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12th Std

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Authors :

Mr. A.Ganesa Moorthy M.Sc., M.Ed., M.phil.
Chennai

Edited by :
Mr. S. Sathish M.Sc., M.Phil.

Reviewed by :
Mr. S. Niranjan, B.Tech., M.B.A(IIM)
Chennai

Head Office:

1620, 'J' Block, 16th Main Road,
Anna Nagar, **Chennai - 600 040.**
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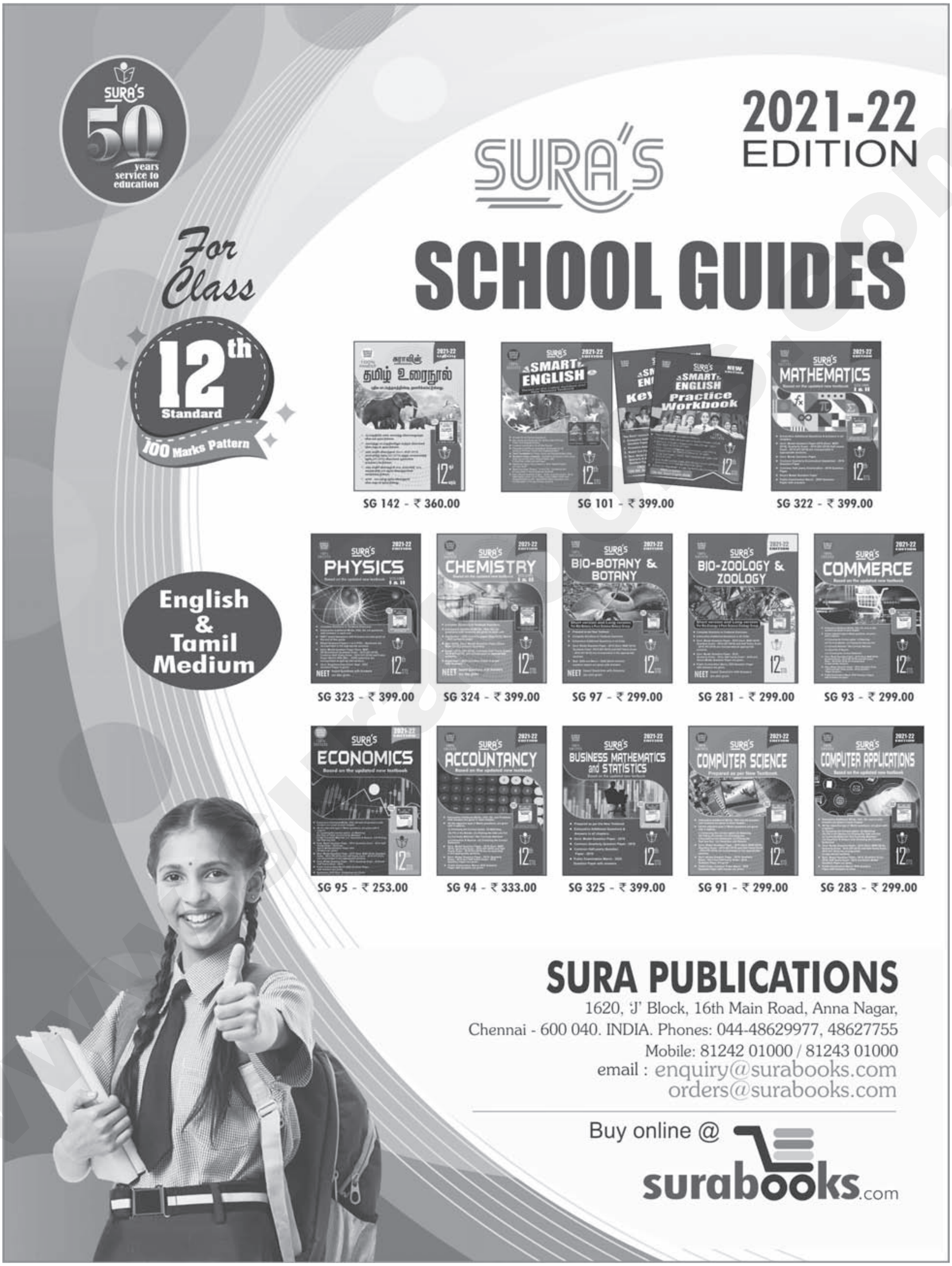
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Chapter 1

APPLICATIONS OF MATRICES AND DETERMINANTS

CHAPTER SNAPSHOT

Rank of a matrix :-

The rank of a matrix A is the order of the largest non-zero minor of A and is denoted by ρ (A).

(i) $\rho(A) \geq 0$.

(ii) If A is a matrix of order $m \times n$, then $\rho(A) \leq \min\{m, n\}$.

(iii) Rank of a zero matrix is 0.

(iv) The rank of a non - singular matrix of order $n \times n$ *is* "n".

Elementary transformations :

(i) Interchange any two rows (or columns)

$$R_i \leftrightarrow R_j \ (C_i \leftrightarrow C_j)$$

(ii) Multiplication of each element of a row (or column) by any non-zero scalar k.

$$R_i \to k\,R_i \ (\text{or } C_i \to k\,C_i)$$

(iii) Addition to the elements of any row (or column) the same scalar multiples of corresponding elements of any other row (or column).

$$R_i \to R_i + k\,R_j \ (\text{or } C_i \to C_i + k\,C_i)$$

Equivalent matrices:

Two matrices A and B are said to be equivalent if one is obtained from the another by applying a finite number of elementary transformations.

$A \cong b$

Echelon form :

A matrix A of order $m \times n$ is said to be in echelon form if

(i) Every row of A which has all its entries 0 occurs below every row which has a non - zero entry.

(ii) The number of zeros before the first non - zero element in a row is less than the number of such zeros in the next row.

Transition matrix :

The transition probabilities P_{jk} satisfy $P_{jk} > 0$, $\sum_{k} P_{jk} = 1$ for all j

FORMULAE TO REMEMBER

1. Linear equations can be written in matrix form AX = B, then the solution is $X = A^{-1}B$, provided $|A| \neq 0$.
2. Consistency of non homogeneous linear equations by rank method.

(i) If $\rho([A,B]) = \rho(A)$, then the equations are consistent.

(ii) If $\rho([A,B]) = \rho(A) = n$, where n is the number of variables then the equations are consistent and have unique solution.

(iii) If $\rho([A,B]) = \rho(A) < n$, then the equations are consistent and have infinitely many solutions.

(iv) If $\rho([A,B]) \neq \rho(A)$, then the equations are inconsistent and has no solution.

TEXTUAL QUESTIONS

EXERCISE 1.1

1. Find the rank of each of the following matrices.

(i) $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 1 & -1 \\ 3 & -6 \end{pmatrix}$

(iii) $\begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$ **(iv)** $\begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$

(v) $\begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$ **(vi)** $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

(vii) $\begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$ **(viii)** $\begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$

Sol : (i) Let A $= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

Order of A is 2 × 2

$\therefore \rho(A) \leq 2$ [Since minimum of (2, 2) is 2]

Consider the second order minor,

$$\begin{vmatrix} 5 & 6 \\ 7 & 8 \end{vmatrix} = 40 - 42 = -2 \neq 0.$$

There is a minor of order 2, which is not zero

$\therefore \rho(A) = 2$

(ii) Let A $= \begin{pmatrix} 1 & -1 \\ 3 & -6 \end{pmatrix}$

Order of A is 2 × 2

$\therefore \rho(A) \leq 2$ [Since minimum of (2, 2) is 2]

Consider the second order minor,

$$\begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -6-(-3) = -6+3 = -3 \neq 0.$$

There is a minor of order 2, which is not zero

$\therefore \rho(A) = 2.$

(iii) Let A $= \begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$ **[QY-2019]**

Order of A is 2 × 2 [Since minimum of (2,2) is 2]

Consider the second order minor $\begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix}$

$= 8 - 8$

$= 0.$

Since the second order minor vanishes, $\rho(A) \neq 2$

Consider a first order minor $|1| \neq 0$

There is a minor of order 1, which is not zero

$\therefore \rho(A) = 1.$

(iv) Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$ **[PTA - 1]**

The order of A is 3 × 3

$\therefore \rho(A) \leq 3$ [Since minimum of (3, 3) is 3]

Let us transform the matrix A to an echelon form

Matrix A	Elementary Transformation
$A = \begin{pmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & -5 \\ 2 & -1 & 1 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -2 & -8 \\ 0 & -3 & -1 \end{pmatrix}$	$R_2 \to R_2 - 3R_1$ $R_3 \to R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -4 \\ 0 & -3 & -1 \end{pmatrix}$	$R_2 \to R_2 \div 2$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & -4 \\ 0 & 0 & 11 \end{pmatrix}$	$R_3 \to R_3 - 3R_2$

This matrix is in echelon from and number of non-zero rows is 3.

$\therefore \rho(A) = 3.$

(v) Let $A = \begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$

The order of A is 3 × 3

$\therefore \rho(A) \leq 3$ [Since minimum of (3, 3) is 3]

Let us transform the matrix to an echelon form.

Matrix A	Elementary Transformation
$A = \begin{pmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & -2 & 2 \\ 4 & -3 & 4 \\ -2 & 4 & -4 \end{pmatrix}$	$R_1 \to R_1 (-1)$
$\sim \begin{pmatrix} 1 & -2 & 2 \\ 0 & 5 & -4 \\ -2 & 4 & -4 \end{pmatrix}$	$R_2 \to R_2 - 4R_1$
$\sim \begin{pmatrix} 1 & -2 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 + 2R_1$

The matrix is in echelon form and the number of non-zero rows is 2.

$\therefore \rho(A) = 2.$

(vi) Let $A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

The order of A is 3 × 4

$\therefore \rho(A) \leq 3$ [Since minimum of (3, 3) is 3]

Let us transform the matrix to an echelon form.

Matrix A	Elementary Transformation
$A = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 3 & 6 & 3 & -7 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 6 & -16 \end{pmatrix}$	$R_3 \to R_3 - 3R_1$
$\sim \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 0 & 3 & -18 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - 2R_2$

The matrix is in echelon form and the number of non- zero rows is 2.

$\therefore \rho(A) = 2.$

(vii) $A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$

The order of A is 3 × 4

$\therefore \rho(A) \leq 3$ [Since minimum of (3, 4) is 3]

Let us transform the matrix A to an echelon form.

Matrix A	Elementary Transformation
$A = \begin{pmatrix} 3 & 1 & -5 & -1 \\ 1 & -2 & 1 & -5 \\ 1 & 5 & -7 & 2 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & -5 & -1 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 3 & 1 & -5 & -1 \end{pmatrix}$	$R_2 \to R_2 - R_1$
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & -14 & 16 & -7 \end{pmatrix}$	$R_3 \to R_3 - 3R_1$
$\sim \begin{pmatrix} 1 & 5 & -7 & 2 \\ 0 & -7 & 8 & -7 \\ 0 & 0 & 0 & 7 \end{pmatrix}$	$R_3 \to R_3 - 2R_2$

The matrix is in echelon form and the number of non-zero matrix is 3.

$\therefore \rho(A) = 3.$

(viii) $A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$

The order of A is 3 × 4

$\therefore \rho(A) \le$ minimum of $(3, 4) \Rightarrow \rho(A) \le 3$

Let us transform the matrix A to an echelon form.

Matrix A	Elementary Transformation
$A = \begin{pmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ -1 & 2 & 7 & 6 \end{pmatrix}$	$R_2 \to R_2 + 2R_1$
$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 10 & 10 \end{pmatrix}$	$R_3 \to R_3 + R_1$
$\sim \begin{pmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - 2R_2$

The matrix is in echelon form and the number of non- zero rows is 2.

$\therefore \rho(A) = 2.$

2. If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ **and** $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$, **then find the rank of AB and the rank of BA.**

Sol :

Given $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2-5 & -2+4-1 & 3-6+1 \\ 2+6+20 & -4-12+4 & 6+18-4 \\ 3+4+15 & -6-8+3 & 9+12-3 \end{pmatrix}$$

$$AB = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$$

Matrix (AB)	Elementary Transformation
$AB = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & -6 & -2 \\ -12 & 28 & 20 \\ -11 & 22 & 18 \end{pmatrix}$	$C_1 \leftrightarrow C_2$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ -11 & 22 & 18 \end{pmatrix}$	$R_2 \to R_2 + 12R_1$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & -44 & -4 \end{pmatrix}$	$R_3 \to R_3 + 11R_1$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

The matrix is in echelon form and the number of non-zero rows is 2.

$\therefore \rho(AB) = 2.$

Now, $BA = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$

$$= \begin{pmatrix} 1-4+9 & 1+6-6 & -1-8+9 \\ -2+8-18 & -2-12+12 & 2+16-18 \\ 5+2-3 & 5-3+2 & -5+4-3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$$

Matrix (BA)	Elementary Transformation
$BA = \begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 6 & 0 \\ -2 & -12 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	$C_1 \leftrightarrow C_2$
$\sim \begin{pmatrix} 1 & 6 & 0 \\ 0 & 0 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	$R_2 \to R_2 + 2R_1$
$\sim \begin{pmatrix} 1 & 6 & 0 \\ 0 & 0 & 0 \\ 0 & -20 & -4 \end{pmatrix}$	$R_3 \to R_3 - 4R_1$

The number of non-zero rows is 2.

$\therefore\ \rho(BA) = 2.$

3. Solve the following system of equations by rank method $x + y + z = 9$, $2x + 5y + 7z = 52$, $2x - y - z = 0$

Sol : The given equations are $x + y + z = 9$, $2x + 5y + 7z = 52$, $2x - y - z = 0$

The matrix equation corresponding to the given system is

$$\underbrace{\begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{pmatrix}}_{A}\underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 9 \\ 52 \\ 0 \end{pmatrix}}_{B}$$

Augmented matrix [AB]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 9 \\ 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & -1 & -3 & -18 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$ $R_3 \to R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{pmatrix}$	$R_3 \to 3R_3 + R_2$
$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -4 \end{pmatrix}$	$\Rightarrow P(A) = 3$

Since augmented matrix $[A, B] \sim \begin{pmatrix} 1 & 1 & 1 & 9 \\ 0 & 3 & 5 & 34 \\ 0 & 0 & -4 & -20 \end{pmatrix}$ has three non-zero rows, $\rho([A,B]) = 3$.

That is, $\rho(A) = \rho([A,B]) = 3$ = number of unknowns.

So the given system is consistent and has unique solution.

To find the solution, we rewrite the echelon form into the matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 34 \\ -20 \end{pmatrix}$$

$\Rightarrow\quad x + y + z = 9 \quad\ldots(1)$

$3y + 5z = 34 \quad\ldots(2)$

$-4z = -20 \quad\ldots(3)$

$(3) \Rightarrow\quad -4z = -20$

$z = \dfrac{-20}{-4} = 5$

$(2) \Rightarrow\quad 3y + 5(5) = 34$

$\Rightarrow\quad 3y + 25 = 34 \Rightarrow 3y = 34 - 25$

$\Rightarrow\quad 3y = 9 \quad \Rightarrow y = \dfrac{9}{3}$

$y = 3.$

$(1) \Rightarrow\quad x + 3 + 5 = 9$

$\Rightarrow\quad x + 8 = 9 \quad \Rightarrow x = 9 - 8 \Rightarrow x = 1$

$\therefore\ x = 1,\ y = 3,\ z = 5$ is the unique solution of the given equations.

4. Show that the equations $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$ are consistent and solve them by rank method.

Sol : Given non-homogeneous equations are

$5x + 3y + 7z = 4$

$$\begin{aligned} 3x + 26y + 2z &= 9 \\ 7x + 2y + 10z &= 5 \end{aligned}$$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 5 \end{pmatrix}$$

$$A \quad X = B$$

Augmented matrix $[A, B] = \begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	
$\sim \begin{pmatrix} 3 & 26 & 2 & 9 \\ 5 & 3 & 7 & 4 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_1 \leftrightarrow R_2$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 5 & 3 & 7 & 4 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_1 \to R_1 \div 3$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-121}{3} & \frac{11}{3} & -11 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_2 \to R_2 - 5R_1$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-121}{3} & \frac{11}{3} & -11 \\ 0 & \frac{-176}{3} & \frac{16}{3} & -16 \end{pmatrix}$	$R_3 \to R_3 - 7R_1$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \end{pmatrix}$	$R_2 \to R_2 \div 11$ $R_3 \to R_3 \div 16$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

Here $\rho(A) = \rho(A, B) = 2 <$ Number of unknowns.

$\therefore$ The system is consistent with infinitely many solutions let us rewrite the above echelon form into matrix form.

$$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} \\ 0 & \frac{-11}{3} & \frac{1}{3} \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$$

$$x + \frac{26}{3}y + \frac{2}{3}z = 3 \qquad \text{... (1)}$$

$$\frac{-11}{3}y + \frac{1}{3}z = -1 \qquad \text{...(2)}$$

let $z = k$; where $k \in R$

$$(2) \Rightarrow \frac{-11}{3}y + \frac{k}{3} = -1$$

$$\Rightarrow \frac{-11}{\cancel{3}}y = -1 - \frac{k}{3} = \frac{-3-k}{\cancel{3}}$$

$$\Rightarrow -11y = -3 - k$$

$$\Rightarrow 11y = 3 + k$$

$$\Rightarrow y = \frac{1}{11}(3 + k)$$

Substituting $y = \frac{1}{11}(3+k)$ and $z = k$ in (1) we get,

$$x + \frac{26}{3}\left(\frac{3+k}{11}\right) + \frac{2}{3}k = 3$$

$$x = -\frac{26}{3}\left(\frac{3+k}{11}\right) - \frac{2k}{3} + 3$$

$$= \frac{-78-26k}{33} - \frac{2k}{3} + 3 = \frac{-78-26k-22k+99}{33}$$

$$= \frac{21-48k}{33} = \frac{3(7-16k)}{33}$$

$$x = \frac{1}{11}(7 - 16k)$$

$\therefore$ Solution set is $\{\frac{1}{11}(7-16k), \frac{1}{11}(3+k), k\}$ $k \in R$.

Hence, for different values of k, we get infinitely many solutions.

5. Show that the following system of equations have unique solution: $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$ by rank method. [QY-2019]

Sol : Given non-homogeneous equations are

$$\begin{aligned} x + y + z &= 3 \\ x + 2y + 3z &= 4 \\ x + 4y + 9z &= 6 \end{aligned}$$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

$$A \quad X = B$$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 6 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & 8 & 3 \end{pmatrix}$	$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \end{pmatrix}$	$R_3 \to R_3 - 3R_2$

Clearly the last equivalent matrix is in echelon form and it has three non-zero rows.

$\therefore$ $\rho(A) = 3$ and $\rho([A, B]) = 3$

$\Rightarrow \rho(A) = \rho([A, B]) = 3 =$ Number of unknowns.

$\therefore$ The given system is consistent and has unique solution.

To find the solution, let us rewrite the above echelon form into the matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad x + y + z = 3 \quad \ldots (1)$$
$$y + 2z = 1 \quad \ldots (2)$$
$$2z = 0 \quad \ldots (3)$$

$(3) \Rightarrow 2z = 0 \Rightarrow z = \frac{0}{2} = 0$

$(2) \Rightarrow y + 2(0) = 1 \Rightarrow y + 0 = 1 \Rightarrow y = 1 - 0 = 1$

$(1) \Rightarrow \quad x + 1 + 0 = 3$

$\Rightarrow \quad x + 1 = 3$

$\Rightarrow \quad x = 3 - 1$

$\Rightarrow \quad x = 2$

$\therefore$ Solution is $\{2, 1, 0\}$

6. For what values of the parameter λ, will the following equations fail to have unique solution: $3x - y + \lambda z = 1$, $2x + y + z = 2$, $x + 2y - \lambda z = -1$ by rank method.

Sol : Given non-homogeneous equations are

$$3x - y + \lambda z = 1$$
$$2x + y + z = 2$$
$$x + 2y - \lambda z = -1$$

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 3 & -1 & \lambda \\ 2 & 1 & 1 \\ 1 & 2 & -\lambda \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$$A \quad X = B$$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 3 & -1 & \lambda & 1 \\ 2 & 1 & 1 & 2 \\ 1 & 2 & -\lambda & -1 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 2 & 1 & 1 & 2 \\ 3 & -1 & \lambda & 1 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 3 & -1 & \lambda & 1 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -3 & 1+2\lambda & 4 \\ 0 & -7 & 4\lambda & 4 \end{pmatrix}$	$R_3 \to R_3 - 3R_1$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -1 & \frac{1+2\lambda}{3} & \frac{4}{3} \\ 0 & -1 & \frac{4\lambda}{7} & \frac{4}{7} \end{pmatrix}$	$R_2 \to R_2 \div 3$ $R_3 \to R_3 \div 7$
$\sim \begin{pmatrix} 1 & 2 & -\lambda & -1 \\ 0 & -1 & \frac{1+2\lambda}{3} & \frac{4}{3} \\ 0 & 0 & \frac{-7-2\lambda}{21} & \frac{-16}{21} \end{pmatrix}$	$R_3 \to R_3 - R_2$

Since

$$\frac{4\lambda}{7} - \frac{1+2\lambda}{3} = \frac{12\lambda - 7 - 14\lambda}{21} = \frac{-7-2\lambda}{21}$$

$$\text{and } \frac{4}{7} - \frac{4}{3} = \frac{12-28}{21} = \frac{-16}{21}$$

Since the system is fail to have unique solution either it can have infinitely many solution or it may be inconsistent.

$\therefore$ This can happen only when $\frac{-7-2\lambda}{21} = 0$.

$$\Rightarrow \quad -7 - 2\lambda = 0$$

$$\Rightarrow \quad -7 = 2\lambda$$

$$\Rightarrow \quad \lambda = \frac{-7}{2}.$$

7. The price of three commodities X,Y and Z are *x*,*y* and *z* respectively Mr. Anand Purchases 6 units of Z and sells 2 units of X and 3 units of Y. Mr. Amar Purchases a unit of Y and sells 3 units of X and 2units of Z. Mr. Amit Purchases a unit of X and sells 3 units of Y and a unit of Z. In the process they earn ₹ 5,000/-, ₹ 2,000/- and ₹ 5,500/- respectively Find the prices per unit of three commodities by rank method. [PTA-5]

Sol : Given that the price of commodities X, Y and Z are *x*, *y* and *z* respectively.

By the given data,

Transaction	x	y	z	Earning
Mr. Anand	+2	+3	–6	Rs. 5000
Mr. Amar	+3	–1	+2	Rs. 2000
Mr. Amit	–1	+3	+1	Rs. 5500

Here, purchasing is taken as negative symbol and selling is taken as positive symbol.

Thus, the non-homogeneous equations are

$$2x + 3y - 6z = 5000$$
$$3x - y + 2z = 2000$$
$$-x + 3y + z = 5500$$

The matrix equation corresponding to the given system is

$$\underset{A}{\begin{pmatrix} 2 & 3 & -6 \\ 3 & -1 & 2 \\ -1 & 3 & 1 \end{pmatrix}} \underset{X}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{B}{\begin{pmatrix} 5000 \\ 2000 \\ 5500 \end{pmatrix}}$$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 2 & 3 & -6 & 5,000 \\ 3 & -1 & 2 & 2,000 \\ -1 & 3 & 1 & 5,500 \end{pmatrix}$	
$\sim \begin{pmatrix} -1 & 3 & 1 & 5500 \\ 3 & -1 & 2 & 2000 \\ 2 & 3 & -6 & 5000 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} -1 & 3 & 1 & 5500 \\ 0 & 8 & 5 & 18500 \\ 0 & 9 & -4 & 16000 \end{pmatrix}$	$R_2 \to R_2 + 3R_1$ $R_3 \to R_3 + 2R_1$
$\sim \begin{pmatrix} -1 & 3 & 1 & 5500 \\ 0 & 72 & 45 & 166500 \\ 0 & 72 & -32 & 128000 \end{pmatrix}$	$R_2 \to 9R_2$ $R_3 \to 8R_3$
$\sim \begin{pmatrix} -1 & 3 & 1 & 5500 \\ 0 & 72 & 45 & 166500 \\ 0 & 0 & -77 & -38500 \end{pmatrix}$	$R_3 \to R_3 - R_2$

$\rho(A) = \rho([A, B]) = 3$ = number of unknowns so the system has unique solution.

$\therefore$ The given system is equivalent to the matrix equation.

$$\begin{pmatrix} -1 & 3 & 1 \\ 0 & 72 & 45 \\ 0 & 0 & -77 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5500 \\ 166500 \\ -38500 \end{pmatrix}$$

$$-x + 3y - z = 5500 \quad \text{... (1)}$$
$$72y + 45z = 166500 \quad \text{... (2)}$$
$$-77z = -38500 \quad \text{... (3)}$$

$$(3) \Rightarrow \quad -77z = -38500$$

$$\Rightarrow \quad z = \frac{\cancel{-}38500}{\cancel{-}77} = 500$$

$$(2) \Rightarrow \quad 72y + 45\,(500) = 166500$$

$$\Rightarrow \quad 72y + 22,500 = 166500$$

$$\Rightarrow \quad 72y = 166500 - 22500$$

$$\Rightarrow \quad 72y = 144000$$

$$\Rightarrow \quad y = \frac{144000}{72} \Rightarrow y = 2,000$$

$$(1) \Rightarrow -x + 3(2000) + 500 = 5500$$

$$\Rightarrow \quad -x \cancel{+} 6500 = 5500$$

$$-x = -5500 - 6500$$

$$\cancel{-} x = \cancel{-} 1000$$

$$x = 1000$$

$\therefore$ The prices per unit of the three commodities are ₹1000, ₹ 2000 and ₹ 500.

8. An amount of ₹ 5,000/- is to be deposited in three different bonds bearing 6%, 7% and 8% per year respectively. Total annual income is ₹358/-. If the income from first two investments is ₹ 70/- more than the income from the third, then find the amount of investment in each bond by rank method.

Sol : Let the amount of investment in each bond be ₹ x, ₹ y and ₹ z respectively.

$$\text{Given } x + y + z = 5000 \quad \ldots (1)$$

$$\text{Also } \frac{6x}{100} + \frac{7y}{100} + \frac{8z}{100} = 358$$

$$\boxed{\because \text{ Interest} = \frac{PNR}{100} = \frac{x \times 1 \times 6}{100} = \frac{6x}{100}}$$

$$\Rightarrow \frac{6x + 7y + 8z}{100} = 358$$

$$\Rightarrow 6x + 7y + 8z = 35800 \quad \ldots (2)$$

$$\text{Given that } \frac{6x}{100} + \frac{7y}{100} = 70 + \frac{8z}{100}$$

$$\Rightarrow \frac{6x + 7y}{\cancel{100}} = \frac{7000 + 8z}{\cancel{100}}$$

$$\Rightarrow 6x + 7y = 700 + 8z$$

$$\Rightarrow 6x + 7y - 8z = 7000 \quad \ldots (3)$$

The matrix equation corresponding to the given system is.

$$\begin{pmatrix} 1 & 1 & 1 \\ 6 & 7 & 8 \\ 6 & 7 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 35800 \\ 7000 \end{pmatrix}$$

$$A \quad X = B$$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 5000 \\ 6 & 7 & 8 & 35800 \\ 6 & 7 & -8 & 7000 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 1 & -14 & -23000 \end{pmatrix}$	$R_2 \to R_2 - 6R_1$ $R_3 \to R_3 - 6R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 5000 \\ 0 & 1 & 2 & 5800 \\ 0 & 0 & -16 & -28800 \end{pmatrix}$	$R_3 \to R_3 - R_2$

The last equivalent matrix is in echelon form and $\rho(A) = \rho([A, B]) = 3$ = Number of unknowns.

Thus, the given system is consistent with unique solution. To find the solution, let us rewrite the above echelon form into the matrix form.

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -16 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5000 \\ 5800 \\ -28800 \end{pmatrix}$$

$$\Rightarrow x + y + z = 5000 \quad \ldots (1)$$

$$\Rightarrow y + 2z = 5800 \quad \ldots (2)$$

$$\Rightarrow -16z = -28800 \quad \ldots (3)$$

$$(3) \Rightarrow -16z = -28800$$

$$\Rightarrow z = -\frac{28800}{-16} = 1800$$

Substituting $z = 1800$ in (2) we get,

$$y + 2(1800) = 5800$$

$$\Rightarrow y + 3600 = 5800$$

$$\Rightarrow y = 5800 - 3600$$

$$\Rightarrow y = 2200$$

Substituting $y = 2200$ and $z = 1800$ in (1) we get,

$$x + 2200 + 1800 = 5000$$

$$\Rightarrow x + 4000 = 5000$$

$$\Rightarrow x = 5000 - 4000$$

$$\Rightarrow x = 1000$$

Hence, the amount of investment in each bond is ₹ 1000, ₹ 2200 and ₹1800 respectively.

EXERCISE 1.3

1. The subscription department of a magazine sends out a letter to a large mailing list inviting subscriptions for the magazine. Some of the people receiving this letter already subscribe to the magazine while others do not. From this mailing list, 45% of those who already subscribe will subscribe again while 30% of those who do not now subscribe will subscribe. On the last letter, it was found that 40% of those receiving it ordered a subscription. What percent of those receiving the current letter can be expected to order a subscription?

Sol : Transition probability matrix

$$T = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.45 & 0.55 \\ 0.30 & 0.70 \end{pmatrix} \end{array}$$

Where A represents the percentage of subscribers and B represents the percentage of non-subscribers. By the given data, 40% received the order of subscription ⇒ 60% are non-subscribers.

$$A = 40\% = 0{\cdot}40$$
$$\text{and } B = 60\% = 0{\cdot}60$$

$$\therefore \begin{array}{cc} A & B \\ (0.40 & 0.60) \end{array} \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.45 & 0.55 \\ 0.30 & 0.70 \end{pmatrix} \end{array} =$$

$$\big((0.40)(0.45)+(0.60)(0.30) \quad (0.40)(0.55)+(0.60)(0.70)\big)$$

$$= (0.18+0.18 \quad 0.22+0.42)$$
$$= (0{\cdot}36 \quad 0{\cdot}64)$$

⇒ 36 % of those receiving the current letter can be expected to order a subscription.

2. A new transit system has just gone into operation in Chennai. Of those who use the transit system this year, 30% will switch over to using metro train next year and 70% will continue to use the transit system. Of those who use metro train this year, 70% will continue to use metro train next year and 30% will switch over to the transit system. Suppose the population of Chennai city remains constant and that 60% of the commuters use the transit system and 40% of the commuters use metro train next year.

(i) What percent of commuters will be using the transit system after the next year?

(ii) What percent of commuters will be using the transit system in the long run? [HY-2019]

Sol : Transition probability matrix

$$T = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.70 & 0.30 \\ 0.30 & 0.70 \end{pmatrix} \end{array}$$

Where A represents the percentage of people using transit system and B represents the percentage of people using metro train.

By the given data

$$A = 60\% = 0{\cdot}60$$
$$\text{and } B = 40\% = 0{\cdot}40$$

$$(0.60 \quad 0.40)\begin{pmatrix} 0.70 & 0.30 \\ 0.30 & 0.70 \end{pmatrix}$$
$$= \big((0.6)(0.7)+(0.4)(0.3) \quad (0.6)(0.3)+(0.4)(0.7)\big)$$
$$= (0.42+0.12 \quad 0.18+0.28) = (0.54 \quad 0.46)$$

$\therefore$ A = 54% and B = 46%

(i) The percent of Commuters using the transit system after one year is 54% and the percent of commuters using the metro train after the next year is 46%

(ii) Equilibrium will be reached in the long run. At equilibrium we must have

$$(A \quad B)\, T = (A \quad B)$$

where $A + B = 1$

$$\Rightarrow (A \quad B)\begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix} = (A \quad B)$$

$$\Rightarrow (0.7A + 0.3B \quad 0.3A + 0.7B) = (A \quad B)$$

Equaling the entries on both sides, we get

$$0.7A + 0.3B = A$$
$$\Rightarrow {\cdot}7A + 0.3\,(1 - A) = A$$
$$[\because A + B = 1 \Rightarrow B = 1 - A]$$
$$\Rightarrow 0.7A + 0.3 - 0.3A = A$$
$$\Rightarrow 0.3 = A - 0.7A + 0.3A$$
$$\Rightarrow 0.3 = A\,(1 - 0.7 + 0.3)$$
$$\Rightarrow 0.3 = A\,(0.3 + 0.3)$$
$$\Rightarrow 0.3 = A\,(0.6)$$
$$\Rightarrow A = \frac{0.3}{0.6} = \frac{1}{2} = 0{\cdot}50$$

$\therefore$ The percent of commuters using the transit system in the long run is 50%.

3. Two types of soaps A and B are in the market. Their present market shares are 15% for A and 85% for B. Of those who bought A the previous year, 65% continue to buy it again while 35% switch over to B. Of those who bought B the previous year, 55% buy it again and 45% switch over to A. Find their market shares after one year and when is the equilibrium reached?

[PTA-2; Govt. MQP & QY- 2019]

Sol : Transition probability matrix

$$T = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} \end{array}$$

Where A represents the percent of people those who bought soap A and B represents the percent of people those who bought soap B.

By the given data,

$$A = 15\% = 0{\cdot}15$$
$$\text{and } B = 85\% = 0{\cdot}85$$

Percentage after one year is

$$(0.15 \quad 0.85)\begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix}$$
$$= (0.15)(0.65)+(0.85)(0.45)\ 0.15(0.35)+0.85(0.55))$$
$$= (0.0975+0.3825 \quad 0.0525+0.4675) = (0.48 \quad 0.52)$$

Hence, market share after one year is 48% and 52%

At equilibrium,

$$(A \quad B)\,T = (A \quad B)$$
$$(A \quad B)\begin{pmatrix} 0.65 & 0.35 \\ 0.45 & 0.55 \end{pmatrix} = (A \quad B)$$
$$(0.65A+0.45B \quad 0.35A+0.55B) = (A \quad B)$$

Equating the corresponding entries on both sides we get,

$$0.65A+0.45B = A$$
$$\Rightarrow 0.65A+0.45(1-A) = A$$
$$[\text{Since } A+B=1 \Rightarrow B = 1-A]$$
$$\Rightarrow 0.65A+0.45-0.45A = A$$
$$\Rightarrow 0.45 = A-0.65A+0.45A$$
$$\Rightarrow 0.45 = A(1-0.65+0.45)$$
$$\Rightarrow 0.45 = A(0.35+0.45)$$
$$\Rightarrow 0.45 = A(0.8)$$
$$\Rightarrow A = \frac{0.45}{0.8} = 0.5625 = 56.25\%$$
$$\therefore B = 1-A$$
$$= 1-0.5625 = 0.4375$$
$$= 43.75\%$$

$\therefore$ Equilibrium is reached when A = 56.25% and B = 43.75%

4. Two products A and B currently share the market with shares 50% and 50% each respectively. Each week some brand switching takes place. Of those who bought A the previous week, 60% buy it again whereas 40% switch over to B. Of those who bought B the previous week, 80% buy it again where as 20% switch over to A. Find their shares after one week and after two weeks. If the price war continues, when is the equilibrium reached? **[Sep. - 2020]**

Sol : Transition probability matrix

$$T = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} \end{array}$$

By the given data

$$A = 50\% = 0.5$$
$$B = 50\% = 0.5$$

Shares after one week

$$(0.5 \quad 0.5)\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$$
$$= ((0.5)(0.6)+(0.5)(0.2) \quad 0.5(0.4)+0.5(0.8))$$
$$= (0.30)+0.10 \quad 0.20+0.40) = (0.40 \quad 0.60)$$

$\therefore$ Shares after one week for products A and B are 40% and 60% respectively.

Shares after two weeks

$$(0.4 \quad 0.6)\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix}$$
$$= ((0.4)(0.6)+(0.6)(0.2) \quad (0.4)(0.4)+0.6(0.8)$$
$$= (0.24+0.12 \quad 0.16+0.48) = 0.36 \quad 0.64)$$

$\therefore$ Shares after two week for products A and B are 36% and 64% respectively.

At equilibrium, we must have

$$(A \quad B)\,T = (A \quad B)$$
$$\text{where} \quad A+B = 1$$
$$(A \quad B)\begin{pmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{pmatrix} = (A \quad B)$$
$$\Rightarrow (0.6A+0.2B \quad 0.4A+0.8B) = (A \quad B)$$

Equating the corresponding entries on both sides we get,

$$0.6A+0.2B = A$$
$$\Rightarrow 0.6A+0.2(1-A) = A$$
$$\Rightarrow 0.6A+0.2-0.2A = A$$
$$\Rightarrow 0.2 = A-0.6A+0.2A$$
$$\Rightarrow 0.2 = A(1-0.6+0.2)$$
$$\Rightarrow 0.2 = A(0.4+0.2)$$
$$\Rightarrow 0.2 = A(0.6)$$
$$\Rightarrow A = \frac{0.2}{0.6} = 0.33$$
$$\Rightarrow A = 33\%$$

and $B = 1-A = 1-0.33 = 0.67 \Rightarrow B = 67\%$

$\therefore$ Equilibrium is reached when A = 33% and B = 67%

EXERCISE 1.4

Choose the correct answer

1. If A = (1 2 3), then the rank of AA^T is

(a) 0 (b) 2 (c) 3 (d) 1

[Ans: (d) 1]

Hint: $AA^T = \begin{pmatrix}1 & 2 & 3\end{pmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = (1+4+9) = 14$

$\therefore$ Rank of AA^T is 1

2. The rank of $m \times n$ matrix whose elements are unity is

(a) 0 (b) 1 (c) m (d) n

[Ans: (b) 1]

Hint: $A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & --- & 1 \\ 1 & 1 & 1 & 1 & 1 & --- & 1 \\ - & - & - & - & - & --- & - \\ - & - & - & - & - & --- & - \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

m rows and n columns applying the elementary transformation.

$R_2 \to R_2 - R_1, R_3 \to R_1 \ldots R_m \to R_m - R_1$ we get

$$A = \begin{pmatrix} 1 & 1 & 1 & --- & --- & 1 \\ 0 & 0 & 0 & --- & --- & 0 \\ 0 & 0 & 0 & --- & --- & 0 \\ - & - & - & --- & --- & - \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\therefore$ Rank is 1

3. If $T = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix} \end{array}$ is a transition probability matrix, then at equilibrium A is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$

[Ans: (a) $\frac{1}{4}$]

Hint: At equilibrium,

$$\begin{pmatrix} A & B \end{pmatrix}\begin{pmatrix} 0\cdot4 & 0\cdot6 \\ 0\cdot2 & 0\cdot8 \end{pmatrix} = \begin{pmatrix} A & B \end{pmatrix}$$

Where A + B = 1

$$\begin{pmatrix} 0\cdot4A + 0\cdot2B & 0\cdot6A + 0\cdot8B \end{pmatrix} = \begin{pmatrix} A & B \end{pmatrix}$$

$0\cdot4A + 0\cdot2B = A$

$0\cdot4A + 0\cdot2\,(1 - A) = A$

[since $A+B = 1 \Rightarrow B = 1-A$]

$0\cdot4A + 0\cdot2 - 0.2A = A$

$\Rightarrow \quad 0\cdot2 = A - 0\cdot4A + 0\cdot2A$

$\Rightarrow \quad 0\cdot2 = A\,(1 - 0\cdot4 + 0\cdot2)$

$= A\,(0\cdot8)$

$\Rightarrow \quad A = \frac{0\cdot2}{0\cdot8} = \frac{1}{4}$

4. If $A = \begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$ then ρ (A) is

(a) 0 (b) 1 (c) 2 (d) n

[Ans: (c) 2]

Hint: If $A = \begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$

$|A| = \begin{vmatrix} 2 & 0 \\ 0 & 8 \end{vmatrix} = 16 - 0 = 16 \neq 0.$

Since the second order determinant does not vanish. ρ (A) = 2

5. The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ is

(a) 0 (b) 1 (c) 2 (d)3

[Ans: (d) 3]

Hint: Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 8 \end{pmatrix} \begin{matrix} R_2 \to R_2 - R \\ R_3 \to R_3 - R \end{matrix}$

$= \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix} R_3 \to R_3 - 3R_2$

Since there are 3 non-zero rows, ρ (A) = 3

$\therefore \quad \rho\,(A) = 3$

6. The rank of the unit matrix of order n is

(a) $n - 1$ (b) n (c) $n + 1$ (d) n^2

[PTA - 3; QY - 2019]

[Ans: (b) n]

Hint: The rank of the unit matrix of order n [Since the rank of a non-singular matrix of order $n \times n$ is 'n']

7. If ρ(A) = r then which of the following is correct? **[Sep-2020]**

(a) all the minors of order r which does not vanish.

(b) A has at least one minor of order r which does not vanish.

(c) A has at least one (r + 1) order minor which vanishes.

(d) all (r + 1) and higher order minors should not vanish.

[Ans: (b) A has at least one minor of order r which does not vanish.]

8. If A = $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ then the rank of AA^T is

(a) 0 (b) 1 (c) 2 (d) 3

[Ans: (b) 1]

Hint: If A = $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ then $A^T = (1\ 2\ 3)$

$$AA^T = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}(1\ 2\ 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \therefore \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\therefore$ Rank of AA^T is 1

9. If the rank of the matrix $\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix}$ is 2, then λ is [Govt. MQP - 2019]

(a) 1 (b) 2

(c) 3 (d) only real number

[Ans: (a) 1]

Hint: Rank of $\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix} = 2$

Since the rank is 2,

$$\begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda \begin{vmatrix} \lambda & -1 \\ 0 & \lambda \end{vmatrix} + 1 \begin{vmatrix} 0 & -1 \\ -1 & \lambda \end{vmatrix} + 0 \begin{vmatrix} 0 & \lambda \\ -1 & 0 \end{vmatrix} = 0$$

$$\lambda(\lambda^2) + 1(0-1) + 0 = 0$$

$$\lambda^3 - 1 = 0 \Rightarrow \lambda^3 = 1$$

$$\lambda = 1$$

10. The rank of the diagonal matrix $\begin{pmatrix} 1 & & & & & \\ & 2 & & & & \\ & & -3 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$

(a) 0 (b) 2 (c) 3 (d) 5

[Ans: (c) 3]

Hint: Since there are 3 non-zero rows, rank is 3

11. If T = $\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0\cdot7 & 0\cdot3 \\ 0\cdot6 & x \end{pmatrix} \end{array}$ is a transition probability matrix, then the value of x is [PTA-6; QY-2019]

(a) 0.2 (b) 0.3 (c) 0.4 (d) 0.7

[Ans: (c) 0·4]

Hint: T = $\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0\cdot7 & 0\cdot3 \\ 0\cdot6 & x \end{pmatrix} \end{array}$

Since this is a transition probability matrix,

$$0\cdot6 + x = 1 \Rightarrow x = 1 - 0\cdot6 = 0\cdot4$$

12. Which of the following is not an elementary transformation? [March - 2020]

(a) $R_i \leftrightarrow R_j$ (b) $R_i \to 2R_i + 2C_j$

(c) $R_i \to 2R_i - 4R_j$ (d) $C_i \to C_i + 5C_j$

[Ans: (b) $R_i \to 2R_i + 2C_j$]

Hint: $R_i \to 2R_i + 2C_j$ is not an elementary transformation since it includes rows and columns.

13. If ρ(A) = ρ(A, B) then the system is [PTA-4]

(a) Consistent and has infinitely many solutions

(b) Consistent and has a unique solution

(c) Consistent (d) inconsistent

[Ans: (c) Consistent]

Hint: If ρ(A)=ρ(A, B) then the system can have a unique solution or infinitely many solutions.

$\therefore$ The system is consistent.

14. If ρ(A) = ρ(A, B) = the number of unknowns, then the system is

(a) Consistent and has infinitely many solutions

(b) Consistent and has a unique solution

(c) inconsistent (d) consistent

[Ans: (b) Consistent and has a unique solution]

Hint: If ρ(A)=ρ(A, B) = Number of unknowns then the system is consistent and has a unique solution.

15. If ρ(A) ≠ ρ(A, B), then the system is

(a) Consistent and has infinitely many solutions

(b) Consistent and has a unique solution

(c) inconsistent (d) consistent

[Ans: (c) inconsistent]

Hint: If ρ(A) ≠ ρ(A, B), then the system is inconsistent.

16. In a transition probability matrix, all the entries are greater than or equal to

(a) 2 (b) 1 (c) 0 (d) 3

[Ans: (c) 0]

Hint: $[\because 0 < p < 1]$

17. If the number of variables in a non-homogeneous system AX = B is n, then the system possesses a unique solution only when

(a) $\rho(A) = \rho(A, B) > n$ **[HY-2019]**

(b) $\rho(A) = \rho(A, B) = n$

(c) $\rho(A) = \rho(A, B) < n$ (d) none of these

[Ans: (b) $\rho(A) = \rho(A, B) = n$]

18. The system of equations $4x + 6y = 5$, $6x + 9y = 7$ has **[PTA-3; Govt. MQP - 2019]**

(a) a unique solution (b) no solution

(c) infinitely many solutions (d) none of these

[Ans: (b) no solution]

Hint: Given

$$4x + 6y = 5$$
$$6x + 9y = 7$$
$$[A, B] = \begin{pmatrix} 4 & 6 & 5 \\ 6 & 9 & 7 \end{pmatrix}$$
$$\sim \begin{pmatrix} 1 & \frac{6}{4} & \frac{5}{4} \\ 6 & 9 & 7 \end{pmatrix} R_1 \to R_1 \div 4$$
$$\sim \begin{pmatrix} 1 & \frac{3}{2} & \frac{5}{4} \\ 0 & 0 & \frac{-1}{2} \end{pmatrix} R_2 \to R_2 - 6R_1$$

Here $\rho(A) = 1$ and $\rho(A, B) = 2$

Since $\rho(A) \neq \rho(A, B)$, the system has no solution

19. For the system of equations $x + 2y + 3z = 1$, $2x + y + 3z = 2$ $5x + 5y + 9z = 4$

(a) there is only one solution

(b) there exists infinitely many solutions

(c) there is no solution (d) None of these

[Ans: (a) there is only one solution.]

Hint: Given $x + 2y + 3z = 1$, $2x + y + 3z = 2$, $5x + 5y + 9z = 4$

$$[A, B] = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 3 & 2 \\ 5 & 5 & 9 & 4 \end{pmatrix}$$
$$\sim \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -3 & -3 & 0 \\ 0 & -5 & -6 & -1 \end{pmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$
$$\sim \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & -5 & -6 & -1 \end{pmatrix} R_2 \to R_2 \div 5$$
$$\sim \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix} R_3 \to R_3 - 5R_2$$

Here $\rho(A) = 3$ and $\rho(A, B) = 3$

$\therefore \rho(A) = \rho(A, B) = 3$ = Number of unknowns.

20. If $|A| \neq 0$, then A is

(a) non- singular matrix (b) singular matrix

(c) zero matrix (d) none of these

[Ans: (a) non- singular matrix]

21. The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + k = 4$ has unique solution, if k is not equal to **[March -2020]**

(a) 4 (b) 0 (c) –4 (d) 1

[Ans: (b) 0]

Hint: Given $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + k = 4$

$$[A, B] = \begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 3 \\ 3 & 2 & k & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -1 \\ 0 & -1 & k-3 & -2 \end{pmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$
$$\sim \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & -1 & -3 & -1 \\ 0 & 0 & k & -1 \end{pmatrix} R_3 \to R_3 - R_2$$

For unique solution $\rho(A) = \rho(A, B) = 3$

This can happen only when $k \neq 0$

22. Cramer's rule is applicable only to get an unique solution when

(a) $\Delta_z \neq 0$ (b) $\Delta_x \neq 0$

(c) $\Delta \neq 0$ (d) $\Delta_y \neq 0$

[Ans: (c) $\Delta \neq 0$]

23. If $\frac{a_1}{x} + \frac{b_1}{y} = c_1$, $\frac{a_2}{x} + \frac{b_2}{y} = c_2$, $\Delta_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$; $\Delta_3 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}$ then (x, y) is

(a) $\left(\frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_1}\right)$ (b) $\left(\frac{\Delta_3}{\Delta_1}, \frac{\Delta_2}{\Delta_1}\right)$

(c) $\left(\frac{\Delta_1}{\Delta_2}, \frac{\Delta_1}{\Delta_3}\right)$ (d) $\left(\frac{-\Delta_1}{\Delta_2}, \frac{-\Delta_1}{\Delta_3}\right)$

[Ans: (d) $\left(\frac{-\Delta_1}{\Delta_2}, \frac{-\Delta_1}{\Delta_3}\right)$]

Hint: Given $\frac{a_1}{x} + \frac{b_1}{y} = c_1$; $\frac{a_2}{x} + \frac{b_2}{y} = c_2$

and $\Delta_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$; $\Delta_2 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$; $\Delta_3 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}$

$$\Rightarrow \quad \Delta_2 = -\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} \quad [\because C_1 \leftrightarrow C_2]$$
$$\Rightarrow \quad -\Delta_2 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

and $\Delta_3 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix} = -\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$ $C_1 \leftrightarrow C_2$

$\therefore \quad \frac{1}{x} = \frac{-\Delta_2}{\Delta_1}$ and $\frac{1}{y} = \frac{-\Delta_3}{+\Delta_1}$

$\Rightarrow \quad x = \frac{-\Delta_1}{\Delta_2}$ and $y = \frac{-\Delta_1}{\Delta_3}$

24. If $|A_{n\times n}| = 3$ and $|\text{adj}A|=243$ then the value n is **[PTA-5; QY-2019]**

(a) 4 (b) 5 (c) 6 (d) 7

[Ans: (c) 6]

Hint: Given $|A_{n\times n}| = 3$, $|\text{adj } A| = 243$.

If A is a square matrix of order n, then

$$|\text{adj } A| = |A|^{n-1}$$
$$243 = 3^{n-1}$$
$$3^5 = 3^{n-1}$$
$$5 = n-1$$
$$n = 5 + 1 = 6$$

25. Rank of a null matrix is **[PTA-2]**

(a) 0 (b) –1 (c) ∞ (d) 1

[Ans: (a) 0]

Miscellaneous problems

1. Find the rank of the matrix $A = \begin{pmatrix} 1 & -3 & 4 & 7 \\ 9 & 1 & 2 & 0 \end{pmatrix}$ **[PTA-2]**

Sol : Given $A = \begin{pmatrix} 1 & -3 & 4 & 7 \\ 9 & 1 & 2 & 0 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & -3 & 4 & 0 \\ 0 & 28 & -34 & -63 \end{pmatrix}$ $R_2 \to R_2 - 9R_1$

$\sim \begin{pmatrix} 1 & -3 & 4 & 0 \\ 0 & 0 & \frac{10}{3} & -63 \end{pmatrix}$ $R_2 \to R_2 + \frac{28}{3}.R_1$

The last equivalent matrix is in echelon form and there are 2 non-zero rows.

$\therefore \quad \rho(A) = 2.$

2. Find the rank of the matrix $A = \begin{pmatrix} -2 & 1 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 1 & 3 & 4 & 7 \end{pmatrix}$

Sol : Given $A = \begin{pmatrix} -2 & 1 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 1 & 3 & 4 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ -2 & 1 & 3 & 4 \end{pmatrix}$ $R_1 \leftrightarrow R_3$

$\sim \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 7 & 11 & 18 \end{pmatrix}$ $R_2 \to R_2 + 2R_1$

$\sim \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 4 & 4 \end{pmatrix}$ $R_3 \to R_3 - 7R_2$

The last equivalent matrix is in echelon form and there are 3 non - zero rows.

$\therefore \quad \rho(A) = 3.$

3. Find the rank of the matrix $A = \begin{pmatrix} 4 & 5 & 2 & 2 \\ 3 & 2 & 1 & 6 \\ 4 & 4 & 8 & 0 \end{pmatrix}$

Sol : Given $A = \begin{pmatrix} 4 & 5 & 2 & 2 \\ 3 & 2 & 1 & 6 \\ 4 & 4 & 8 & 0 \end{pmatrix}$

$\sim \begin{pmatrix} 4 & 5 & 2 & 2 \\ 3 & 2 & 1 & 6 \\ 1 & 1 & 2 & 0 \end{pmatrix}$ $R_3 \to R_3 \div 4$

$\sim \begin{pmatrix} 1 & 1 & 2 & 0 \\ 3 & 2 & 1 & 6 \\ 4 & 5 & 2 & 2 \end{pmatrix}$ $R_1 \leftrightarrow R_3$

$\sim \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & -1 & -5 & 6 \\ 0 & 1 & -6 & 2 \end{pmatrix}$ $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 4R_1$

$\sim \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & -1 & -5 & 6 \\ 0 & 0 & -11 & 8 \end{pmatrix}$ $R_3 \to R_3 + R_2$

The last equivalent matrix is in echelon form and there are 3 non-zero rows.

$\therefore \rho(A) = 3.$

4. Examine the consistency of the system of equations:
$x + y + z = 7, x + 2y + 3z = 18, y + 2z = 6$.

Sol : Given non homogeneous equations are

$x + y + z = 7, x + 2y + 3z = 18, y + 2z = 6.$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 7 \\ 1 & 2 & 3 & 18 \\ 0 & 1 & 2 & 6 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 11 \\ 0 & 1 & 2 & 6 \end{pmatrix}$	$R_2 \to R_2 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 7 \\ 0 & 1 & 2 & 11 \\ 0 & 0 & 0 & -5 \end{pmatrix}$	$R_3 \to R_3 - R_2$

Here $\rho(A) = 2$ and $\rho(A, B) = 3$

Since $\rho(A) \neq \rho(A, B)$, the given system is inconsistent and has no solution.

5. Find k if the equations $2x + 3y - z = 5$, $3x - y + 4z = 2$, $x + 7y - 6z = k$ are consistent.

Sol : Given non-homogeneous equations are

$2x + 3y - z = 5,\ 3x - y + 4z = 2,\ x + 7y - 6z = k$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 2 & 3 & -1 & 5 \\ 3 & -1 & 4 & 2 \\ 1 & 7 & -6 & k \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 7 & -6 & k \\ 3 & -1 & 4 & 2 \\ 2 & 3 & -1 & 5 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim \begin{pmatrix} 1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & -11 & 11 & 5-2k \end{pmatrix}$	$R_2 \to R_2 - 3R_1$ $R_3 \to R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & 0 & 0 & 2(5-2k)-(2-3k) \end{pmatrix}$	$R_3 \to 2R_3 - R_2$
$\sim \begin{pmatrix} 1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & 0 & 0 & 10-4k-2+3k \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 7 & -6 & k \\ 0 & -22 & 22 & 2-3k \\ 0 & 0 & 0 & 8-k \end{pmatrix}$	

Here $\rho(A) = 2$

Since the given system is consistent, $\rho(A, B)$ must be equal to 2.

This can happen only when $8 - k = 0 \Rightarrow k = 8$

6. Find k if the equations $x + y + z = 1$, $3x-y-z = 4$, $x + 5y + 5z = k$ are inconsistent.

Sol : $x + y + z = 1,\ 3x - y - z = 4,\ x + 5y + 5z = k$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & -1 & -1 & 4 \\ 1 & 5 & 5 & k \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -4 & 1 \\ 0 & 4 & 4 & k-1 \end{pmatrix}$	$R_2 \to R_2 - 3R_1$ $R_3 \to R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -4 & 1 \\ 0 & 0 & 0 & k \end{pmatrix}$	$R_3 \to R_3 + R_2$

Here clearly $\rho(A) = 2$.

Since the given system is inconsistent,

$\rho(A) \neq \rho(A, B)$

This can happen only when $k \neq 0$.

7. The subscription department of a magazine sends out a letter to a large mailing list inviting subscriptions for the magazine. Some of the people receiving this letter already subscribe to the magazine while others do not. From this mailing list, 60% of those who already subscribe will subscribe again while 25% of those who do not now subscribe will subscribe. On the last letter it was found that 40% of those receiving it ordered a subscription. What percent of those receiving the current letter can be expected to order a subscription?

Sol : Let A represents the percent of people who subscribe the magazine and B represents the percent of people who do not subscribe the magazine.

Given 60% of people subscribe again implies 40% of people do not subscribe. And 25% of people are going to subscribe implies 75% of people are not going to subscribe.

$\therefore$ Transition probability matrix

$$T = \begin{array}{c} \\ A \\ B \end{array} \begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.6 & 0.4 \\ 0.25 & 0.75 \end{pmatrix} \end{array}$$

Also, it is given that 40 % of those received the order of subscription implies 60% are not going to receive the order.

$\therefore (0.4 \quad 0.6)\begin{pmatrix} 0.6 & 0.4 \\ 0.25 & 0.75 \end{pmatrix}$

$= ((0.4)(0.6)+(0.6)(0.25) \quad (0.4)(0.4)+(0.6)(0.75))$

$= (0.24+0.15 \quad 0.16+0.45) = (0.39 \quad 0.61)$

$\therefore$ 39% of people who received the current letter can be expected to order a subscription.

PTA Questions & Answers

1 MARK

1. If O(A) = 3 × 3 and $\rho(A) = 2$ then $\rho(\text{adj}A)$ is________. **[PTA-1]**

(a) 1 (b) 2 (c) 3 (d) 0

[Ans: (a) 1**]**

2. **If A is matrix [A,B] is the augmented matrix then which of the following is true?** [PTA-2]

(a) $\rho([A,B]) = \rho(A)$

(b) $\rho([A,B]) \geq \rho(A)$

(c) $\rho([A,B]) = \rho(A) > n$

(d) $\rho([A,B]) < \rho(A)$

[Ans: (b) $\rho([A,B]) \geq \rho(A)$]

3. If $A = \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix}$ then the rank of AA^T is : [PTA-4] [HY-2019]

(a) 1 (b) 2 (c) 3 (d) 0

[Ans: (a) 1]

4. **If A is matrix of order 4 and |A| = –2 then the value of |adj (A)| is _____** [PTA-6]

(a) –4 (b) 4 (c) –8 (d) 8

[Ans: (c) – 8]

Hint: $|\text{adj } A| = |A|^{n-1} = (-2)^{4-1} = (-2)^3 = -8$

2 MARKS

1. **If** $A = \begin{bmatrix} x & x & x \\ 4 & -2 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ **find** x **if** $\rho(A) = 3$ [PTA - 3]

Sol : $\begin{vmatrix} x & x & x \\ 4 & -2 & 1 \\ 2 & 3 & 4 \end{vmatrix} = x[-8-3] - x[16-2] + x[12+4]$

$= -11x - 14x + 16x$

$= -9x - 9x \neq 0$

$x \neq 0$

2. **Find the rank of the matrix** $\begin{bmatrix} 2 & -4 & x \\ 4 & -8 & 2x \\ 5 & 7 & 1 \end{bmatrix}$ [PTA - 4]

Sol : $\begin{bmatrix} 2 & -4 & x \\ 4 & -8 & 2x \\ 5 & 7 & 1 \end{bmatrix} \xrightarrow{R_1 \to \frac{R_1}{2}} \begin{bmatrix} 1 & -2 & \frac{x}{2} \\ 4 & -8 & 2x \\ 5 & 7 & 1 \end{bmatrix}$

$\xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 4R1} \begin{bmatrix} 1 & -2 & \frac{x}{2} \\ 0 & 0 & 0 \\ 0 & 17 & \frac{2-5x}{2} \end{bmatrix}$

Rank 2 $\rho(A) = 2$

3. **Parithi is either Sad (S) or happy (H) each day. If he is happy in one day, he is sad on the next day by four times out of five. If he is sad on one day, he his happy on the next day by two times out of three. Over a long run, what are the chances that Parithi is happy on any given day?** [PTA - 5; QY - 2019]

Sol : The transition probability matrix is $T = \begin{pmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$

At equilibrium, $(S\ H)\begin{pmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix} = (S\ H)$

where S + H = 1

$\frac{4}{5}S + \frac{2}{3}H = S \Rightarrow \frac{4}{5}S + \frac{2}{3}(1-S) = S$

On solving this, we get

$S = \frac{10}{13}$ and $H = \frac{3}{13}$

In the long run, on a randomly selected day, his chances of being happy is $\frac{10}{13}$.

4. **Find the rank of the matrix** $\begin{bmatrix} 2 & 3 & 4 \\ -1 & 2 & 1 \\ 1 & 2 & -2 \end{bmatrix}$ [PTA - 5]

Sol : $\begin{vmatrix} 2 & 3 & 4 \\ -1 & 2 & 1 \\ 1 & 2 & -2 \end{vmatrix} = 2[-4-2] - 3[2-1] + 4[-2-2]$

$= 2[-6] - 3[1] + 4[-4] = -12 - 3 - 16 = -31$

$\therefore \rho(A) = 3$

5. **Six men and five women can jointly finish of work in 12 days, whereas five men and seven women can jointly finish the same work in 10 days, represent data as a system of linear equations.** [PTA - 6]

Sol : One man finish work in x days

One woman finish work in y days.

Work done by one man in one day $\frac{1}{x}$

Work done by one woman in one day $\frac{1}{y}$

$\frac{6}{x} + \frac{5}{y} = \frac{1}{12}$

$\Rightarrow \frac{5}{x} + \frac{7}{y} = \frac{1}{10}$

3 MARKS

1. Consider the matrix of transition probabilities of a product available in the market in two brands A and B.

$$\begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} \end{array}$$

Determine the market share of each brand in equilibrium position. **[PTA - 1]**

Sol : Transition probability matrix

$$T = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} \end{array}$$

At equilibrium, (A B) T=(AB) where A+B=1

$$\begin{pmatrix} A & B \end{pmatrix}\begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} = \begin{pmatrix} A & B \end{pmatrix}$$

$$\begin{aligned} 0.9A + 0.3B &= A \\ 0.9A + 0.3(1-A) &= A \\ 0.9A - 0.3A + 0.3 &= A \\ 0.6A + 0.3 &= A \\ 0.4A &= 0.3 \\ A &= \frac{0.3}{0.4} = \frac{3}{4} \\ B &= 1 - \frac{3}{4} = \frac{1}{4} \end{aligned}$$

Hence the market share of brand A is 75% and the market share of brand B is 25%

5 MARKS

1. Solve by using rank method $x + y + 2z = 4$, $2x + 2y + 4z = 8$, $3x + 3y + 6z = 12$ **[PTA - 3]**

Sol : The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 2 & 4 \\ 3 & 3 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 12 \end{pmatrix}$$

$$[A\ \ B] = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 2 & 2 & 4 & 8 \\ 3 & 3 & 6 & 12 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Back substituting we get

$x + y + 2z = 4$

$x = 4 - y - 2z$, where y and z are arbitrary

2. Investigate for what values of 'a' and 'b' the following system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + az = b$ have (i) no solution (ii) a unique solution (iii) an infinite number of solutions. **[PTA-3, March - 2020]**

Sol : The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & a \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ b \end{pmatrix}$$

$$A \qquad X = B$$

Augmented matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & a & b \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & a-1 & b-6 \end{pmatrix}$	$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & a-3 & b-10 \end{pmatrix}$	$R_3 \to R_3 - R_2$

Case (i) For no solution:

The system possesses no solution only when $\rho(A) \neq \rho([A,B])$ which is possible only when $a-3=0$ and $b-10 \neq 0$

Hence for $a=3$, $b \neq 10$, the system possesses no solution.

Case (ii) For a unique solution:

The system possesses a unique solution only when $\rho(A) = \rho([A,B])$ = number of unknowns.

i.e when $\rho(A) = \rho([A,B]) = 3$

Which is possible only when a−30 and b may be any real number as we can observe .

Hence for $a \neq 3$ and $b \in R$, the system possesses a unique solution.

Case (iii) For an infinite number of solutions:

The system possesses an infinite number of solutions only when

$\rho(A) = \rho([A,B]) <$ number of unknowns

i,e when $\rho(A) = \rho([A,B]) = 2 < 3$ (number of unknowns) which is possible only

when $a-3=0$, $b-10=0$

Hence for $a = 3$, $b = 10$, the system possesses infinite number of solutions.

3. Metro rail transit system has just gone into operation in a city. Of those who use the transit system this year 15% will switch over to using their own car next year and 85% will continue to use the transit system. Of those who use their cars this year, 70% will continue to use their cars next year and 30% will switch over to the transit system. Suppose the population of the city remains constant and that 60% of the commuters use the transit system and 40% of the commuters use their own car this year,

(i) What will be the change in commuter's usage after one year.

(ii) What percent of commuters will be using the Metro train system in the long run?

Sol : Transition probability matrix **[PTA - 4]**

$$T = \begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} .85 & .15 \\ .30 & .70 \end{pmatrix} \end{array}$$

Where A represent the percentage of people using metro rail transit system and B represents the percentage of people using car.

$A = 60\% = 0.60$ and $B = 40\% = 0.40$

$$\begin{pmatrix} 0.60 & 0.40 \end{pmatrix}\begin{pmatrix} 0.85 & 0.15 \\ 0.30 & 0.70 \end{pmatrix}$$

$= [(0.6)(0.85) + (0.4)(0.3) \;\; (0.6)(0.15) + (0.4)(0.7)]$

$= (0.63 \quad 0.37)$

$A = 63\% \qquad B = 37\%$

(i) The percentage of commuters using the metro rail system after one year is 63% and the percent of commuters using the car after one year is 37%

(ii) Equilibrium will be reached in the long run. At equilibrium we must have

$$(A \quad B)T = (A \quad B)$$

Where $\quad A + B = 1$

$$\Rightarrow (A \quad B)\begin{pmatrix} 0.85 & 0.15 \\ 0.30 & 0.70 \end{pmatrix} = (A \quad B)$$

$(0.85A + 0.30\,B \;\; 0.15A + 0.70B) = (A \quad B)$

$$\begin{aligned} 0.85A + 0.30B &= A \\ 0.85A + (0.3)(1 - A) &= A \\ 0.85A + 0.3 - 0.3A &= A \\ 0.3 &= A + 0.3A - 0.85\,A \\ 0.3 &= A(1 + .3 - 0.85) \\ 0.3 &= A\,(0.45) \\ A &= \frac{.3}{.45} = \frac{30}{45} = \frac{2}{3} \end{aligned}$$

The percent of commuters using the transit system in the long run is $66\frac{2}{3}\%$

4. Find x, y, z for the following system of equations **[PTA-5]**

$$\frac{2}{x} + \frac{3}{y} + \frac{4}{z} = 14,\ \frac{3}{x} - \frac{2}{y} + \frac{1}{z} = 3\ \ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 5$$

Sol : Put $\frac{1}{x} = a, \frac{1}{y} = b, \frac{1}{z} = c$

$2a + 3b + 4c = 14 \qquad ...(1)$

$3a - 2b + c = 3 \qquad ...(2)$

$a + b + c = 5 \qquad ...(3)$

Matrix form of the system

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 14 \\ 3 \\ 5 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 2 & 3 & 4 & 14 \\ 3 & -2 & 1 & 3 \\ 1 & 1 & 1 & 5 \end{bmatrix} \xrightarrow{R_1 \to R_3} \begin{bmatrix} 2 & 3 & 4 & 14 \\ 3 & -2 & 1 & 3 \\ 1 & 1 & 1 & 5 \end{bmatrix}$$

$$\xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & 2 & 4 \\ 0 & -5 & -2 & -12 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 + 5R_2} \begin{bmatrix} 0 & 1 & 1 & 5 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 8 & 8 \end{bmatrix}$$

Equivalent system

$$\begin{aligned} 8c &= 8 \\ \Rightarrow \quad c &= 1 \\ b + 2c &= 4 \\ \Rightarrow \quad b &= 4-2 = 2 \\ a + b + c &= 5 \\ \Rightarrow \quad a &= 5 - 2 - 1 = 2 \\ \frac{1}{x} &= 2 \\ \Rightarrow \quad x &= \frac{1}{2} \\ \frac{1}{y} &= 2 \quad \Rightarrow y = \frac{1}{2} \\ \frac{1}{z} &= 1 \\ \Rightarrow \quad z &= 1 \end{aligned}$$

5. 80% of students who do maths work during one study period, will do the maths work at the next study period. 30% of students who do english work during one study period, will do the english work at the next study period. Initially there were 60 students do maths work and 40 students do english work. Calculate,

(i) The transition probability matrix **[PTA-6]**

(ii) The number of students who do maths work, english work for the next subsequent 2 study periods.

Sol : (i) Transition probability matrix

$$T = \begin{array}{c} \\ M \\ E \end{array} \begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix} \end{array}$$

After one study period,

$$\begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{pmatrix} 60 & 40 \end{pmatrix} \end{array} \begin{array}{c} \\ M \\ E \end{array} \begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{pmatrix} 76 & 24 \end{pmatrix} \end{array}$$

So in the very next study period, there will be 76 students do maths work and

24 students do the English work.

After two study periods,

$$\begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{pmatrix} 76 & 24 \end{pmatrix} \end{array} \begin{array}{c} \\ M \\ E \end{array} \begin{array}{c} \begin{array}{cc} M & E \end{array} \\ \begin{pmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{pmatrix} \end{array} = (60.8 + 16.8 \quad 15.2+7.2)$$

$= 77.6 \quad 22.4$

After two study periods there will be 78 (approx) students do maths work and 22 (approx) students do English work.

Govt. Exam Questions & Answers

1 MARK

1. The system of equations $2x - y = 1$, $3x + 2y = 12$ has **[QY-2019]**

(a) a unique solution
(b) no solution
(c) infinitely many solution
(d) none of these **[*Ans.* (a) a unique solution]**

2. If |A| = 13 and |Adj A| = $\begin{vmatrix} 4 & x \\ 5 & 7 \end{vmatrix}$, then the value of x is : **[March - 2020]**

(a) 3 (b) 4 (c) 2 (d) –5

[*Ans.* (a) 3]

Hint: $[\because |\text{Adj A}| = |A|^{n-1}$ order $n \times n$

$$\begin{aligned} \therefore |A|^{2-1} &= \begin{vmatrix} 4 & x \\ 5 & 7 \end{vmatrix} \\ |A| &= 28 - 5x \\ 5x &= 28 - 13 = 15 \\ 5x &= 15 \\ x &= \frac{15}{5} = 3 \end{aligned}$$

3. The system has a unique solution when two lines : **[Sep.-2020]**

(a) L_1 and L_2 intersect exactly at one point
(b) L_1 and L_2 coincides
(c) L_1 and L_2 are parallel and distinct
(d) Both (a) and (b) **[*Ans.* (d) Both (a) and (b)]**

2 MARKS

1. If the rank of the matrix $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & K \\ 9 & 10 & 11 & 12 \end{bmatrix}$ is 2.

Find the value of 'K' **[HY - 2019]**

Sol : $\begin{vmatrix} 2 & 3 & 5 \\ 3 & 4 & 6 \\ 4 & 5 & k \end{vmatrix} = 0 \Rightarrow k = 7$

2. Find the rank of the matrix A = $\begin{vmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{vmatrix}$

[Sep. - 2020]

Sol : The order of A is 3×4

$\therefore \rho(A) \leq 3$

Let us transform the matrix A to an echelon form.

Matrix A	Elementary Transformation
$A = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{pmatrix}$	
$A \sim \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 3 & 1 & 1 & 3 \end{pmatrix}$	$R_1 \leftrightarrow R_2$
$\sim \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & -5 & -8 & -3 \end{pmatrix}$	$R_3 \to R_3 - 3R_1$
$\sim \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 2 & 2 \end{pmatrix}$	$R_3 \to R_3 - 5R_2$

The number of non zero rows is 3. $\therefore$ $\rho(A) = 3$.

3 MARKS

1. Find the rank of the matrix

$$A = \begin{pmatrix} 1 & 2 & -4 & 5 \\ 2 & -1 & 3 & 6 \\ 8 & 1 & 9 & 7 \end{pmatrix}$$ **[Govt. MQP - 2019]**

Sol : The order of A is 3×4

$$\therefore \rho(A \leq \min (3, 4)$$

$$\rho(A) \leq 3$$

Consider the third order minor,

$$\begin{vmatrix} 1 & 2 & -4 \\ 2 & -1 & 3 \\ 8 & 1 & 9 \end{vmatrix} = 1\begin{vmatrix} -1 & 3 \\ 1 & 9 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} - 4\begin{vmatrix} 2 & -1 \\ 8 & 1 \end{vmatrix}$$

$= 1(-9-3) - 2(18-24) - 4(2+8)$

$= 1(-12) - 2(-6) - 4(10)$

$= -12 + 12 - 40 = -40 \neq 0$.

There is a minor of order 3, which is not zero

$\therefore \quad \rho(A) = 3$.

2. Show that the equations $x + y = 5$, $2x + y = 8$ are consistent and solve them. **[QY - 2019]**

Sol : The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

Matrix A	Augment matrix [A, B]	Elementary Transformation
$\begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 & 5 \\ 2 & 1 & 8 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$	$\sim \begin{pmatrix} 1 & 1 & 5 \\ 1 & -1 & -2 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\rho(A) = 2$	$\rho([A,B]) = 2$	

Number of non-zero rows is 2.

$\rho(A) = \rho([A,B]) = 2$ = Number of unknowns.

The given system is consistent and has unique solution.

Now, the given system is transformed into

$$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

$$\begin{aligned} x + y &= 5 \\ y &= 2 \\ \therefore (1)\ x + 2 &= 5 \\ x &= 3 \\ \text{Solution is } x = 3, y &= 2 \end{aligned}$$

3. Show that the equations $x - 4y + 7z = 14$, $3x + 8y - 2z = 13$, $7x - 8y + 26z = 5$ are inconsistent. **[HY - 2019]**

Sol : $[A, B] = \begin{bmatrix} 1 & -4 & 7 & 14 \\ 3 & 8 & -2 & 13 \\ 7 & -8 & 26 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 1 & -4 & 7 & 14 \\ 0 & 20 & -23 & -29 \\ 0 & 0 & 0 & 64 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \\ R_3 \to R_3 - R_2 \end{matrix}$$

$\to \rho(A) = 2$, $\rho(A, B) = 3$

$\Rightarrow \rho(A) \neq \rho(A, B)$

The system is inconsistent and had no solution.

4. If $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ **and** $B^T = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$, **then find the rank of AB.** **[March - 2020]**

Sol : $A = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1-2-5 & -2+4-1 & 3-6+1 \\ 2+6+20 & -4-12+4 & 6+18-4 \\ 3+4+15 & -6-8+3 & 9+12-3 \end{pmatrix}$$

$$= \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix} = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$$

Matrix (AB)	Elementary Transformation
$AB = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & -6 & -2 \\ -12 & 28 & 20 \\ -11 & 22 & 18 \end{pmatrix}$	$C_1 \leftrightarrow C_2$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ -11 & 22 & 18 \end{pmatrix}$	$R_2 \to R_2 + 12R_1$

Matrix (AB)	Elementary Transformation
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & -44 & -4 \end{pmatrix}$	$R_3 \to R_3 + 11R_1$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

The matrix is in echelon form and the number of non-zero rows is 2.

$\therefore \rho(AB) = 2.$

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