $696 \neq 0$.

Again using Euclid's algorithm

$$1218 = 696 \times 1 + 522$$

The remainder $522 \neq 0$.

Again using Euclid's algorithm.

$$696 = 522 \times 1 + 174$$

The remainder 174 $\neq$ 0.

Again by Euclid's algorithm

$$522 = 174 \times 3 + 0$$

The remainder is zero.

$\therefore$ The HCF of 1218 and 1914 is 174.

$\therefore$ The required number is 174.

8. If d is the Highest Common Factor of 32 and 60, find x and y satisfying $d = 32x + 60y$.

Sol. Applying Euclid's division lemma to 32 and 60, we get

$$60 = 32 \times 1 + 28 \quad \text{...(i)}$$

The remainder is 28 $\neq$ 0.

Again applying division lemma

$$32 = 28 \times 1 + 4 \quad \text{...(ii)}$$

The remainder 4 $\neq$ 0.

Again applying division lemma

$$28 = \boxed{4} \times 7 + 0 \quad \text{...(iii)}$$

The remainder zero.

$\therefore$ HCF of 32 and 60 is 4.

From (ii), we get

$$32 = 28 \times 1 + 4$$
$$\Rightarrow \quad 4 = 32 - 28 \times 1$$
$$\Rightarrow \quad 4 = 32 - (60 - 32 \times 1) \times 1 \quad [\because 28 = (60 - 32) \times 1]$$
$$\Rightarrow \quad 4 = 32 - 60 + 32$$
$$\Rightarrow \quad 4 = 32 \times \underline{2} + (-1) \times \underline{60}$$
$$\therefore \quad x = 2 \text{ and } y = -1$$

9. A positive integer when divided by 88 gives the remainder 61. What will be the remainder when the same number is divided by 11?

Sol. Let the positive integer be x.

$$x = 88 \times y + 61$$
$$61 = 11 \times 5 + 6$$
$$(\because 88 \text{ is multiple of } 11)$$

$\therefore$ 6 is the remainder. (When the number is divided by 88 giving the remainder 61 and when divided by 11 giving the remainder 6).

10. Prove that two consecutive positive integers are always coprime.

Sol. Let the two consecutive integers be n and $n + 1$.

Suppose HCF $(n, n + 1) = p$

$\Rightarrow p$ divides n ... (1)

and p divides $(n + 1)$ ... (2)

$\Rightarrow p$ divides $(n + 1 - n)$ [From (2) – (1)]

$\Rightarrow p$ divides 1

There is no number which divides 1 except 1

$\Rightarrow p = 1$ $\therefore$ HCF $(n, n + 1) = 1$.

$\Rightarrow n$ and $(n + 1)$ are Coprime.

EXERCISE 2.2

1. For what values of natural number n, 4^n can end with the digit 6?

Sol. $4^n = (2 \times 2)^n = 2^n \times 2^n$

2 is a factor of 4^n.

So, 4^n is always even and end with 4 and 6.

When n is an even number say 2, 4, 6, 8 then 4^n can end with the digit 6.

Example:

$4^2 = 16$	$4^3 = 64$
$4^4 = 256$	$4^5 = 1{,}024$
$4^6 = 4{,}096$	$4^7 = 16{,}384$
$4^8 = 65{,}536$	$4^9 = 262{,}144$

2. If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5?

Sol. $2^n \times 5^m$

2^n is always even for all values of n.

5^m is always odd and ends with 5 for all values of m.

But $2^n \times 5^m$ is always even and ends in 0.

[$\because$ even number X odd number = even number]

$\therefore$ $2^n \times 5^m$ cannot end with the digit 5 for any values of m. No value of m will satisfy $2^n \times 5^m$ ends in 5.

3. Find the HCF of 252525 and 363636.

Sol. To find the HCF of 252525 and 363636

Using Euclid's Division algorithm

$$363636 = 252525 \times 1 + 111111$$

The remainder 111111 $\neq$ 0.

$\therefore$ Again by division algorithm

$$252525 = 111111 \times 2 + 30303$$

The remainder 30303 $\neq$ 0.

$\therefore$ Again by division algorithm.

$$111111 = 30303 \times 3 + 20202$$

The remainder 20202 $\neq$ 0.

$\therefore$ Again by division algorithm

$$30303 = 20202 \times 1 + 10101$$

The remainder 10101 $\neq$ 0.

$\therefore$ Again using division algorithm

$$20202 = \boxed{10101} \times 2 + 0$$

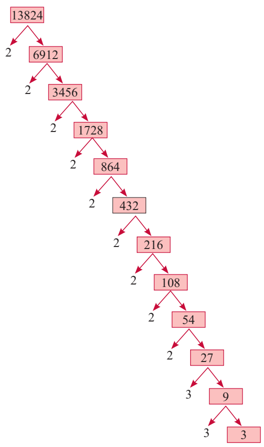
The remainder is 0.

Therefore HCF. of 252525 and 363636 is 10101.

4. If $13824 = 2^a \times 3^b$ then find a and b.

Sol. If $13824 = 2^a \times 3^b$

Using the prime factorisation tree



$$13824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$= 2^9 \times 3^3 = 2^a \times 3^b$$

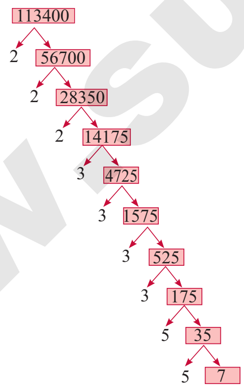
$\therefore \quad a = 9, b = 3.$

5. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4.

Sol. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$

p_1, p_2, p_3, p_4 are primes in ascending order, x_1, x_2, x_3, x_4 are integers.

Using prime factorisation tree.



$$113400 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$$

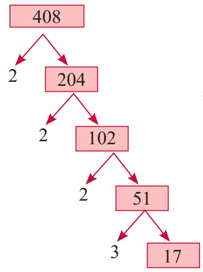
$$= 2^3 \times 3^4 \times 5^2 \times 7^1$$

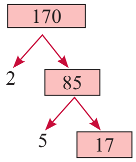
$$= p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4}$$

$\therefore \quad p_1 = 2, p_2 = 3, p_3 = 5, p_4 = 7, x_1 = 3, x_2 = 4, x_3 = 2, x_4 = 1.$

6. Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.

Sol. 408 and 170.





$408 = 2^3 \times 3^1 \times 17^1$

$170 = 2^1 \times 5^1 \times 17^1$

[By fundamental arithmetic theorem, every natural no except, can be factorized as a product of primes and it is unique]

Common Prime Factors	Least Exponents
2	1
17	1

$\therefore$ HCF $= 2^1 \times 17^1 = 34.$

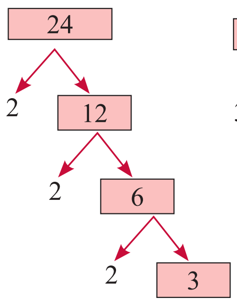
To find LCM, we list all prime factors of 408 and 170, and their greatest exponents as follows.

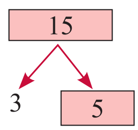
Prime factors of 408 and 170	Greatest Exponents
2	3
3	1
5	1
17	1

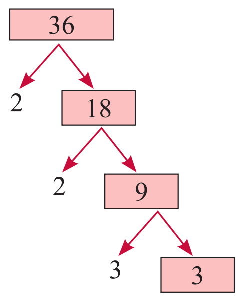
$\therefore$ LCM $= 2^3 \times 3^1 \times 5^1 \times 17^1 = 2040.$

7. Find the greatest number consisting of 6 digits which is exactly divisible by 24,15,36?

Sol. To find LCM of 24, 15, 36







$$24 = 2^3 \times 3$$
$$15 = 3 \times 5$$
$$36 = 2^2 \times 3^2$$

Prime factors of 24, 15, 36	Greatest Exponents
2	3
3	2
5	1

$\therefore$ LCM $= 2^3 \times 3^2 \times 5^1 = 8 \times 9 \times 5 = 360$

If a number has to be exactly divisible by 24, 15, and 36, then it has to be divisible by 360. Greatest 6 digit number is 999999.

Common multiples of 24, 15, 36 with 6 digits are 103680, 116640, 115520, ...933120, 999720 with six digits.

$\therefore$ The greatest number consisting 6 digits which is exactly divisible by 24, 15, 36 is 999720.

8. What is the smallest number that when divided by three numbers such as 35, 56 and 91 leaves remainder 7 in each case?

Sol.

$$\begin{aligned} 35 &= 5 \times 7 \\ 56 &= 2 \times 2 \times 2 \times 7 \\ 91 &= 7 \times 13 \\ \text{LCM of } 35, 56, 91 &= 5 \times 7 \times 2 \times 2 \times 2 \times 13 \\ &= 3640 \end{aligned}$$

$\therefore$ Required number = 3647 which leaves remainder 7 in each case.

9. Find the least number that is divisible by the first ten natural numbers.

Sol. The least number that is divisible by the first ten natural numbers is 2520.

Hint:
1, 2, 3, 4, 5, 6, 7, 8, 9, 10
The least multiple of 2 & 4 is 8
The least multiple of 3 is 9
The least multiple of 7 is 7
The least multiple of 5 is 5
LCM of $8 \times 9 \times 7 \times 5 = 40 \times 63 = 2520$.

EXERCISE 2.4

1. Find the next three terms of the following sequence.
(i) 8, 24, 72, … (ii) 5, 1,–3,… (iii) $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \ldots$

Sol. (i) 8, 24, 72...

In an arithmetic sequence $a = 8$,

$$\begin{aligned} d &= t_2 - t_1 = t_3 - t_2 \\ &= 24 - 8 \neq 72 - 24 \\ &= 16 \neq 48 \end{aligned}$$

So, it is not an arithmetic sequence. In a geometric sequence,

$$r = \frac{t_2}{t_1} = \frac{t_3}{t_2}$$

$$\Rightarrow \frac{24}{8} = \frac{72}{24} \Rightarrow 3 = 3$$

$\therefore$ It is a geometric sequence.

$\therefore$ The n^{th} term of a G.P is $t_n = ar^{n-1}$

[Here $a = 8$; $r = 3$]

$$\begin{aligned} \therefore t_4 &= 8 \times 3^{4-1} & t_5 &= 8 \times 3^{5-1} \\ &= 8 \times 3^3 & &= 8 \times 3^4 \\ &= 8 \times 27 & &= 8 \times 81 \\ &= 216 & &= 648 \end{aligned}$$

$$\begin{aligned} t_6 &= 8 \times 3^{6-1} = 8 \times 3^5 \\ &= 8 \times 243 = 1944 \end{aligned}$$

The next 3 terms are 8, 24, 72, <u>216</u>, <u>648</u>, <u>1944</u>.

(ii) 5, 1, –3, ...

$$\begin{aligned} d &= t_2 - t_1 = t_3 - t_2 \\ \Rightarrow 1 - 5 &= -3 - 1 \\ -4 &= -4 \quad \therefore \text{It is an A.P.} \\ t_n &= a + (n-1)d \end{aligned}$$

[$a = 5$, $d = -4$]

$$\begin{aligned} t_4 &= 5 + 3 \times -4 = 5 - 12 = -7 \\ t_5 &= a + 4d = 5 + 4 \times -4 \\ &= 5 - 16 = -11 \\ t_6 &= a + 5d = 5 + 5 \times -4 \\ &= 5 - 20 = -15 \end{aligned}$$

$\therefore$ The next three terms are 5, 1, –3, <u>–7</u>, <u>–11</u>, <u>–15</u>.

(iii) $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \ldots$

General term is $a_n = \frac{n}{(n+1)^2}$

$$a_4 = \frac{4}{(4+1)^2} = \frac{4}{5^2} = \frac{4}{25}$$

$$a_5 = \frac{5}{(5+1)^2} = \frac{5}{6^2} = \frac{5}{36}$$

$$a_6 = \frac{6}{(6+1)^2} = \frac{6}{7^2} = \frac{6}{49}$$

$\therefore$ The next three terms are $\frac{4}{25}, \frac{5}{36}, \frac{6}{49}$.

2. Find the first four terms of the sequences whose n^{th} terms are given by
(i) $a_n = n^3 - 2$ (ii) $a_n = (-1)^{n+1} n(n+1)$
(iii) $a_n = 2n^2 - 6$

Sol. $t_n = a_n = n^3 - 2$

(i)
$$\begin{aligned} a_1 &= 1^3 - 2 = 1 - 2 = -1 \\ a_2 &= 2^3 - 2 = 8 - 2 = 6 \\ a_3 &= 3^3 - 2 = 27 - 2 = 25 \\ a_4 &= 4^3 - 2 = 64 - 2 = 62 \end{aligned}$$

$\therefore$ The first four terms are –1, 6, 25, 62, ...

(ii) $a_n = (-1)^{n+1}\, n(n+1)$

$a_1 = (-1)^{1+1}(1)(1+1) = (-1)^2(1)(2) = 2$

$a_2 = (-1)^{2+1}(2)(2+1) = (-1)^3(2)(3) = -6$

$a_3 = (-1)^{3+1}(3)(3+1) = (-1)^4(3)(4) = 12$

$a_4 = (-1)^{4+1}(4)(4+1) = (-1)^5(4)(5) = -20$

$\therefore$ The first four terms are 2, –6, 12, –20, ...

(iii) $a_n = 2n^2 - 6$

$a_1 = 2(1)^2 - 6 = 2 - 6 = -4$

$a_2 = 2(2)^2 - 6 = 8 - 6 = 2$

$a_3 = 2(3)^2 - 6 = 18 - 6 = 12$

$a_4 = 2(4)^2 - 6 = 32 - 6 = 26$

$\therefore$ The first four terms are –4, 2, 12, 26, ...

3. Find the n^{th} term of the following sequences

(i) 2, 5, 10, 17,.... (ii) 0, $\frac{1}{2}, \frac{2}{3}$,.....

(iii) 3, 8, 13, 18,...

Sol. (i) $2, 5, 10, 17 = 1^2+1, 2^2+1, 3^2+1, 4^2+1 \ldots$

$\therefore n^{th}$ term is $n^2 + 1$

(ii) $0, \frac{1}{2}, \frac{2}{3}, \ldots = \frac{1-1}{1}, \frac{2-1}{2}, \frac{3-1}{3} \ldots \Rightarrow \frac{n-1}{n}$

$\therefore n^{th}$ term is $\frac{n-1}{n}$

(iii) 3, 8, 13, 18 forms A.P since common difference is same.

$a = 3$

$d = 5$

$t_n = a + (n-1)d = 3 + (n-1)5 = 3 + 5n - 5 = 5n - 2$

$\therefore n^{th}$ term is $5n - 2$

4. Find the indicated terms of the sequences whose n^{th} terms are given by

(i) $a_n = \frac{5n}{n+2}$; a_6 and a_{13}

(ii) $a_n = -(n^2 - 4)$; a_4 and a_{11}

Sol. (i) $a_n = \frac{5n}{n+2}$

$a_{13} = \frac{5\times13}{13+2} = \frac{\cancel{65}^{13}}{\cancel{15}^{3}} = \frac{13}{3}$

$a_6 = \frac{5\times6}{6+2} = \frac{\cancel{30}^{15}}{\cancel{8}^{4}} = \frac{15}{4}$

(ii) $a_n = -(n^2-4)$; a_4 and a_{11}

$a_4 = -(4^2 - 4) = -(16-4) = -12$

$a_{11} = -(11^2 - 4) = -(121-4) = -117$

5. Find a_8 and a_{15} whose n^{th} term is

$$a_n = \begin{cases} \dfrac{n^2-1}{n+3}; & n \text{ is even}, n \in \mathbb{N} \\ \dfrac{n^2}{2n+1}; & n \text{ is odd}, n \in \mathbb{N} \end{cases}$$

Sol. $$a_n = \begin{cases} \dfrac{n^2-1}{n+3}, & n \text{ is even} \\ \dfrac{n^2}{2n+1}, & n \text{ is odd} \end{cases}$$

$a_8 = \frac{n^2-1}{n+3} = \frac{8^2-1}{8+3} = \frac{64-1}{11} = \frac{63}{11}$ [$\because n = 8$ is even]

$a_{15} = \frac{n^2}{2n+1} = \frac{15^2}{2\times15+1} = \frac{225}{30+1} = \frac{225}{31}$ [$\because n = 15$ is odd]

6. If $a_1 = 1$, $a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$, $n \geq 3$, $n \in \mathbb{N}$, then find the first six terms of the sequence.

Sol. $a_1 = 1, a_2 = 1,\ a_n = 2a_{n-1} + a_{n-2}$

$a_3 = 2a_{(3-1)} + a_{(3-2)} = 2a_2 + a_1 = 2\times1+1 = 3$ [$\because a_1 = 1, a_2 = 1$]

$a_4 = 2a_{(4-1)} + a_{(4-2)} = 2a_3 + a_2 = 2\times3+1 = 7$ [$\because a_3 = 3, a_2 = 1$]

$a_5 = 2a_{(5-1)} + a_{(5-2)} = 2a_4 + a_3 = 2\times7+3 = 17$ [$\because a_4 = 7, a_3 = 3$]

$a_6 = 2a_{(6-1)} + a_{(6-2)} = 2a_5 + a_4 = 2\times17+7 = 34+7 = 41$ [$\because a_5 = 17, a_4 = 7$]

$\therefore$ The first six terms of the sequence are 1, 1, 3, 7, 17, 41, ...

EXERCISE 2.5

1. Check whether the following sequences are in A.P.

(i) $a-3, a-5, a-7, \ldots$ **(ii)** $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

(iii) $9, 13, 17, 21, 25, \ldots$ **(iv)** $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$

(v) $1, -1, 1, -1, 1, -1, \ldots$

Sol. To prove it is an A.P, we have to show $d = t_2 - t_1 = t_3 - t_2$.

(i) $\underset{t_1}{a-3}, \underset{t_2}{a-5}, \underset{t_3}{a-7}, \ldots$

$d = t_2 - t_1 = a - 5 - (a-3) = \cancel{a} - 5 - \cancel{a} + 3 = -2$

$d = t_3 - t_2 = a - 7 - (a-5) = \cancel{a} - 7 - \cancel{a} + 5 = -2$

$\therefore d = -2$

$\therefore a-3, a-5, a-7, \ldots$ is an A.P.

(ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

$d = t_2 - t_1 \Rightarrow \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = \frac{-1}{6}$

$d = t_3 - t_2 \Rightarrow \frac{1}{4} - \frac{1}{3} = \frac{3-4}{12} = \frac{-1}{12}$

$\frac{-1}{6} \neq \frac{-1}{12}$

$\Rightarrow t_2 - t_1 \neq t_3 - t_2$

$\therefore$ The given sequence is not an A.P.

(iii) $9, 13, 17, 21, 25, \ldots$

$$d = t_2 - t_1 = 13 - 9 = 4$$
$$d = t_3 - t_2 = 17 - 13 = 4$$
$$4 = 4$$

$\therefore$ The given sequence is an A.P

(iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$

$$d = t_2 - t_1 = 0 - \left(-\frac{1}{3}\right) = \frac{1}{3}$$
$$d = t_3 - t_2 = \frac{1}{3} - 0 = \frac{1}{3}$$
$$\frac{1}{3} = \frac{1}{3}$$

$\therefore \frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$ is an A.P.

(v) $1, -1, 1, -1, 1, -1, \ldots$

$$d = t_2 - t_1 = -1 - 1 = -2$$
$$d = t_3 - t_2 = 1 - (-1) = 2$$
$$-2 \neq 2$$

$\therefore 1, -1, 1, -1, \ldots$ is not an A.P.

2. First term a and common difference d are given below. Find the corresponding A.P.

(i) $a = 5, d = 6$ **(ii)** $a = 7, d = -5$

(iii) $a = \frac{3}{4}, d = \frac{1}{2}$

Sol. (i) $a = 5, d = 6$

A.P is $a, a+d, a+2d, \ldots$

$= 5, 5+6, 5+2\times 6, \ldots$

$= 5, 11, 17, \ldots$

(ii) $a = 7, d = -5$

A.P is $a, a+d, a+2d, \ldots$

$= 7, 7+(-5), 7+2(-5), \ldots$

$= 7, 2, -3, \ldots$

(iii) $a = \frac{3}{4}, d = \frac{1}{2}$

A.P is $a, a+d, a+2d, \ldots$

$= \frac{3}{4}, \frac{3}{4}+\frac{1}{2}, \frac{3}{4}+\cancel{2}\left(\frac{1}{\cancel{2}}\right), \ldots = \frac{3}{4}, \frac{3+2}{4}, \frac{3+4}{4}, \ldots$

A.P is $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots$.

3. Find the first term and common difference of the Arithmetic Progressions whose n^{th} terms are given below

(i) $t_n = -3 + 2n$ (ii) $t_n = 4 - 7n$

Sol. (i)

$$a = t_1 = -3 + 2(1) = -3 + 2 = -1$$
$$\text{Here } t_2 = -3 + 2(2) = -3 + 4 = 1$$
$$\therefore d = t_2 - t_1 = 1 - (-1) = 2$$

First term $= -1$, common difference $= 2$

(ii)

$$a = t_1 = 4 - 7(1) = 4 - 7 = -3$$
$$d = t_2 - t_1$$
$$\text{Here } t_2 = 4 - 7(2) = 4 - 14 = -10$$
$$\therefore d = t_2 - t_1 = -10 - (-3) = -7$$

First term $= -3$, common difference $= -7$

4. Find the 19th term of an A.P. $-11, -15, -19, \ldots$

Sol. Given A.P is $-11, -15, -19, \ldots$

$$\text{Here } a = -11$$
$$d = t_2 - t_1 = -15 - (-11) = -15 + 11 = -4$$
$$n = 19$$
$$\therefore t_n = a + (n-1)d$$

$$\begin{aligned} t_{19} &= -11 + (19-1)(-4) \\ &= -11 + 18 \times -4 = -11 - 72 \\ &= -83 \end{aligned}$$

Therefore 19^{th} term is -83.

5. Which term of an A.P. 16, 11, 6, 1,... is –54 ? **[Qy - 2019]**

Sol. Given A.P is 16, 11, 6, 1, ...

It is given that

$$\begin{aligned} t_n &= -54 \\ \text{Here } a = 16, d &= t_2 - t_1 = 11 - 16 = -5 \\ \therefore t_n &= a + (n-1)d \\ -54 &= 16 + (n-1)(-5) \\ -54 &= 16 - 5n + 5 \\ 21 - 5n &= -54 \\ -5n &= -54 - 21 \\ -5n &= -75 \\ n &= \frac{75}{5} = 15 \end{aligned}$$

$\therefore$ 15th term is -54.

6. Find the middle term(s) of an A.P. 9, 15, 21, 27,…,183. **[PTA - 1]**

Sol. Given A.P is 9, 15, 21, 27,..., 183

No. of terms in an A.P. is

$$n = \frac{l-a}{d} + 1$$

$$\begin{aligned} \text{Here } a = 9, l = 183, d &= 15 - 9 = 6 \\ \therefore n &= \frac{183-9}{6} + 1 = \frac{174}{6} + 1 \\ &= 29 + 1 = 30 \end{aligned}$$

$\therefore$ No. of terms = 30. The middle must be 15th term and 16th term.

[$\because n = 30$ is even, the middle terms are $t_{\frac{30}{2}}$ and $t_{\frac{30}{2}+1}$]

$$\begin{aligned} \therefore t_{15} &= a + (n-1)d \\ &= 9 + 14 \times 6 \\ &= 9 + 84 = 93 \\ t_{16} &= a + 15d \\ &= 9 + 15 \times 6 = 9 + 90 = 99 \end{aligned}$$

$\therefore$ The middle terms are 93, 99.

7. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.

Sol. Let a and d be the first term and common difference of the A.P.

$$\begin{aligned} \text{Given that } 9t_9 &= 15t_{15} \\ \text{Since } t_n &= a + (n-1)d, \text{ we get} \\ 9(a+8d) &= 15(a+14d) \\ 9a + 72d &= 15a + 210d \\ 9a + 72d - 15a - 210d &= 0 \\ \Rightarrow -6a + 138d &= 0 \\ \Rightarrow 6a + 138d &= 0 \quad [\text{Dividing by } -1] \quad \ldots(1) \end{aligned}$$

$$\begin{aligned} \text{To prove that } 6t_{24} &= 0 \\ \text{Consider } 6t_{24} &= 6(a+23d) \\ &= 6a + 138d = 0 \quad [\text{by (1)}] \\ \Rightarrow 6t_{24} &= 0 \end{aligned}$$

$\Rightarrow$ 6 times 24^{th} term is 0. Hence proved.

8. If 3 + k, 18 – k, 5k + 1 are in A.P. then find k. **[PTA - 3 & 5]**

Sol. Given $3 + k$, $18 - k$, $5k + 1$ are in A.P

$\Rightarrow$ Common difference is same

$$\begin{aligned} \Rightarrow t_2 - t_1 &= t_3 - t_2 \\ \Rightarrow 18 - k - (3+k) &= 5k + 1 - (18 - k) \\ \Rightarrow 18 - k - 3 - k &= 5k + 1 - 18 + k \\ \Rightarrow 15 - 2k &= 6k - 17 \\ \Rightarrow 15 + 17 &= 6k + 2k \\ \Rightarrow 32 &= 8k \\ \Rightarrow k &= \frac{32}{8} = 4 \\ \therefore k &= 4 \end{aligned}$$

9. Find x, y and z, given that the numbers x, 10, y, 24, z are in A.P.

Sol. Given A.P is x, 10, y, 24, z, ...

$\Rightarrow$ Common difference is same.

$$\begin{aligned} d &= t_2 - t_1 = 10 - x \quad &\ldots(1) \\ &= t_3 - t_2 = y - 10 \quad &\ldots(2) \\ &= t_4 - t_3 = 24 - y \quad &\ldots(3) \\ &= t_5 - t_4 = z - 24 \quad &\ldots(4) \end{aligned}$$

(2) and (3)

$$\begin{aligned} \Rightarrow y - 10 &= 24 - y \\ 2y &= 24 + 10 = 34 \\ y &= \frac{34}{2} = 17 \end{aligned}$$

(1) and (2)

$$\begin{aligned} \Rightarrow 10 - x &= y - 10 \\ 10 - x &= 17 - 10 = 7 \\ -x &= 7 - 10 \\ \cancel{-}x &= \cancel{-}3 \Rightarrow x = 3. \end{aligned}$$

From (3) and (4)

$$24 - y = z - 24$$
$$24 - 17 = z - 24$$
$$7 = z - 24$$
$$\therefore \quad z = 7 + 24 = 31$$

$\therefore$ The required values are

$$x = 3$$
$$y = 17$$
$$z = 31$$

10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row? **[PTA - 4]**

Sol. Since there are 20 seats in the first row,

$$\text{let } a = 20 = t_1$$

Each successive row contains 2 additional seats than its previous row.

$$t_2 = t_1 + 2 = 20 + 2 = 22$$
$$\therefore \quad t_3 = t_2 + 2 = 22 + 2 = 24$$

and so on

$\therefore$ The A.P is 20, 22, 24, 26, ...

Since there are 30 rows, $n = 30$

$$\text{Here } a = 20, d = t_2 - t_1$$
$$= 22 - 20 = 2$$
$$\therefore t_{30} = 20 + 29(2)$$
$$[\because t_n = a + (n - 1) d]$$
$$= 20 + 58 = 78$$

$\therefore$ There will be 78 seats in the last row.

11. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.

Sol. Let the three consecutive terms be $a - d$, a, $a + d$

$$\text{Their sum} = a - \cancel{d} + a + a + \cancel{d} = 27$$
$$3a = 27$$
$$a = \frac{27}{3} = 9$$
$$\text{Their product} = (a - d)(a)(a + d) = 288$$
$$= 9(a^2 - d^2)$$
$$\Rightarrow \quad 9(9^2 - d^2) = 288$$
$$\Rightarrow \quad \cancel{9}(81 - d^2) = \cancel{288}^{32}$$
$$81 - d^2 = 32$$
$$-d^2 = 32 - 81$$
$$-d^2 = -49$$
$$d^2 = 49 \quad \Rightarrow d = \pm 7$$

$\therefore$ The three terms are if $a = 9$, $d = 7$

$$a - d, a, a + d = 9 - 7, 9 + 7$$
$$\text{A.P} = 2, 9, 16$$
$$\text{if } a = 9, d = -7,$$
$$\text{A.P} = 9 - (-7), 9, 9 + (-7)$$
$$= 16, 9, 2$$

12. The ratio of 6th and 8th term of an A.P is 7:9. Find the ratio of 9th term to 13th term.

Sol.

$$\frac{t_6}{t_8} = \frac{7}{9}$$
$$\frac{a+5d}{a+7d} = \frac{7}{9} \quad [\because t_n = a + (n-1)d]$$
$$9a + 45d = 7a + 49d$$
$$9a + 45d - 7a - 49d = 0$$
$$2a - 4d = 0 \Rightarrow \quad 2a = 4d$$
$$a = 2d$$

Substitute $a = 2d$ in

$$\frac{t_9}{t_{13}} = \frac{a+8d}{a+12d} = \frac{2d+8d}{2d+12d}$$
$$= \frac{10\cancel{d}}{14\cancel{d}} = \frac{5}{7}$$
$$\therefore t_9 : t_{13} = 5:7.$$

13. In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is 0° C and the sum of the temperatures from Wednesday to Friday is 18° C. Find the temperature on each of the five days.

Sol. Let the five days temperature be $(a - d)$, a, $a + d$, $a + 2d$, $a + 3d$.

The three days sum $= a - \cancel{d} + a + a + \cancel{d} = 0$

$\Rightarrow 3a = 0 \Rightarrow a = 0.$ (given)

$a + d + a + 2d + a + 3d = 18$

[$\because$ Sum of the last 3 days = 18° C]

$$3a + 6d = 18$$
$$3(0) + 6d = 18$$
$$6d = 18$$
$$d = \frac{18}{6} = 3$$

$\therefore$ The temperature of each five days is $a - d$, a, $a + d$, $a + 2d$, $a + 3d$

$0 - 3, 0, 0 + 3, 0 + 2(3), 0 + 3(3)$

$= -3$°C, 0°C, 3°C, 6°C, 9°C

14. Priya earned ₹15,000 in the first month. Thereafter her salary increased by ₹1500 per year. Her expenses are ₹13,000 during the first year and the expenses increases by ₹900 per year. How long will it take for her to save ₹20,000 per month.

Sol.

	Yearly Salary	Yearly expenses	Yearly savings
1^{st} year	15000	13000	2000
2^{nd} year	16500	13900	2600
3^{rd} year	18000	14800	3200

We find that the yearly savings is in A.P with $a_1 = 2000$ and $d = 600$.

We are required to find how many years are required to save 20,000 a year .....

$$\begin{aligned} \text{Given } a_n &= 20,000 \\ a_n &= a + (n-1)d \\ 20000 &= 2000 + (n-1)600 \\ (n-1)\,600 &= 18000 \\ n-1 &= \frac{18000}{600} = 30 \\ n &= 31 \text{ years.} \end{aligned}$$

EXERCISE 2.10

Multiple choice questions

1. Euclid's division lemma states that for positive integers *a* and *b*, there exist unique integers *q* and *r* such that *a* = *bq* + *r*, where *r* must satisfy.

(A) $1 < r < b$ (B) $0 < r < b$
(C) $0 \le r < b$ (D) $0 < r \le b$

[Ans. (C) $0 \le r < b$]

2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are [Sep.-2020; PTA - 5]

(A) 0, 1, 8 (B) 1, 4, 8
(C) 0, 1, 3 (D) 1, 3, 5

[Ans. (A) 0, 1, 8]

Hint: Cube of any +ve integers $1^3, 2^3, 3^3, 4^3$, ...
1, 8, <u>27</u>, <u>64</u>, <u>125</u>, 216 ...
Remainders when 27, 64, 125 are divided by 9.

3. If the HCF of 65 and 117 is expressible in the form of 65*m* – 117 , then the value of *m* is [PTA -1; Hy - 2019]

(A) 4 (B) 2 (C) 1 (D) 3

[Ans. (B) 2]

Hint: HCF of 65 and 117

$$\begin{aligned} 117 &= 65 \times 1 + 52 \\ 65 &= 52 \times 1 + 13 \\ 52 &= 13 \times 4 + 0 \end{aligned}$$

$\therefore$ 13 is the HCF of 65 and 117.

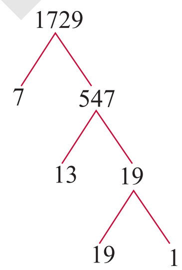
$$\begin{aligned} 65m - 117 &= 65 \times 2 - 117 \\ 130 - 117 &= 13 \\ \therefore m &= 2 \end{aligned}$$

4. The sum of the exponents of the prime factors in the prime factorization of 1729 is [PTA - 4]

(A) 1 (B) 2 (C) 3 (D) 4

[Ans. (C) 3]

Hint: $1729 = 7^1 \times 13^1 \times 19^1$



5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(A) 2025 (B) 5220 (C) 5025 (D) 2520

[Ans. (D) 2520]

Hint:

2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10
2	1, 1, 3, 2, 5, 3, 7, 4, 9, 5
3	1, 1, 3, 1, 5, 1, 7, 2, 9, 5
5	1, 1, 1, 1, 5, 1, 7, 2, 3, 5
7	1, 1, 1, 1, 1, 1, 7, 2, 3, 1
2	1, 1, 1, 1, 1, 1, 1, 2, 3, 1
3	1, 1, 1, 1, 1, 1, 1, 1, 3, 1
	1, 1, 1, 1, 1, 1, 1, 1, 1, 1

$\therefore$ LCM of 1, 2, 3, 4, ...,10 is $2 \times 2 \times 3 \times 5 \times 7 \times 2 \times 3 = 2520$

6. $7^{4k} \equiv$ _____ (mod 100) [PTA - 1 ; Qy - 2019]

(A) 1 (B) 2 (C) 3 (D) 4

Hint: $7^{4k} \equiv$ _____ (mod 100) **[Ans. (A) 1]**

$7^{4k} = 7^{4 \times 11} \equiv$ _____ (mod 100)

7. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is

(A) 3 (B) 5 (C) 8 (D) 11

[Ans. (D) 11]

Hint:

$$F_1 = 1, F_2 = 3$$
$$F_n = F_{n-1} + F_{n-2}$$
$$F_5 = F_{5-1} + F_{5-2} = F_4 + F_3$$
$$= F_3 + F_2 + F_2 + F_1$$
$$= F_2 + F_1 + F_2 + F_2 + F_1$$
$$= 3 + 1 + 3 + 3 + 1 = 11$$

8. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P

(A) 4551 (B) 10091
(C) 7881 (D) 13531

[Ans. (C) 7881]

Hint:

$$t_1 = 1$$
$$d = 4$$
$$t_n = a + (n-1)d$$
$$= 1 + 4n - 4$$
$$4n - 3 = 4551$$
$$4n = 4554$$
$$n = \text{will be a fraction}$$

It is not possible.

$$4n - 3 = 10091$$
$$4n = 10091 + 3 = 10094$$
$$n = \text{a fraction}$$
$$4n - 3 = 7881$$
$$4n = 7881 + 3 = 7884$$
$$n = \frac{\cancel{7884}^{1971}}{\cancel{4}_1}, n \text{ is a whole number.}$$
$$4n - 3 = 13531$$
$$4n = 13531 + 3 = 13534$$

n is a fraction.

$\therefore$ 7881 will be 1971[th] term of A.P.

9. If 6 times of 6[th] term of an A.P is equal to 7 times the 7th term, then the 13th term of the A.P. is [PTA - 4]

(A) 0 (B) 6 (C) 7 (D) 13

[Ans. (A) 0]

Hint:

$$6t_6 = 7t_7$$
$$6(a + 5d) = 7(a + 6d)$$
$$6a + 30d = 7a + 42d$$
$$7a + 42d - 6a - 30d = 0$$
$$a + 12d = 0 = t_{13}$$

10. An A.P consists of 31 terms. If its 16th term is *m*, then the sum of all the terms of this A.P. is [PTA - 5]

(A) 16 m (B) 62 m (C) 31 m (D) $\frac{31}{2}$ m

[Ans. (C) 31 m]

Hint:

$$t_{16} = m$$
$$S_{31} = \frac{31}{2}(2a + 30d)$$
$$= \frac{31}{\cancel{2}}(\cancel{2}(a + 15d)) \quad (\because t_{16} = a + 15d)$$
$$= 31(t_{16}) = 31\,m$$

11. In an A.P., the first term is 1 and the common difference is 4. How many terms of the A.P must be taken for their sum to be equal to 120? [Govt. MQP - 2019]

(A) 6 (B) 7 (C) 8 (D) 9

[Ans. (C) 8]

Hint:

$$a = 1, d = 4$$
$$S_n = 120 = \frac{n}{2}(2a + (n-1)d)$$
$$120 = \frac{n}{2}(2\times1 + (n-1)4)$$
$$120 = \frac{n}{2}(2 + 4n - 4) = \frac{n}{2}(4n - 2)$$
$$= \frac{n}{\cancel{2}}.\cancel{2}(2n-1)$$

$$= n(2n-1)$$

$$120 = 2n^2 - n$$

$$2n^2 - n - 120 = 0$$

$$(n-8)(2n+15) = 0$$

$$\therefore n = 8,$$

$$n = \frac{-15}{2}$$

$$-240 \to \frac{-16}{2},\ \frac{15}{2} \qquad (n-8)\left(n+\frac{15}{2}\right)$$

12. If A = 2^{65} and B = $2^{64}+2^{63}+2^{62}+ ...+2^0$ which of the following is true? **[Sep. - 2020; PTA - 6]**

(A) B is 2^{64} more than A

(B) A and B are equal

(C) B is larger than A by 1

(D) A is larger than B by 1

[Ans. (D) A is larger than B by 1]

Hint:

$$A = 2^{65}$$

$$B = 2^{64} + 2^{63} + 2^{62} + ...+ 2^0$$

$$B = 2^0 + 2^1 + 2^2 + ...+ 2^{64}$$

G.P $= 1 + 2^1 + 2^2 + ...+ 2^{64}$ it is a G.P

Here $a = 1$, $r = 2$, $n = 65$

$\therefore$ Sum of the G.P $= S_{65} = \dfrac{a(r^n-1)}{r-1}$

$$= \frac{1(2^{65}-1)}{2-1} = 2^{65}-1$$

$$A = 2^{65},\ B = 2^{65}-1$$

$\therefore$ B is smaller. A is larger than B by 1.

13. The next term of the sequence $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, ...$ is **[PTA - 2 ; Qy - 2019]**

(A) $\frac{1}{24}$ (B) $\frac{1}{27}$ (C) $\frac{2}{3}$ (D) $\frac{1}{81}$

[Ans. (B) $\frac{1}{27}$]

Hint: $\frac{3}{16}, \frac{1}{8}, \frac{1}{12}, \frac{1}{18}, ...$

$$r = \frac{\frac{1}{8}}{\frac{3}{16}} = \frac{1}{\cancel{8}} \times \frac{\cancel{16}^{\,2}}{3} = \frac{2}{3}$$

$$r = \frac{\frac{1}{12}}{\frac{1}{8}} = \frac{1}{\cancel{12}_{\,3}} \times \frac{\cancel{8}^{\,2}}{1} = \frac{2}{3}$$

$\therefore$ The next term is $\frac{1}{18} \times \frac{2}{3} = \frac{2}{54} = \frac{1}{27}$

14. If the sequence $t_1, t_2, t_3, ...$ are in A.P. then the sequence $t_6, t_{12}, t_{18}, ...$ is **[Hy - 2019]**

(A) a Geometric Progression

(B) an Arithmetic Progression

(C) neither an Arithmetic Progression nor a Geometric Progression

(D) a constant sequence

[Ans. (B) an Arithmetic Progression]

Hint: If $t_1, t_2, t_3, ...$is 1, 2, 3, ...

If $t_6 = 6$, $t_{12} = 12$, $t_{18} = 18$ then 6, 12, 18 ...is an arithmetic progression

15. The value of $(1^3 + 2^3 + 3^3 + ... + 15^3) - (1 + 2 + 3 + ... + 15)$ is **[PTA - 3]**

(A) 14400 (B) 14200

(C) 14280 (D) 14520

[Ans. (C) 14280]

Hint: $\left(\dfrac{15\times16}{2}\right)^2 - \dfrac{15\times16}{2} = (120)^2 - 120$

$$= 14280.$$

Unit Exercise - 2

1. Prove that $n^2 - n$ divisible by 2 for every positive integer n.

Sol. To prove $n^2 - n$ divisible by 2 for every positive integer n.

We know that any positive integer is of the form $2q$ or $2q + 1$, for some integer q.

So, following cases arise:

Case I. When $n = 2q$.

In this case, we have

$$n^2 - n = (2q)^2 - 2q = 4q^2 - 2q = 2q(2q-1)$$

$\Rightarrow n^2 - n = 2r$ where $r = q(2q - 1)$

$\Rightarrow n^2 - n$ is divisible by 2.

Case II. When $n = 2q + 1$.

In this case, we have

$n^2 - n = (2q + 1)^2 - (2q + 1)$

$= (2q + 1)(2q + 1 - 1) = (2q + 1)2q$

$\Rightarrow n^2 - n = 2r$ where $r = q(2q + 1)$

$\Rightarrow n^2 - n$ is divisible by 2.

Hence $n^2 - n$ is divisible by 2 for every positive integer n.

2. A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Calculate the following (i) Capacity of a can (ii) Number of cans of cow's milk (iii) Number of cans of buffalow's milk.

Sol. 175 litres of cow's milk.

105 litres of goat's milk.

HCF of 175 & 105 by using Euclid's division algorithm.

$175 = 105 \times 1 + 70$, the remainder $70 \neq 0$

Again using division algorithm,

$105 = 70 \times 1 + 35$, the remainder $35 \neq 0$

Again using division algorithm.

$70 = 35 \times 2 + 0$, the remainder is 0.

$\therefore$ 35 is the HCF of 175 & 105.

(i) $\therefore$ The milk man's milk can's capacity is 35 litres.

(ii) No. of cow's milk obtained $= \dfrac{175}{35}$

$= 5$ cans

(iii) No. of buffalow's milk obtained $= \dfrac{105}{35}$

$= 3$ cans.

3. When the positive integers a, b and c are divided by 13 the respective remainders are 9, 7 and 10. Find the remainder when $a + 2b + 3c$ is divided by 13.

Sol. Let the positive integers be a, b, and c.

$a = 13q + 9$

$b = 13q + 7$

$c = 13q + 10$

$a + 2b + 3c = 13q + 9 + 2(13q + 7) + 3(13q + 10)$

$= 13q + 9 + 26q + 14 + 39q + 30$

$= 78q + 53 = (13 \times 6)q + 53$

The remainder is 53.

But $53 = 13 \times 4 + 1$

$\therefore$ The remainder is 1

4. Show that 107 is of the form $4q + 3$ for any integer q.

Sol. $107 = 104 + 3$

$107 = 4 \times 26 + 3$. This is of the form $a = bq + r$. Where $q = 26$. Hence it is proved.

5. If $(m + 1)^{\text{th}}$ term of an A.P. is twice the $(n + 1)^{\text{th}}$ term, then prove that $(3m + 1)^{\text{th}}$ term is twice the $(m + n + 1)^{\text{th}}$ term.

Sol.

$$
\begin{aligned}
t_n &= a + (n-1)d \\
t_{m+1} &= a + (m + 1 - 1)d \\
&= a + md \\
t_{n+1} &= a + (n + 1 - 1)d \\
&= a + nd \\
2(t_{n+1}) &= 2(a + nd) \\
t_{m+1} &= 2t_{n+1} \qquad \text{...(1)} \\
\Rightarrow a + md &= 2(a + nd) \\
2a + 2nd - a - md &= 0 \\
a + (2n - m)d &= 0 \\
t_{(3m+1)} &= a + (3m + 1 - 1)d \\
&= a + 3md \\
t_{(m+n+1)} &= a + (m + n + 1 - 1)d \\
&= a + (m + n)d \\
2(t_{(m+n+1)}) &= 2(a + (m + n)d) \\
&= 2a + 2md + 2nd
\end{aligned}
$$

$$t_{(3m+1)} = 2t_{(m+n+1)} \quad ...(2)$$
$$a + 3md = 2a + 2md + 2nd$$
$$2a + 2md + 2nd - a - 3md = 0$$
$$a - md + 2nd = 0$$
$$a + (2n - m)d = 0$$
$$\therefore \text{It is proved that } t_{(3m+1)} = 2t_{(m+n+1)}$$

6. Find the 12th term from the last term of the A.P –2, –4, –6,... –100 .

Sol.
$$n = \frac{l-a}{d} + 1 = \frac{-100-(-2)}{-2} + 1$$
$$= \frac{-100+2}{-2} + 1 = \frac{-98}{-2} + 1$$
$$n = 49 + 1 = 50$$

12^{th} term from the last $= 39^{th}$ term from the beginning
$$\therefore t_{39} = a + 38d$$
$$= -2 + 38(-2) = -2 - 76$$
$$= -78$$

7. Two A.P's have the same common difference. The first term of one A.P is 2 and that of the other is 7. Show that the difference between their 10^{th} terms is the same as the difference between their 21^{st} terms, which is the same as the difference between any two corresponding terms.

Sol. Let the two A.Ps be
$$AP_1 = a_1, a_1 + d, a_1 + 2d, ...$$
$$AP_2 = a_2, a_2 + d, a_2 + 2d, ...$$

In AP_1 we have $a_1 = 2$
In AP_2 we have $a_2 = 7$
t_{10} in $AP_1 = a_1 + 9d = 2 + 9d$ ..(1)
t_{10} in $AP_2 = a_2 + 9d = 7 + 9d$ ..(2)
The difference between their 10th terms
$$= (1) - (2) = 2 + \cancel{9d} - 7 - \cancel{9d}$$
$$= -5 \quad ...(I)$$
$$t_{21} \text{ in } AP_1 = a_1 + 20d = 2 + 20d \quad ..(3)$$
$$t_{21} \text{ in } AP_2 = a_2 + 20d = 7 + 20d \quad ..(4)$$

The difference between their 21st terms is (3) – (4)
$$= 2 + \cancel{20d} - 7 - \cancel{20d}$$
$$= -5 \quad ...(II)$$
$$\text{I} = \text{II}$$

Hence it is proved.

PTA EXAM QUESTION & ANSWERS

1 MARK

1. The sequence – 3, – 3, – 3,.... is **[PTA - 1]**
(A) an A.P only
(B) a G. P only
(C) neither A.P nor G.P
(D) both A.P and G.P

[Ans. (D) both A.P and G.P]

Hint: Common difference = 0
Common ratio = 1

2. If a and b are two positive integers where $a > 0$ an b is a factor of a then HCF of a and b is **[PTA - 4]**

(A) b (B) a (C) $3ab$ (D) $\frac{a}{b}$

[Ans. (A) b]

3. If a, b, c are in A.P then $\frac{a-b}{b-c}$ is equal to **[PTA - 6]**

(A) $\frac{a}{b}$ (B) $\frac{b}{c}$ (C) $\frac{a}{c}$ (D) 1

[Ans. (D) 1]

Hint: $a - b = b - c$

2 MARKS

1. Is 7 × 5 × 3 × 2 + 3, a composite number? Justify your answer. **[PTA - 3]**

Sol. Yes, the given number is a composite number, because

$7 \times 5 \times 3 \times 2 + 3 = 3 \times (7 \times 5 \times 2 + 1) = 3 \times 71$

Since the given number can be factorized in terms of two primes, it is a composite number.

2. Which term of the A.P 21, 18, 15, ... is – 81? State with reason is there any term 0 in this A.P? **[PTA - 5]**

Sol.
$$a = 21$$
$$a = 18 - 21 = -3$$
$$t_n = a + (n-1)d$$

$$
\begin{aligned}
-81 &= 21+(n-1)\times -3 \\
-81-21 &= (n-1)\times -3 \\
-102 &= (n-1)\times -3 \\
\frac{-102}{-3} &= (n-1) \\
34 &= n-1 \\
n &= 34+1 = 35 \\
0 &= 21+(n-1)\times -3 \\
21 &= (n-1)\times -3 \\
\frac{-21}{-3} &= n-1 \\
7 &= n-1 \\
7+1 &= n \\
n &= 8
\end{aligned}
$$

$$3\overline{)102} = 34 \quad (102 - 9 = 12)$$

8^{th} term is 0

3. Find the common difference of an A.P in which $t_{18} - t_{14} = 32$. [PTA - 6]

Sol.

$$
\begin{aligned}
t_n &\equiv a+(n-1)d \\
t_{18} &= a+17d \\
t_{14} &= a+13d \\
a+17d-a-13d &= 32 \\
4d &= 32 \\
d &= \frac{32}{4} = 8
\end{aligned}
$$

5 MARKS

1. If the sum of the first p terms of an A.P is $ap^2 + bp$. Find its common difference. [PTA - 6]

Sol.

$$
\begin{aligned}
a &= a(1)^2 + b(1) = a+b, \\
t_1 + t_2 &= 2^2a + 2b \\
&= 4a+2b \\
t_2 &= 4a+2b-(a+b) \\
&= 3a+b \\
d &= t_2 - t_1 \\
&= 3a+b-(a+b) \\
&= 2a
\end{aligned}
$$

GOVT. EXAM QUESTION & ANSWERS

1 MARK

Multiple choice questions

1. A sequence is a function defined on the set of ______. [Qy - 2019]

(A) Real numbers (B) Natural numbers

(C) Whole numbers (D) Integers

[Ans. (B) Natural numbers]

2 MARKS

1. Find the 3^{rd} and 4^{th} terms of a sequence, if [Sep. - 2020]

$$a_n = \begin{cases} n^2 & \text{if } n \text{ is odd} \\ \dfrac{n^2}{2} & \text{if } n \text{ is even} \end{cases}$$

Sol.

$$a_n = \begin{cases} n^2 & \text{if } n \text{ is odd} \\ \dfrac{n^2}{2} & \text{if } n \text{ is even} \end{cases}$$

$$a_4 = 3^2 = 9 \quad [\because n = 9 \text{ is odd}]$$

$$a_4 = \frac{42}{2} = \frac{16}{2} = 8 \; [\because n = 8 \text{ is even}]$$

2. Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively. [Qy - 2019]

Sol. Since the remainders are 4, 5 respectively the required number is the HCF of the number 445–4 = 441, 572 – 5 = 567.

Hence, we will determine the HCF of 441 and 567. Using Euclid's Division algorithm,

We have

$$
\begin{aligned}
567 &= 441 \times 1 + 126 \\
441 &= 126 \times 3 + 63 \\
126 &= 63 \times 2 + 0
\end{aligned}
$$

Therefore HCF of 441, 567 = 63 and so the required number is 63.

3. a and b are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'. [Hy - 2019]

Sol. The number 800 can be factorized as

$800 = 2\times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^5\times 5^2$

Hence, $a^b \times b^a = 2^5 \times 5^2$

This implies that $a = 2$ and $b = 5$ (or) $a = 5$ and $b = 2$

4. Show that the sequence described by $a_n = \frac{1}{3}n + \frac{1}{6}$ is an A.P. [Hy - 2019]

Sol. $a_1 = \frac{3}{6}$ $\quad a_2 = \frac{5}{6}$ $\quad a_3 = \frac{7}{6}$

$t_2 - t_1 = t_3 - t_2$

$\frac{5}{6} - \frac{3}{6} = \frac{7}{6} - \frac{5}{6} \Rightarrow \frac{2}{6} = \frac{2}{6} \Rightarrow \frac{1}{3} = \frac{1}{3}$

$\therefore$ It is an A.P

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