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Chapter 1

RELATIONS AND FUNCTIONS

FORMULAE TO REMEMBER

- **Vertical line test :**

  A curve drawn in a graph represents a functions, if every vertical line intersects the curve in at most one point.

- **Horizontal line test :**

  A function represented in a graph is one - one, if every horizontal line intersect the curve in at most one point.

- Linear functions has applications in Cryptography as well as in several branches of Science and Technology.

EXERCISE 1.1

1. Find A× B, A×A and B ×A
(i) $A = \{2, -2, 3\}$ and $B = \{1, -4\}$ (ii) $A = B = \{p, q\}$
(iii) $A = \{m, n\}$; $B = \phi$ **[PTA - 1; FRT - 2022]**

Sol. (i) $A = \{2, -2, 3\}, B = \{1, -4\}$

$A \times B = \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$

$A \times A = \{(2, 2), (2, -2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3, 3)\}$

$B \times A = \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$

(ii) $A = B = \{(p, q)\}$

$A \times B = \{(p, p), (p, q), (q, p), (q, q)\}$

$A \times A = \{(p, p), (p, q), (q, p), (q, q)\}$

$B \times A = \{(p, p), (p, q), (q, p), (q, q)\}$

(iii) $A = \{m, n\}$, $B = \phi$

$A \times B = \{\ \}$

$A \times A = \{(m, m), (m, n), (n, m), (n, n)\}$

$B \times A = \{\ \ \}$

2. Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A× B and B × A. **[May - 2022]**

Sol. $A = \{1, 2, 3\}, B = \{2, 3, 5, 7\}$

$A \times B = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$

$B \times A = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$

3. If B × A={(−2, 3),(−2, 4),(0, 3),(0, 4),(3, 3), (3, 4)} find A and B. **[Qy. - 2019; April - 2023]**

Sol. Given $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$

Here $B = \{-2, 0, 3\}$

[All the first elements of the order pair] and $A = \{3, 4\}$

[All the second elements of the order pair]

4. If A ={5, 6}, B = {4, 5, 6} , C ={5, 6, 7}, Show that A×A = (B × B)∩(C ×C). **[FRT & Aug. - 2022]**

Sol. $A = \{5, 6\}, B = \{4, 5, 6\}, C = \{5, 6, 7\}$

$A \times A = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(1)

$B \times B = \{(4, 4), (4, 5), (4, 6), (5, 4), (\underline{5, 5}), (\underline{5, 6}), (6, 4), (\underline{6, 5}), (\underline{6, 6})\}$ ...(2)

$C \times C = \{(\underline{5, 5}), (\underline{5, 6}), (5, 7), (\underline{6, 5}), (\underline{6, 6}), (6, 7), (7, 5), (7, 6), (7, 7)\}$ ...(3)

$(B \times B) \cap (C \times C) = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(4)

(1) = (4)

$A \times A = (B \times B) \cap (C \times C)$. It is proved.

5. Given A ={1, 2, 3}, B = {2, 3, 5}, C = {3, 4} and D = {1, 3, 5}, check if (A∩C) × (B∩D) = (A×B)∩(C × D) is true? **[Qy. - 2019]**

Sol. LHS $= \{(A \cap C) \times (B \cap D)$

$A \cap C = \{3\}$

$B \cap D = \{3, 5\}$

$(A \cap C) \times (B \cap D) = \{(3, 3), (3, 5)\}$ ...(1)

RHS $= (A \times B) \cap (C \times D)$

$A \times B = \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), (\underline{3, 3}), (\underline{3, 5})\}$

$C \times D = \{(3, 1), (\underline{3, 3}), (\underline{3, 5}), (4, 1), (4, 3), (4, 5)\}$

$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ ...(2)

$\therefore$ (1) = (2) $\therefore$ It is true.

6. Let A={$x \in \mathbb{W}$ | $x < 2$}, B = {$x \in \mathbb{N}$ | $1 < x \leq 4$} and C = {3, 5} . Verify that
(i) A × (B∪C) = (A × B)∪(A × C) **[PTA - 2; FRT - 2022; June & Hy. - 2023]**
(ii) A×(B∩C) = (A × B)∩(A × C) **[PTA - 5; Sep. - 2021]**
(iii) (A∪B) × C = (A×C)∪(B × C) **[Qy. - 2023]**

(i) A × (B ∪ C) = (A × B) ∪ (A × C)

Sol. $A = \{x \in \mathbb{W} \mid x < 2\} = \{0, 1\}$

[Whole numbers less than 2]

$B = \{x \in \mathbb{N} \mid 1 < x \leq 4\} = \{2, 3, 4\}$

[Natural numbers from 2 to 4]

$C = \{3, 5\}$

LHS $= A \times (B \cup C)$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\} = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(1)

RHS $= (A \times B) \cup (A \times C)$

$(A \times B) = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$(A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(2)

(1) = (2), LHS = RHS Hence it is proved.

(ii) A × (B ∩ C) = (A × B) ∩ (A × C)

LHS $= A \times (B \cap C)$

$(B \cap C) = \{3\}$

$A \times (B \cap C) = \{(0, 3), (1, 3)\}$ ...(1)

RHS $= (A \times B) \cap (A \times C)$

$(A \times B) = \{(0, 2), (\underline{0, 3}), (0, 4), (1, 2), (\underline{1, 3}), (1, 4)\}$

$(A \times C) = \{(\underline{0, 3}), (0, 5), (\underline{1, 3}), (1, 5)\}$

$(A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ...(2)

(1) = (2) $\Rightarrow$ LHS = RHS.

Hence it is verified.

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

LHS $= (A \cup B) \times C$

$A \cup B = \{0, 1, 2, 3, 4\}$

$(A \cup B) \times C = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$ ...(1)

RHS $= (A \times C) \cup (B \times C)$

$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$(B \times C) = \{(2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$

$(A \times C) \cup (B \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$...(2)

(1) = (2) ∴ LHS = RHS. Hence it is verified.

7. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ [Sep. - 2020]

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ [PTA - 1] [May - 2022]

$A = \{1, 2, 3, 4, 5, 6, 7\}$

$B = \{2, 3, 5, 7\}$

$C = \{2\}$

[∵ 2 is the only even prime number]

Sol. **(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$**

LHS $= (A \cap B) \times C$

$A \cap B = \{2, 3, 5, 7\}$

$(A \cap B) \times C = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ...(1)

RHS $= (A \times C) \cap (B \times C)$

$(A \times C) = \{(1, 2), \underline{(2, 2)}, \underline{(3, 2)}, (4, 2), \underline{(5, 2)}, (6, 2), \underline{(7, 2)}\}$

$(B \times C) = \{\underline{(2, 2)}, \underline{(3, 2)}, \underline{(5, 2)}, \underline{(7, 2)}\}$

$(A \times C) \cap (B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ...(2)

(1) = (2) ∴ LHS = RHS. Hence it is verified.

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

LHS $= A \times (B - C)$

$(B - C) = \{3, 5, 7\}$

$A \times (B - C) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\}$ ...(1)

RHS $= (A \times B) - (A \times C)$

$(A \times B) = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7), (7, 2), (7, 3), (7, 5), (7,7)\}$

$(A \times C) = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$

$(A \times B) - (A \times C) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\}$ ...(2)

(1) = (2) ⇒ LHS = RHS. Hence it is verified.

EXERCISE 1.2

1. Let A = {1,2,3,7} and B = {3,0,–1,7}, which of the following are relation from A to B ?

(i) $\mathbb{R}_1 = \{(2,1), (7,1)\}$

(ii) $\mathbb{R}_2 = \{(-1,1)\}$

(iii) $\mathbb{R}_3 = \{(2,-1), (7,7), (1,3)\}$ [FRT - 2022]

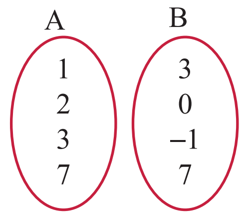
(iv) $\mathbb{R}_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$

Sol. Given A = {1, 2, 3, 7} and B = {3, 0, –1, 7}

(i) $R_1 = \{(2, 1), (7, 1)\}$

2 and 7 cannot be related to 1 since $1 \notin B$

∴ R_1 is not a relation.

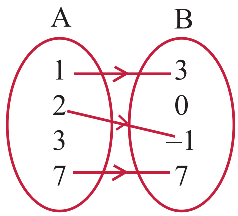


(ii) $R_2 = \{(-1, 1)\}$

–1 cannot be related to 1 since $-1 \notin A$ and $1 \notin B$

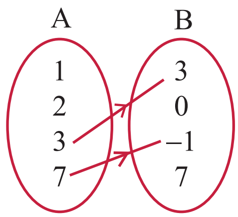
∴ R_2 is not a relation.

(iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$



R_3 is a relation since 2 is related to –1, 7 is related to 7 and 1 is related to 3.

(iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$



7 is related to –1

3 is related to 3

Since $0 \notin A$, 0 cannot be related to 3 and 7.

∴ R_4 is not a relation.

2. Let A={1, 2, 3, 4,...,45} and R be the relation defined as "square is of a number" on A. Write $\mathbb{R}$ as a subset of A × A. Also, find the domain and range of $\mathbb{R}$. [Sep. - 2021]

Sol. Given A = {1, 2, 3, 4, . . . 45}

∴ $A \times A = \{(1, 1) (1, 2) (1,3) ... (1, 45) (2, 1) (2, 2) ... (2, 45) (45, 1) (45, 2) (45, 3) ... (45, 45)\}$ ... (1)

R is defined as "is square of "

∴ R = {(1,1) (2,4) (3,9) (4,16) (5, 25) (6,36)}... (2)

[$\because$ 1 is the square of 1, 2 is the square of 4 and so on]

From (1) and (2), R is the subset of A × A

$\therefore R \subset A \times A$

Domain of R = {1, 2, 3, 4, 5, 6}

[All the first elements of the order pair in (2)]

Range of R = {1, 4, 9, 16, 25, 36}

[All the second elements of the order pair in (2)]

3. A Relation $\mathbb{R}$ is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range. **[PTA - 5; June - 2023]**

Sol. Given $\mathbb{R} = \{(x, y) / y = x + 3\}$ and $x \in \{0, 1, 2, 3, 4, 5\}$

When $x = 0$, $y = 0 + 3 = 3$ [$\because y = x + 3$]

When $x = 1$, $y = 1 + 3 = 4$

When $x = 2$, $y = 2 + 3 = 5$

When $x = 3$, $y = 3 + 3 = 6$

When $x = 4$, $y = 4 + 3 = 7$

When $x = 5$, $y = 5 + 3 = 8$

$\therefore \mathbb{R} = \{(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)\}$

$\therefore$ Domain of $\mathbb{R}$ = {0, 1, 2, 3, 4, 5}

[All the first element in $\mathbb{R}$]

Range of $\mathbb{R}$ = {3, 4, 5, 6, 7, 8}

[All the second element in $\mathbb{R}$]

4. Represent each of the given relation by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$

(ii) $\{(x, y) | y = x + 3, x, y$ are natural numbers $< 10\}$ **[Aug. - 2022]**

Sol. (i) $R = \{(x, y) | x = 2y, x \in \{2, 3, 4, 5\}$ and $y \in \{1, 2, 3, 4\}\}$

When $x = 2$, $y = \frac{x}{2} = \frac{2}{2} = 1$

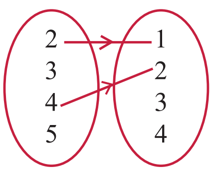
[$\because x = 2y \Rightarrow y = \frac{x}{2}$]

When $x = 3$, $y = \frac{3}{2}$

When $x = 4$, $y = \frac{4}{2} = 2$

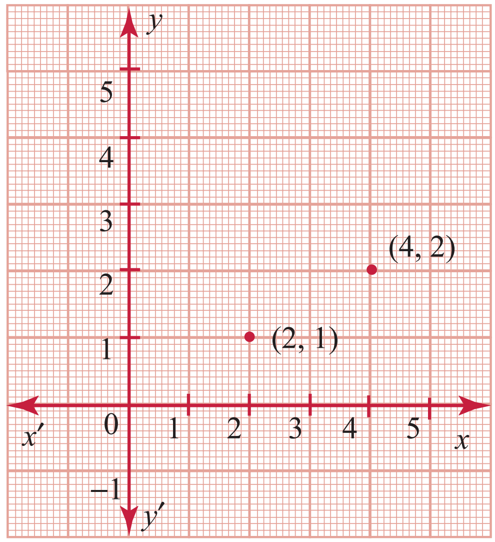
When $x = 5$, $y = \frac{5}{2}$

(a) an arrow diagram



3 cannot be related to $\frac{3}{2}$ and 5 cannot be related to $\frac{5}{2}$.

(b) a graph



(c) Roster form : R = {(2, 1), (4, 2)}

(ii) $R = \{(x, y) | y = x + 3$, x and y are natural numbers $< 10\}$

$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9$

$y = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

[$\because x$ and y are natural numbers less than 10]

Given $y = x + 3$

When $x = 1$, $y = 1 + 3 = 4$

When $x = 2$, $y = 2 + 3 = 5$

When $x = 3$, $y = 3 + 3 = 6$

When $x = 4$, $y = 4 + 3 = 7$

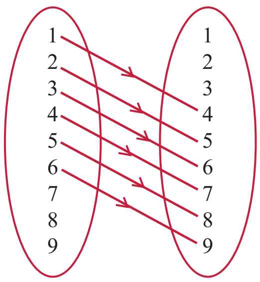
When $x = 5$, $y = 5 + 3 = 8$

When $x = 6$, $y = 6 + 3 = 9$

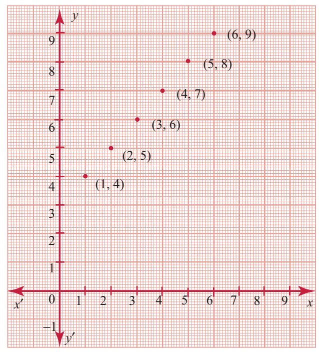
$\left.\begin{array}{ll} \text{When } x = 7, & y = 7 + 3 = 10 \\ \text{When } x = 8, & y = 8 + 3 = 11 \\ \text{When } x = 9, & y = 9 + 3 = 12 \end{array}\right\}$ [10, 11, 12 $\notin y$]

R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

(a) an arrow diagram



(b) a graph



(c) Roster form :

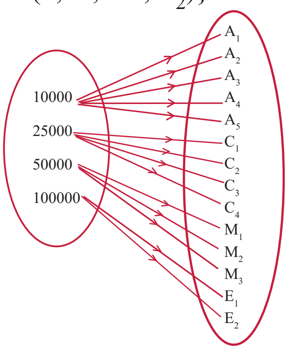
R = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)}

5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation $\mathbb{R}$ is defined by $x\mathbb{R}y$, where x is the salary given to person y, express the relation $\mathbb{R}$ through an ordered pair and an arrow diagram. [FRT - 2022]

Sol.
A – Assistants $\rightarrow$ A_1, A_2, A_3, A_4, A_5
C – Clerks $\rightarrow$ C_1, C_2, C_3, C_4
M – Managers $\rightarrow$ M_1, M_2, M_3
E – Executive officer $\rightarrow$ E_1, E_2

xRy is defined as x is the salary for assistants is ₹10,000, clerks is ₹25,000, Manger is ₹50,000 and for the executing officer ₹1,00,000.

(a) $\therefore$ R = {(10,000, A_1), (10,000, A_2), (10,000, A_3), (10,000, A_4), (10,000, A_5), (25,000, C_1), (25,000, C_2), (25,000, C_3), (25,000, C_4) (50,000, M_1), (50,000, M_2), (50,000, M_3), (1,00,000, E_1), (1,00,000, E_2)}

(b)


EXERCISE 1.3

1. Let $f = \{(x, y) | x, y \in \mathbb{N}$ and $y = 2x\}$ be a relation on $\mathbb{N}$. Find the domain, co-domain and range. Is this relation a function?

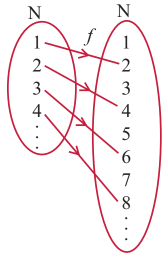
Sol. Given $f = \{(x, y) | x, y \in \mathbb{N}$ and $y = 2x\}$

When $x = 1$, $y = 2(1) = 2$
When $x = 2$, $y = 2(2) = 4$
When $x = 3$, $y = 2(3) = 6$
When $x = 4$, $y = 2(4) = 8$ and so on.

R = {(1, 2), (2, 4), (3, 6), (4, 8), (5, 10),...}

Domain of R = {1, 2, 3, 4,...},

Range of R = {2, 4, 6, 8,...}

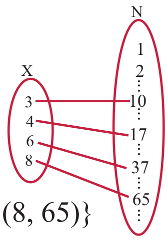


Since all the elements of domain are related to some elements of co-domain, this relation f is a function.

2. Let X = {3, 4, 6, 8}. Determine whether the relation $\mathbb{R} = \{(x, f(x)) | x \in X, f(x) = x^2 + 1\}$ is a function from X to $\mathbb{N}$?

Sol.
x = {3, 4, 6, 8}
R = $((x, f(x))| x \in X, f(x) = x^2 + 1\}$

$f(x) = x^2 + 1$
$f(3) = 3^2 + 1 = 10$
$f(4) = 4^2 + 1 = 17$
$f(6) = 6^2 + 1 = 37$
$f(8) = 8^2 + 1 = 65$



R = {(3, 10), (4, 17), (6, 37), (8, 65)}

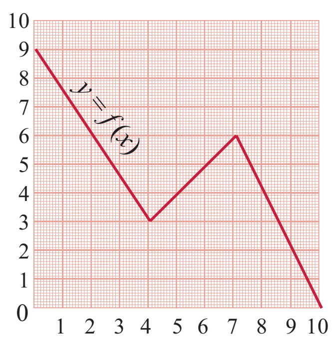
Yes, R is a function from X to $\mathbb{N}$.
Since all the elements of X are related to some elements of $\mathbb{N}$.

3. Given the function $f: x \rightarrow x^2 - 5x + 6$, evaluate
(i) $f(-1)$ (ii) $f(2a)$
(iii) $f(2)$ (iv) $f(x-1)$

Give the function $f: x \rightarrow x^2 - 5x + 6$.

(i) $f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$
(ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$
(iii) $f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$
(iv) $f(x - 1) = (x - 1)^2 - 5(x - 1) + 6$
$= x^2 - 2x + 1 - 5x + 5 + 6$
$= x^2 - 7x + 12$

4. A graph representing the function $f(x)$ is given in figure it is clear that $f(9) = 2$.



(i) Find the following values of the function
(a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$
(ii) For what value of x is $f(x) = 1$?

(iii) Describe the following
(i) Domain (ii) Range.
(iv) What is the image of 6 under f ?

Sol. (i) From the graph

(a) $f(0) = 9$ (c) $f(2) = 6$

(b) $f(7) = 6$ (d) $f(10) = 0$

(ii) At $x = 9.5, f(x) = 1$

(iii) Domain $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$= \{x \mid 0 \le x \le 10, x \in \mathbb{R}\}$
Range $= \{x \mid 0 \le x \le 9, x \in \mathbb{R}\}$
$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(iv) The image of 6 under f is 5. Since when you draw a line at $x = 6$, it meets the graph at 5.

5. Let $f(x) = 2x + 5$. If $x \neq 0$ then find $\dfrac{f(x+2) - f(2)}{x}$.

Sol. Given $f(x) = 2x + 5, x \neq 0$.

$$f(x) = 2x + 5$$
$$\Rightarrow f(x+2) = 2(x+2) + 5$$
$$= 2x + 4 + 5 = 2x + 9$$
$$\Rightarrow f(2) = 2(2) + 5 = 4 + 5 = 9$$
$$\therefore \frac{f(x+2) - f(2)}{x} = \frac{2x + 9 - 9}{x} = \frac{2x}{x} = 2$$

6. A function f is defined by $f(x) = 2x - 3$

(i) find $\dfrac{f(0) + f(1)}{2}$
(ii) find x such that $f(x) = 0$.
(iii) find x such that $f(x) = x$.
(iv) find x such that $f(x) = f(1 - x)$.

Sol. Given $f(x) = 2x - 3$

(i) $\dfrac{f(0) + f(1)}{2}$

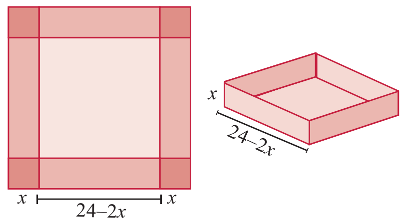
$$f(0) = 2(0) - 3 = -3$$
$$f(1) = 2(1) - 3 = -1$$
$$\therefore \frac{f(0) + f(1)}{2} = \frac{-3 - 1}{2} = \frac{-4}{2} = -2$$

(ii) $f(x) = 0 \Rightarrow 2x - 3 = 0$
$$2x = 3$$
$$x = \frac{3}{2}$$

(iii) $f(x) = x \Rightarrow 2x - 3 = x \Rightarrow 2x - x = 3$
$$x = 3$$

(iv) $f(x) = f(1 - x)$
$$2x - 3 = 2(1 - x) - 3$$
$$2x - 3 = 2 - 2x - 3$$
$$2x + 2x = 2 - \cancel{3} + \cancel{3}$$
$$4x = 2$$
$$x = \frac{\cancel{2}}{\cancel{4}\,2} \Rightarrow x = \frac{1}{2}$$

7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown in figure. Express the volume V of the box as a function of x.



Sol. Volume of the box = Volume of the cuboid
$= l \times b \times h$ cu. units

Here $l = 24 - 2x$
$b = 24 - 2x$
$h = x$
$\therefore V = (24 - 2x)(24 - 2x) \times x$
$= (576 - 48x - 48x + 4x^2)x$
$V = 4x^3 - 96x^2 + 576x$

8. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$. **[Qy.& Hy - 2023]**

Sol. Given $f(x) = 3 - 2x$

Also, it is given that $f(x^2) = [f(x)]^2$

$f(x^2) = 3 - 2x^2$ [Replacing x by x^2] ... (1)

$[f(x)]^2 = (3 - 2x)^2 = 9 - 12x + 4x^2$ ... (2)

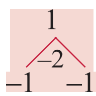
$[\because (a - b)^2 = a^2 - 2ab + b^2]$

From (1) and (2),

$\Rightarrow 9 - 12x + 4x^2 = 3 - 2x^2$

$\Rightarrow 9 - 12x + 4x^2 - 3 + 2x^2 = 0$

$\Rightarrow 6x^2 - 12x + 6 = 0$



Dividing by 6, we get $x^2 - 2x + 1 = 0$

On factorizing we get, $(x - 1)(x - 1) = 0$

$\Rightarrow x = 1$

9. A plane is flying at a speed of 500 km per hour. Express the distance d travelled by the plane as function of time t in hours.

Sol. Speed $= \dfrac{\text{distance covered}}{\text{time taken}}$

$\Rightarrow$ distance = Speed × time

$\Rightarrow d = 500 \times t$ [$\because$ time $= t$ hrs]

$\Rightarrow d = 500\,t$

10. The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length(x) as $y = ax + b$, where a, b are constants. **[PTA - 4]**

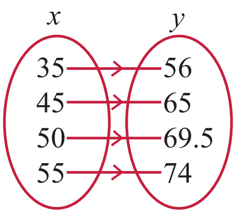
Length 'x' of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

(i) Check if this relation is a function.
(ii) Find a and b.
(iii) Find the height of a person whose forehand length is 40 cm.
(iv) Find the length of forehand of a person if the height is 53.3 inches.

Sol. Given relation is $y = ax + b$ ...(1)

(i) The given ordered pairs are

R = { (35, 56) (45, 65) (50, 69.5) (55, 74)}



Since all the elements of x are related to some elements of y, the given relation is a function.

(ii) Consider any two ordered pairs (35, 56) and (45, 65)

Substitute $\underset{x\ \ y}{(35, 56)}$ in $y = ax + b$ we get,

$$56 = a(35) + b \quad \text{... (1)}$$

Similarly substitute (45, 65) in $y = ax + b$, we get

$$65 = a(45) + b \quad \text{...(2)}$$

$$(2) \rightarrow \quad 65 = 45a + \cancel{b} \quad \text{....(2)}$$

$$(1) \rightarrow \quad \underline{56 = 35a + \cancel{b}} \quad \text{...(3)}$$

Substituting, $9 = 10a$

$$\Rightarrow \quad a = \frac{9}{10} = 0.9$$

Substituting $a = 0.9$ in (1) we get

$$56 = 35(0.9) + b$$

$$\Rightarrow \quad 56 = 31.5 + b$$

$$\Rightarrow \quad b = 56 - 31.5 = 24.5$$

$$\text{Since } y = ax + b$$

$$\text{We get } y = 0.9x + 24.5$$

(iii) When the length of the forehand $x = 40$ cm,

$$y = 0.9(40) + 24.5$$

$$\Rightarrow \quad y = 36 + 24.5 = 60.5 \text{ inches}$$

$\therefore$ The required height of the person is 60.5 inches.

(iv) When the height of the person $y = 53.3$ inches,

$$53.3 = 0.9x + 24.5 \quad [\because y = 0.9x + 24.5]$$

$$\Rightarrow \quad 53.3 - 24.5 = 0.9x$$

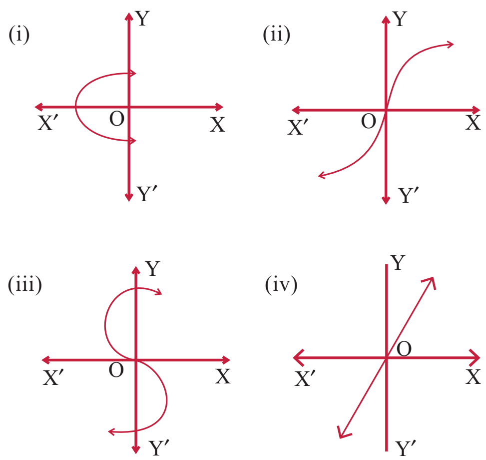
$$\Rightarrow \quad 28.8 = 0.9x$$

$$\Rightarrow \quad x = \frac{28.8 \times 10}{0.9 \times 10}$$

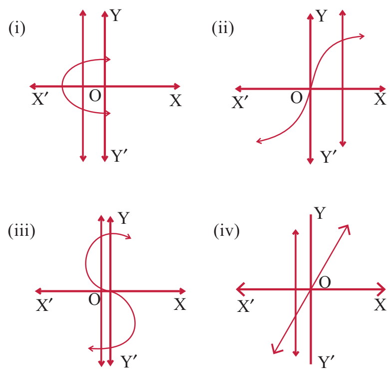
$$\Rightarrow \quad x = \frac{288}{9} = 32 \text{ cm}$$

EXERCISE 1.4

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.



Sol.



(i) It is not a function. The graph meets the vertical line at more than one points.

(ii) It is a function as the curve meets the vertical line at only one point.

(iii) It is not a function as it meets the vertical line at more than one points.

(iv) It is a function as it meets the vertical line at only one point.

2. Let f : A → B be a function defined by $f(x) = \frac{x}{2} - 1$, where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}. Represent f by [Govt. MQP - 2019; April - 2023]

(i) set of ordered pairs;
(ii) a table;
(iii) an arrow diagram;
(iv) a graph

Sol. $f : A \to B$

A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}

$$f(x) = \frac{x}{2} - 1, \qquad f(2) = \frac{2}{2} - 1 = 0$$

$$f(4) = \frac{4}{2} - 1 = 1 \qquad f(6) = \frac{6}{2} - 1 = 2$$

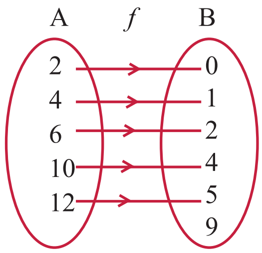
$$f(10) = \frac{10}{2} - 1 = 4 \qquad f(12) = \frac{12}{2} - 1 = 5$$

(i) Set of ordered pairs
={(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)}

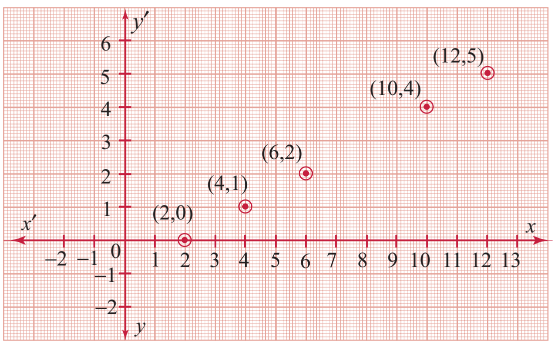
(ii) a table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iii) an arrow diagram;



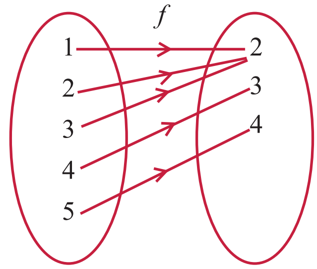
(iv) a graph



3. Represent the function f ={(1, 2),(2, 2),(3, 2), (4, 3), (5, 4)} through
(i) an arrow diagram
(ii) a table form
(iii) a graph

Sol. f = {(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)}

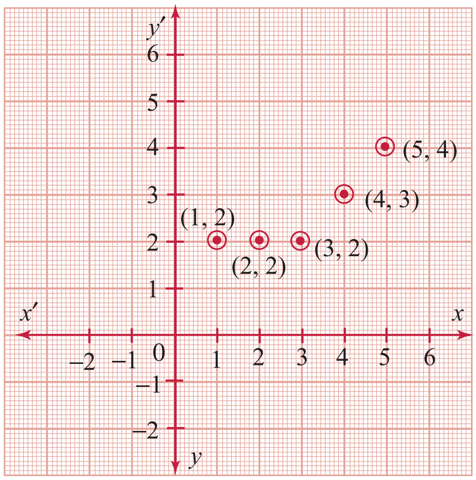
(i) An arrow diagram.



(ii) a table form

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

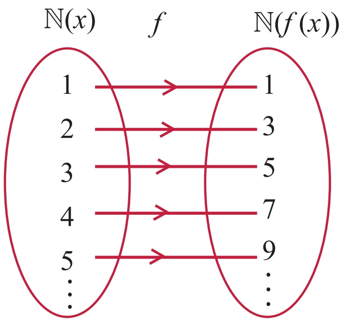
(iii) A graph representation.



4. Show that the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x - 1$ is one - one but not onto.

Sol. $f : \mathbb{N} \to \mathbb{N}$

$$f(x) = 2x - 1$$
$$\mathbb{N} = \{1, 2, 3, 4, 5, ...\}$$
$$f(1) = 2(1) - 1 = 1$$
$$f(2) = 2(2) - 1 = 3$$
$$f(3) = 2(3) - 1 = 5$$
$$f(4) = 2(4) - 1 = 7$$
$$f(5) = 2(5) - 1 = 9$$



In the figure, for different elements in x, there are different images in $f(x)$.

Hence $f : \mathbb{N} \to \mathbb{N}$ is a one-one function.

A function $f : \mathbb{N} \to \mathbb{N}$ is said to be onto function if the range of f is equal to the co-domain of f.

Range = {1, 3, 5, 7, 9, ...}

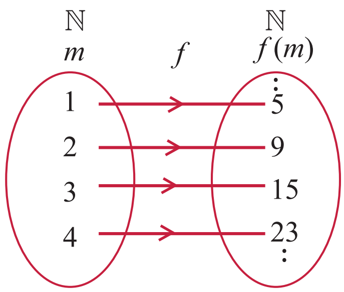
Co-domain = {1, 2,3,..}

But here the range is not equal to co-domain. Therefore it is one-one but not onto function.

5. Show that the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one - one function.

Sol. $f : \mathbb{N} \to \mathbb{N}$

$$\begin{aligned} f(m) &= m^2 + m + 3 \\ \mathbb{N} &= \{1, 2, 3, 4, 5, \ldots\}, m \in \mathbb{N} \\ f(m) &= m^2 + m + 3 \\ f(1) &= 1^2 + 1 + 3 = 5 \\ f(2) &= 2^2 + 2 + 3 = 9 \\ f(3) &= 3^2 + 3 + 3 = 15 \\ f(4) &= 4^2 + 4 + 3 = 23 \end{aligned}$$



In the figure, for different elements in the domain, there are different images in codomain. Hence $f : \mathbb{N} \to \mathbb{N}$ is a one to one but not onto function as the range of f is not equal to co-domain.

Co-domain = $\mathbb{N}$

Range = {5, 9, 15, 23, ...}

Hence it is proved.

6. Let A ={1, 2, 3, 4} and B = $\mathbb{N}$. Let $f : A \to B$ be defined by $f(x) = x^3$ then, [Hy. - 2019 & 2023]

(i) find the range of f

(ii) identify the type of function

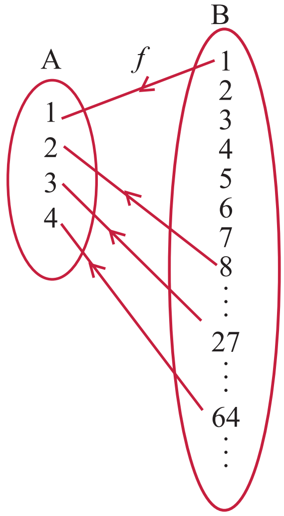
Sol.
$$\begin{aligned} A &= \{1, 2, 3, 4\} \\ B &= \mathbb{N} \end{aligned}$$

$f : A \to B, f(x) = x^3$

$$\begin{aligned} f(1) &= 1^3 = 1 \\ f(2) &= 2^3 = 8 \\ f(3) &= 3^3 = 27 \\ f(4) &= 4^3 = 64 \end{aligned}$$

(i) The range of f = {1, 8, 27, 64,....}

(ii)



Here co-domain = B = {1, 2, 3 ...}

Range = {1, 8, 27, 64}

Different elements have different images and co-domain ≠ Range.

∴ The given function is one - one and function.

7. In each of the following cases state whether the function is bijective or not. Justify your answer.

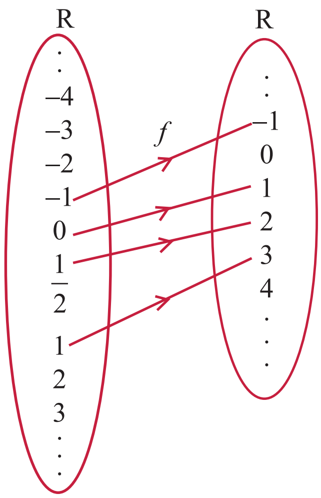
(i) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 1$

(ii) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3 - 4x^2$

Sol. Given $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 2x + 1$

(i) When $x = 1$,

$$\begin{aligned} f(1) &= 2(1) + 1 = 3 \\ f(2) &= 2(2) + 1 = 5 \\ f(0) &= 2(0) + 1 = 1 \\ f(-1) &= 2(-1) + 1 = -2 + 1 = -1 \\ f\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2 \text{ and so on} \end{aligned}$$

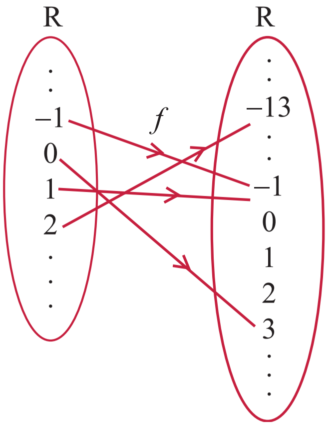


Here, different element in domain have different images in B and Co-domain = Range = R.

∴ f is a bijective function.

(ii) Given $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3 - 4x^2$

$$\begin{aligned} f(1) &= 3-4(1^2) = 3-4\,(1) \\ &= 3-4 = -1 \\ f(2) &= 3-4(2^2) = 3-4\,(4) \\ &= 3-16 = -13 \\ f(0) &= 3-4(0)^2 = 3-0 = 3 \\ f(-1) &= 3-4(-1)^2 = 3-4(1) \\ &= 3-4 = -1 \end{aligned}$$



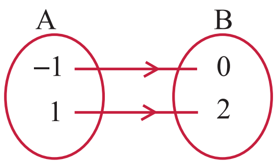
Here, different element in domain do not have different images in B. Since 1 and −1 are related to −1.

$\therefore f$ is not one - one.

Hence, f is not a bijective function.

8. Let A = {−1, 1}and B ={0, 2}. If the function f :A → B defined by $f(x) = ax + b$ is an onto function? Find a and b.

Sol. Given A = {−1, 1}, B = {0, 2} and $f : A \to B$ is defined by $f(x) = ax + b$ is an onto function.



$$\begin{aligned} f(-1) &= 0 \\ \Rightarrow \quad a(-1)+b &= 0 \quad [\because \text{Sub } x=-1, y=0 \text{ in } y = ax+b] \\ \Rightarrow \quad -a+b &= 0 \quad \ldots (1) \\ \text{Also } f(1) &= 2 \\ \Rightarrow \quad a(1)+b &= 2 \quad [\because \text{Sub } x=1, y=2 \text{ in } y = ax+b] \\ \Rightarrow \quad \cancel{a}+b &= 2 \quad \ldots (2) \\ (1) \Rightarrow \quad -\cancel{a}+b &= 0 \\ \text{Adding,} \quad 2b &= 2 \\ \Rightarrow \quad b &= \frac{2}{2} = 1 \end{aligned}$$

Substituting $b = 1$ in (2) we get

$$a+1 = 2 \quad \Rightarrow a = 2-1 = 1$$

$$\therefore a = 1, b = 1$$

9. If the function f is defined by

$$f(x) = \begin{cases} x+2; & x>1 \\ 2; & -1 \le x \le 1 \\ x-1; & -3 < x < -1 \end{cases}$$ **find the values of**

(i) $f(3)$ (ii) $f(0)$

(iii) $f(-1.5)$ (iv) $f(2) + f(-2)$

Sol. (i) $f(3) \Rightarrow f(x) = x + 2 \Rightarrow 3 + 2 = 5$ $[\because x = 3]$

(ii) $f(0) \Rightarrow 2$ $[\because 0 \in -1 \le x \le 1]$

(iii) $f(-1.5) = x - 1 = -1.5 - 1 = -2.5$

(iv) $f(2) + f(-2)$

$$\begin{aligned} f(2) &= 2+2 = 4 \quad [\because f(x) = x+2] \\ f(-2) &= -2-1 = -3 \quad [\because f(x) = x-1] \\ f(2)+f(-2) &= 4-3 = 1 \end{aligned}$$

10. A function $f : [-5, 9] \to \mathbb{R}$ is defined as follows:

$$f(x) = \begin{cases} 6x+1; & -5 \le x < 2 \\ 5x^2-1; & 2 \le x < 6 \\ 3x-4; & 6 \le x \le 9 \end{cases}$$

Find (i) $f(-3) + f(2)$ (ii) $f(7) - f(1)$ [PTA - 4]

(iii) $2f(4) + f(8)$ (iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$ [PTA- 4]

Sol. $f : [-5, 9] \to \mathbb{R}$

(i) $f(-3) + f(2)$

$$\begin{aligned} f(-3) &= 6x+1 = 6(-3)+1 = -17 \\ f(2) &= 5x^2-1 = 5(2^2)-1 = 19 \\ \therefore f(-3)+f(2) &= -17+19 = 2 \end{aligned}$$

(ii) $f(7) - f(1)$

$$\begin{aligned} f(7) &= 3x-4 = 3(7)-4 = 17 \\ f(1) &= 6x+1 = 6(1)+1 = 7 \\ f(7)-f(1) &= 17-7 = 10 \end{aligned}$$

(iii) $2f(4) + f(8)$

$$\begin{aligned} f(4) &= 5x^2-1 = 5 \times 4^2 - 1 = 79 \\ f(8) &= 3x-4 = 3 \times 8 - 4 = 20 \\ \therefore 2f(4)+f(8) &= 2 \times 79 + 20 = 178 \end{aligned}$$

(iv) $\dfrac{2f(-2) - f(6)}{f(4) + f(-2)}$

$$\begin{aligned} f(-2) &= 6x+1 = 6(-2)+1 = -11 \\ f(6) &= 3x-4 = 3(6)-4 = 14 \\ f(4) &= 5x^2-1 = 5(4^2)-1 = 79 \\ f(-2) &= 6x+1 = 6(-2)+1 = -11 \end{aligned}$$

$$\frac{2f(-2)-f(6)}{f(4)+f(-2)} = \frac{2(-11)-14}{79+(-11)} = \frac{-22-14}{68} = \frac{-36}{68} = \frac{-9}{17}$$

11. The distance S an object travels under the influence of gravity in time t seconds is given by $S(t) = \frac{1}{2}gt^2 + at + b$ where, (g is the acceleration due to gravity), a, b are constants. Verify whether the function S (t) is one-one or not. [PTA - 3]

Sol. $S(t) = \frac{1}{2}gt^2 + at + b$

Let t be 1, 2, 3, . . ., seconds.

$$S(1) = \frac{1}{2}g(1^2) + a(1) + b = \frac{1}{2}g + a + b$$

$$S(2) = \frac{1}{2}g(2^2) + a(2) + b$$

$$= 2g + 2a + b$$

Yes, for every different values of t, there will be different values as images. And there will be different pre-images for the different values of the range. Therefore it is one-one function.

12. The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by $t(C) = F$ where $F = \frac{9}{5}C + 32$. Find,

(i) $t(0)$ [PTA - 1]

(ii) $t(28)$

(iii) $t(-10)$

(iv) the value of C when $t(C) = 212$ [PTA - 1]

(v) the temperature when the Celsius value is equal to the Fahrenheit value. [PTA - 1]

Sol. (i) $t(0) = F$

$$F = \frac{9}{5}(C) + 32 = \frac{9}{5}(0) + 32 = 32°F$$

(ii) $$t(28) = F = \frac{9}{5}(28) + 32 = \frac{252}{5} + 32$$

$$= 50.4 + 32 = 82.4°F$$

(iii) $$t(-10) = F = \frac{9}{5}(-10) + 32 = 14°F$$

(iv) $t(C) = 212$

$$\text{i.e} \frac{9}{5}(C) + 32 = 212 \Rightarrow \frac{9}{5}C = 212 - 32 = 180$$

$$\frac{9}{5}C = 180 \Rightarrow C = \frac{\overset{20}{\cancel{180}} \times 5}{\cancel{9}} = 100°C$$

$$C = 100°C.$$

(v) when $C = F$

$$\frac{9}{5}C + 32 = C$$

$$32 = C - \frac{9}{5}C$$

$$32 = C\left(1 - \frac{9}{5}\right)$$

$$32 = C\left(\frac{5-9}{5}\right)$$

$$32 = C\left(\frac{-4}{5}\right)$$

$$C = \overset{8}{\cancel{32}} \times \frac{-5}{\cancel{4}}$$

$$C = -40°$$

EXERCISE 1.5

1. Using the functions f and g given below, find fog and gof. Check whether $fog = gof$.

(i) $f(x) = x - 6, g(x) = x^2$ [June - 2023]

(ii) $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$

(iii) $f(x) = \frac{x+6}{3}, g(x) = 3 - x$

(iv) $f(x) = 3 + x, g(x) = x - 4$ [Govt. MQP - 2019]

(v) $f(x) = 4x^2 - 1, g(x) = 1 + x$

Sol. (i) Given $f(x) = x - 6, g(x) = x^2$

$$fog(x) = f(g(x)) = f(x^2) \quad [\because g(x) = x^2]$$

$$= x^2 - 6$$

[In $f(x) = x - 6$, Replace x by x^2] ...(1)

$$gof(x) = g(f(x)) = g(x-6)$$

$[\because f(x) = x - 6]$

$$= (x-6)^2$$

[In $g(x) = x^2$, Replace x by $x - 6$]

$$= x^2 - 12x + 36$$

$[\because (a-b)^2 = a^2 - 2ab + b^2]$ ... (2)

From (1) and (2),

$$fog(x) \neq gof(x)$$

(ii) Given $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$

$$fog(x) = f(g(x)) = f(2x^2 - 1)$$

$[\because g(x) = 2x^2 - 1]$

$$= \frac{2}{2x^2 - 1}$$

[In $f(x) = \frac{2}{x}$. Replace x by $2x^2 - 1$] ...(1)

$$gof(x) = g(f(x)) = g\left(\frac{2}{x}\right) \quad [\because f(x) = \frac{2}{x}]$$

$$= 2\left(\frac{2}{x}\right)^2 - 1$$

[In $g(x) = 2x^2 - 1$, Replace x by $\frac{2}{x}$]

$$= 2\left(\frac{4}{x^2}\right) - 1 = \frac{8}{x^2} - 1 \quad ...(2)$$

From (1) and (2),

$$fog(x) \neq gof(x)$$

(iii) $x^2 + x + 7 = 0$

Let $y = x^2 + x + 7$

Step 1:

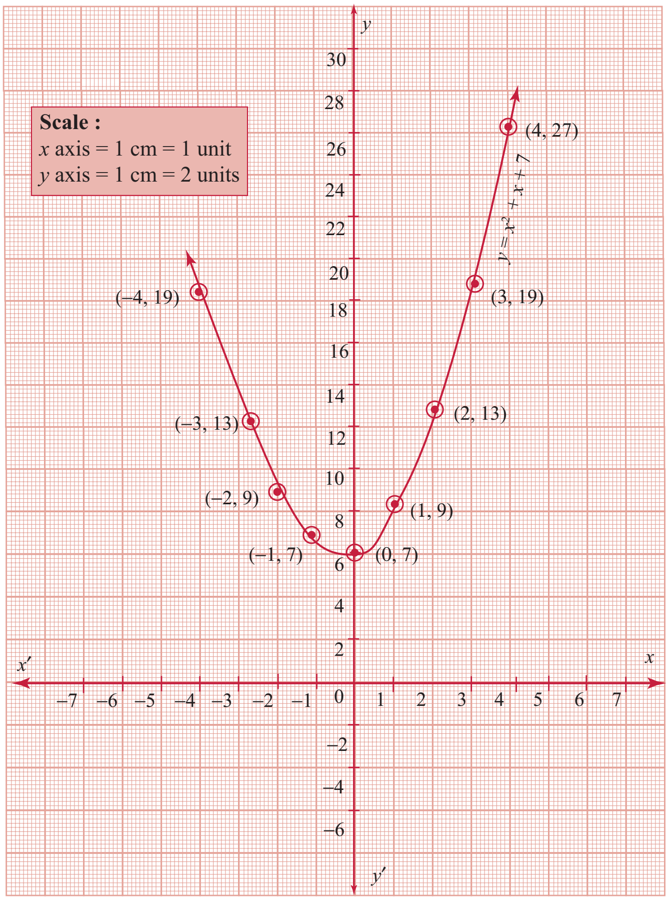
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
7	7	7	7	7	7	7	7	7	7
$y = x^2 + x + 7$	19	13	9	7	7	9	13	19	27

Step 2:

Points to be plotted: (–4, 19), (–3, 13), (–2, 9), (–1, 7), (0, 7), (1, 9), (2, 13), (3, 19), (4, 27)

Step 3:

Draw the parabola and mark the co-ordinates of the parabola which intersect with the x-axis.



Step 4:

The roots of the equation are the points of intersection of the parabola with the x axis. Here the parabola does not intersect the x axis at any point.

So, we conclude that there is no real roots for the given quadratic equation.

(iv) $x^2 - 9 = 0$

Let $y = x^2 - 9$

Step 1:

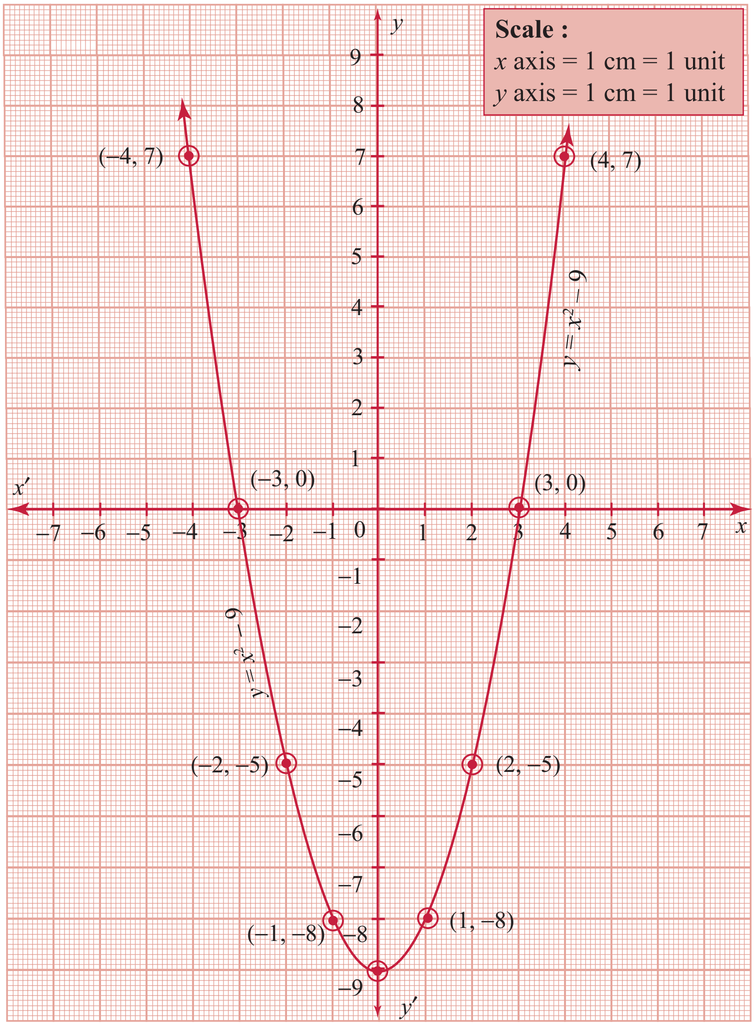
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
–9	–9	–9	–9	–9	–9	–9	–9	–9	–9
$y = x^2 - 9$	7	0	–5	–8	–9	–8	–5	0	7

Step 2:

The points to be plotted: (–4, 7), (–3, 0), (–2, –5), (–1, –8), (0, –9), (1, –8), (2, –5), (3, 0), (4, 7)

Step 3:

Draw the parabola and mark the co-ordinates of the parabola which intersect the x-axis.



Step 4:

The roots of the equation are the co-ordinates of the intersecting points (–3, 0) and (3, 0) of the parabola with the x-axis which are –3 and 3 respectively.

Step 5:

Since there are two points of intersection with the x axis, the quadratic equation has real and unequal roots.

$\therefore$ Solution $\{-3, 3\}$

(v) $x^2 - 6x + 9 = 0$

Let $y = x^2 - 6x + 9$

Step 1:

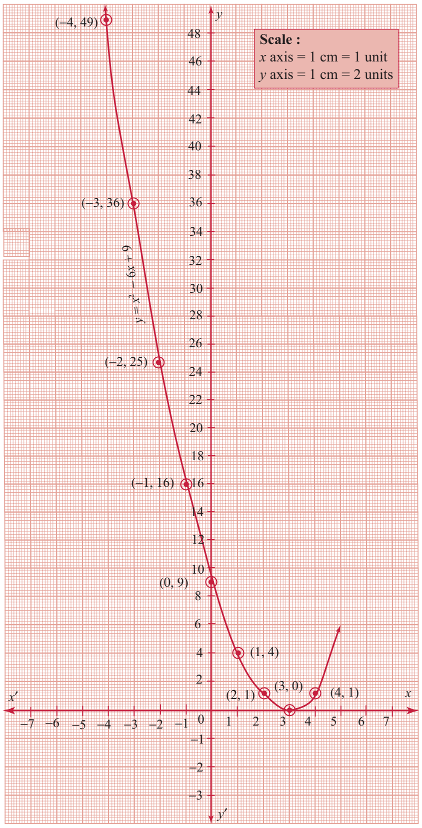
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-6x$	24	18	12	6	0	–6	–12	–18	–24
9	9	9	9	9	9	9	9	9	9
$y = x^2 - 6x + 9$	49	36	25	16	9	4	1	0	1

Step 2:

Points to be plotted: (–4, 49), (–3, 36), (–2, 25), (–1, 16), (0, 9), (1, 4), (2, 1), (3, 0), (4, 1)

Step 3:

Draw the parabola and mark the co-ordinates of the intersecting points.



Step 4:

The point of intersection of the parabola with x axis is (3, 0)

Since there is only one point of intersection with the x-axis, the quadratic equation has real and equal roots.

∴ Solution (3, 3)

(vi) $(2x - 3)(x + 2) = 0$

$2x^2 - 3x + 4x - 6 = 0$; $2x^2 + 1x - 6 = 0$

Let $y = 2x^2 + x - 6= 0$

Step 1:

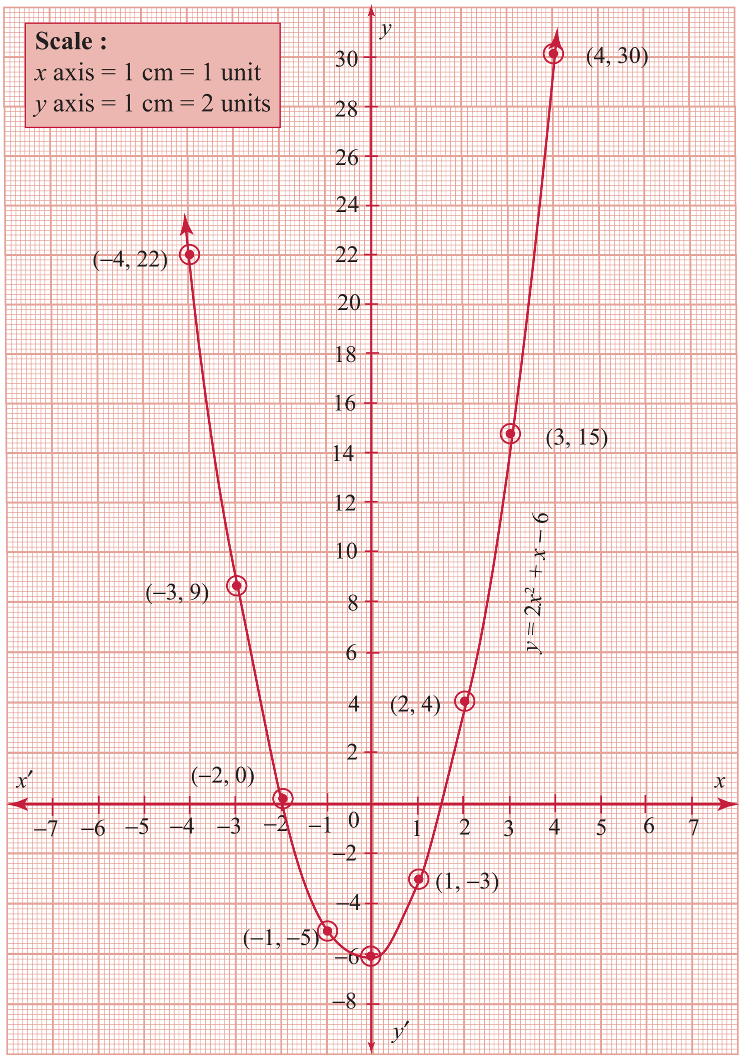
x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
x	–4	–3	–2	–1	0	1	2	3	4
–6	–6	–6	–6	–6	–6	–6	–6	–6	–6
$y = 2x^2 + x - 6$	22	9	0	–5	–6	–3	4	15	30

Step 2:

The points to be plotted: (–4, 22), (–3, 9), (–2, 0), (–1, –5), (0, –6), (1, –3), (2, 4), (3, 15), (4, 30)

Step 3:

Draw the parabola and mark the co-ordinates of the intersecting point of the parabola with the x-axis.



Step 4:

The points of intersection of the parabola with the x-axis are (–2, 0) and (1.5, 0).

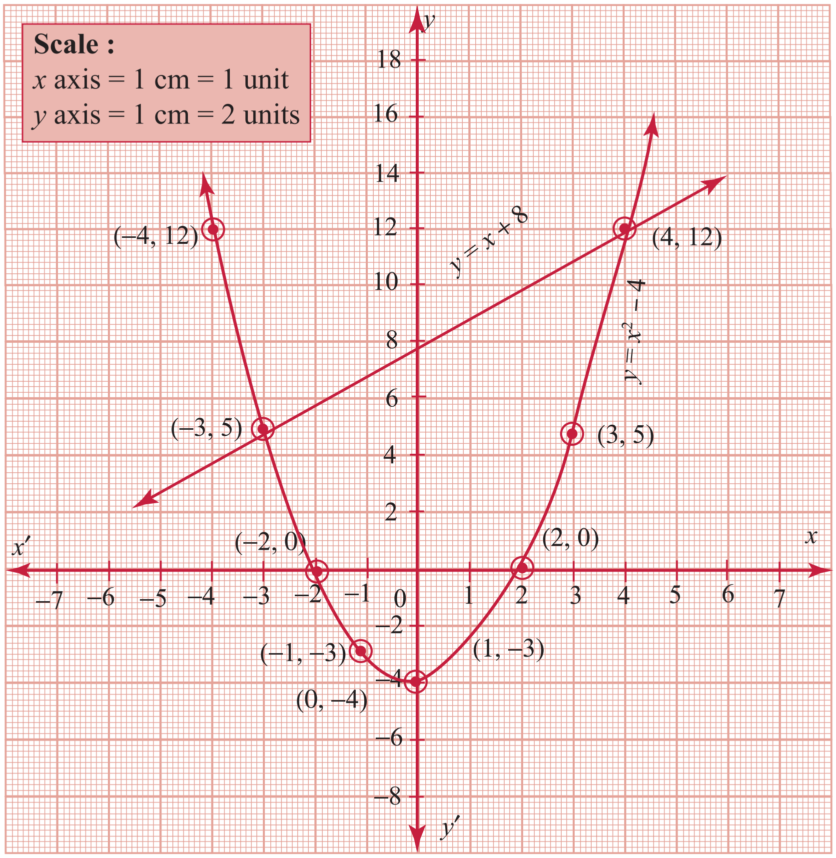
Since the parabola intersects the x-axis at two points, the equation has real and unequal roots.

$\therefore$ Solution {–2, 1.5}

2. Draw the graph of $y = x^2 - 4$ and hence solve $x^2 - x - 12 = 0$. **[June - 2023]**

Sol.

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
– 4	–4	–4	–4	–4	–4	–4	–4	–4	–4
$x^2 - 4$	12	5	0	–3	–4	–3	0	5	12



To solve $x^2 - x - 12 = 0$

$$\cancel{x^2} + 0x - 4 = y$$
$$\cancel{x^2} - x - 12 = 0$$
$$(-) \quad (+) \quad (+) \qquad (-)$$
$$x + 8 = y$$
$$y = x + 8$$

x	–4	–3	–2	–1	0	1	2	3	4
8	8	8	8	8	8	8	8	8	8
$x - 8$	4	5	6	7	8	9	10	11	12

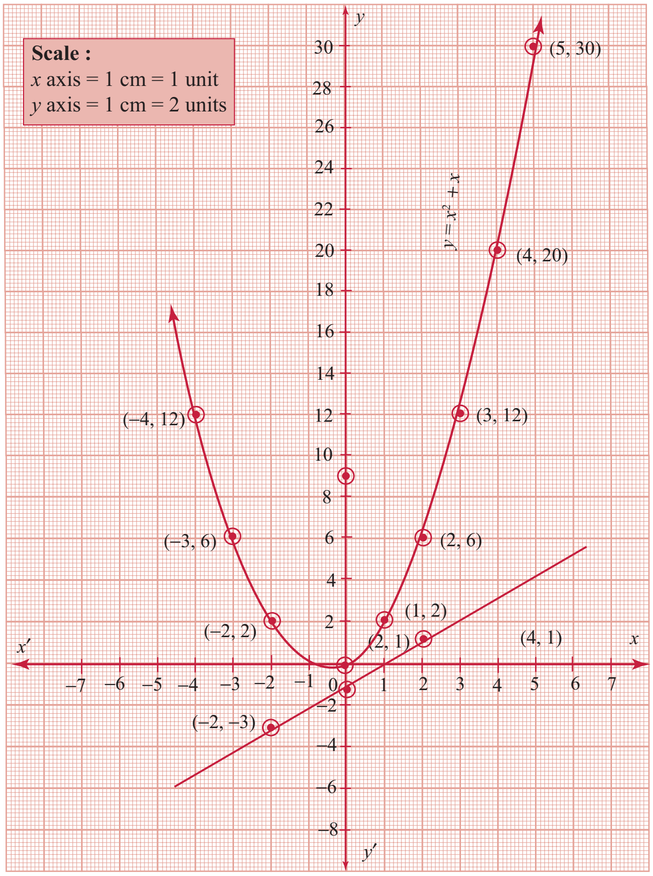
Point of intersection (–3, 5), (4, 12) solution of $x^2 - x - 12 = 0$ is –3, 4

3. Draw the graph of $y = x^2 + x$ and hence solve $x^2 + 1 = 0$.

Sol.

x	–4	–3	–2	–1	0	1	2	3	4	5
x^2	16	9	4	1	0	1	4	9	16	25
$+x$	–4	–3	–2	–1	0	1	2	3	4	5
$y = x^2 + x$	12	6	2	0	0	2	6	12	20	30

Draw the parabola by the plotting the points (–4, 12), (–3, 6), (–2, 2), (–1, 0), (0, 0), (1, 2), (2, 6), (3, 12), (4, 20), (5, 30)



To solve: $x^2 + 1 = 0$, subtract $x^2 + 1 = 0$ from $y = x^2 + x$.

$x^2 + 1 = 0$ from $y = x^2 + x$

i.e.
$$\begin{array}{rcl} y & = & \cancel{x^2} + x \\ 0 & = & \cancel{x^2} + 1 \\ (-) & (-) & (-) \\ \hline y & = & x - 1 \end{array}$$

This is a straight line.
Draw the line $y = x - 1$.

x	–2	0	2
–1	–1	–1	–1
y	–3	–1	1

Plotting the points (–2, –3), (0, –1), (2, 1) we get a straight line. This line does not intersect the parabola. Therefore there is no real roots for the equation $x^2 + 1 = 0$.

(ii) Any line parallel to y axis will be of the form $x = c$.

$\Rightarrow \quad x = \frac{5}{2}$

$\Rightarrow \quad 2x = 5$

$\Rightarrow \quad 2x - 5 = 0$

2. The equation of a straight line is $2(x - y) + 5 = 0$. Find its slope, inclination and intercept on the Y axis.

Sol. $2(x - y) + 5 = 0$

$\Rightarrow \quad 2x - 2y + 5 = 0$

$\Rightarrow \quad 2y = 2x + 5$

$\Rightarrow \quad y = x + \frac{5}{2}$ ... (1) $\quad \therefore$ slope = 1

Inclination, $m = 1$

[Equating (1) with $y = mx + c$]

$\Rightarrow \quad \tan\theta = 1 = \tan 45°$

$\theta = 45°$

$\Rightarrow y$ intercept $\Rightarrow y = \frac{5}{2} \quad [\because C = \frac{5}{2}]$

3. Find the equation of a line whose inclination is 30° and making an intercept – 3 on the Y axis.

Sol. $\theta = 30°$

$\Rightarrow \quad c = -3$

y intercept is $\Rightarrow c = -3$ (i.e) let the equation of line be : $y = mx + c$

$m = \tan\theta$

$\Rightarrow \quad m = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore y = \frac{1}{\sqrt{3}} \cdot x + c$

When $x = 0, y = -3$

$\Rightarrow \quad -3 = 0 + c \Rightarrow c = -3$

$\therefore$ Equation of line: $\boxed{y = \frac{x}{\sqrt{3}} - 3}$

$\Rightarrow \quad x - \sqrt{3}y - 3\sqrt{3} = 0 \quad$ [Multiplied by $\sqrt{3}$]

4. Find the slope and y intercept of $\sqrt{3}x + (1 - \sqrt{3})y = 3$.

Sol. Given equation is $x\sqrt{3} + y(1 - \sqrt{3}) = 3$

$\Rightarrow \quad y(1 - \sqrt{3}) = -x\sqrt{3} + 3$

Dividing by $(1 - \sqrt{3})$ we get,

$\Rightarrow \quad y = \frac{-x\sqrt{3}}{1 - \sqrt{3}} + \frac{3}{1 - \sqrt{3}}$

$= x\left(\frac{-\sqrt{3}}{1 - \sqrt{3}}\right) + \frac{3}{1 - \sqrt{3}}$

$\Rightarrow \quad y = x\left(\frac{\sqrt{3}}{\sqrt{3} - 1}\right) + \frac{3}{1 - \sqrt{3}}$ ... (1)

Comparing (1) with $y = mx + c$ we get,

$m = \frac{\sqrt{3}}{\sqrt{3} - 1}$ and $c = \frac{3}{1 - \sqrt{3}}$

$\Rightarrow \quad$ Slope $= m = \frac{\sqrt{3}}{\sqrt{3} - 1} = \frac{\sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$

[Rationalising the denominator]

$\Rightarrow \quad m = \frac{\sqrt{3}(\sqrt{3} + 1)}{(\sqrt{3} - 1)(\sqrt{3} + 1)}$

$\Rightarrow \quad m = \frac{3 + \sqrt{3}}{3 - 1}$

$[\because (\sqrt{3} - 1)(\sqrt{3} + 1) = (\sqrt{3})^2 - (1)^2 = 3 - 1]$

$\Rightarrow m = \frac{3 + \sqrt{3}}{2}$ and $c = \frac{3}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$

[Rationalising the denominator]

$\Rightarrow \quad c = \frac{3(1 + \sqrt{3})}{1 - 3}$

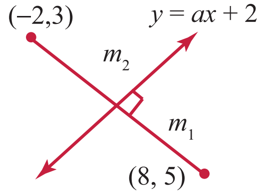
$[\because (1 - \sqrt{3})(1 + \sqrt{3}) = 1^2 - (\sqrt{3})^2 = 1 - 3 = -2]$

$\Rightarrow \quad c = \frac{3 + 3\sqrt{3}}{-2}$

Hence slope is $\frac{3 + \sqrt{3}}{2}$ and y intercept is $\left(\frac{3 + 3\sqrt{3}}{-2}\right)$.

5. Find the value of 'a', if the line through (–2, 3) and (8, 5) is perpendicular to $y = ax + 2$.

Sol.



Let m_1 be slope of line joining (–2, 3) and (8, 5).

let m_2 be slope of $y = ax + 2 \Rightarrow ax - y + 2 = 0$.

$m_1 = \frac{5 - 3}{8 - (-2)} = \frac{\cancel{2}}{\cancel{10}_{5}} = \frac{1}{5}$

$\left[\because \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}\right]$

$m_2 = a \quad \left[\because \text{Slope} = -\frac{\text{co-efficient } x}{\text{co-efficient } y}\right]$

$\because$ The lines are perpendicular $\Rightarrow m_1 \times m_2 = -1$

$\Rightarrow \frac{1}{5} \times a = -1 \Rightarrow \boxed{a = -5}$

6. The hill in the form of a right triangle has its foot at (19, 3)The inclination of the hill to the ground is 45°. Find the equation of the hill joining the foot and top. **[PTA - 2]**

Sol. Let AB be the hill and B (19, 3) be the foot of the hill

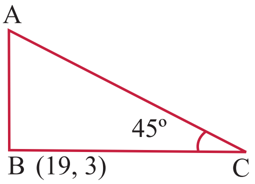
Given $\angle ACB = 45°$

Slope $= m = \tan\theta = \tan 45° = 1$ and (x_1, y_1) is (19, 3)

Equation of AB is $y - y_1 = m(x - x_1)$

[Slope - point form]

$$\Rightarrow \quad y - 3 = 1(x - 19)$$
$$\Rightarrow \quad y - 3 = x - 19$$
$$\Rightarrow \quad x - 19 - y + 3 = 0$$
$$\Rightarrow \quad x - y - 16 = 0$$



Which is the equation of the hill joining the foot and top.

7. Find the equation of a line through the given pair of points

(i) $\left(2, \frac{2}{3}\right)$ **and** $\left(\frac{-1}{2}, -2\right)$

(ii) (2, 3) and (–7, –1)

Sol. (i) Equation of the line in two point form is

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y - \frac{2}{3}}{-2 - \frac{2}{3}} = \frac{x - 2}{-\frac{1}{2} - 2} \quad [\because (x_1, y_1) \text{ is } (2, \tfrac{2}{3})$$
$$(x_2, y_2) \text{ is } -\tfrac{1}{2}, -2)]$$

$$\Rightarrow \frac{\frac{3y - 2}{\cancel{3}}}{\frac{-6 - 2}{\cancel{3}}} = \frac{x - 2}{\frac{-1 - 4}{2}} \Rightarrow \frac{3y - 2}{-8} = \frac{2(x - 2)}{-5}$$

$$\Rightarrow -15y + 10 = -16x + 32$$

[Cross multiplying]

$$\Rightarrow 16x - 15y + 10 - 32 = 0$$
$$\Rightarrow 16x - 15y - 22 = 0$$

(ii) (x_1, y_1) is (2, 3) (x_2, y_2) is (–7, – 1)

$$\therefore \text{ Equation is } \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

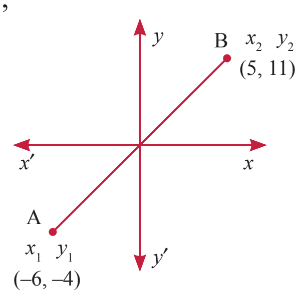
$$\Rightarrow \frac{y - 3}{-1 - 3} = \frac{x - 2}{-7 - 2} \Rightarrow \frac{y - 3}{-4} = \frac{x - 2}{-9}$$

$$-9(y - 3) = -4(x - 2)$$
$$-9y + 27 = -4x + 8$$
$$4x - 9y + 27 - 8 = 0 \qquad \therefore 4x - 9y + 19 = 0$$

8. A cat is located at the point(–6, –4) in *xy* plane. A bottle of milk is kept at (5, 11). The cat wish to consume the milk traveling through shortest possible distance. Find the equation of the path it needs to take its milk. **[SRT - 2022; April - 2023]**

Sol. Let A be the position of the cat and B be the position of the bottle of milk.

Shortest path between A and B is a line joining A and B,



$$\therefore \text{ Equation of AB is } \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y - (-4)}{11 - (-4)} = \frac{x - (-6)}{5 - (-6)}$$
$$\Rightarrow \frac{y + 4}{11 + 4} = \frac{x + 6}{5 + 6}$$
$$\Rightarrow \frac{y + 4}{15} = \frac{x + 6}{11}$$
$$\Rightarrow 11y + 44 = 15x + 90$$

(by cross multiplication)

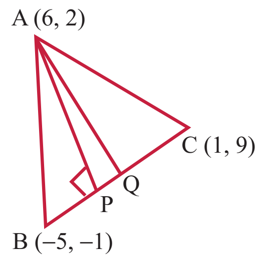
$$\Rightarrow 15x + 90 - 11y - 44 = 0$$
$$\Rightarrow 15x - 11y + 46 = 0$$

9. If the vertices of a ΔABC are A(6, 2) B(–5,–1) and C(1,9). **[PTA - 6; Sep. - 2021]**

i) find the equation of median

ii) find the equation of altitude

Sol.



Let AP be the altitude

$\Rightarrow AP \perp BC$

$$\text{Slope of BC} = \frac{9-(-1)}{1-(-5)} \quad \left[\because \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}\right]$$
$$= \frac{10}{6} = \frac{5}{3}$$

$$\text{Slope of AP} = \frac{-1}{\text{Slope of (BC)}} = \frac{-3}{5}$$

[∵ They are perpendicular]

Equation of line AP: $y - 2 = \frac{-3}{5}(x-6)$

$[\because y - y_1 = M(x - x_1)]$

$\Rightarrow 5y - 10 = -3x + 18$

$\Rightarrow 3x + 5y = 28$

$\Rightarrow 3x + 5y - 28 = 0$

Let AQ be the median ⇒ Q is mid point of BC

$$\Rightarrow Q = \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right) = (-2, 4)$$
$$\left[Q \text{ is } \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$$

$$\text{Slope of AQ} = \frac{4-2}{-2-6} = \frac{2}{-8} = \frac{-1}{4}$$

$\therefore$ Equation of line AQ: $y - 2 = \frac{-1}{4} \times (x-6)$

$$\Rightarrow y - 2 = \frac{-1}{4}(x-6)$$
$$4y - 8 = -x + 6$$
$$\Rightarrow x + 4y = 14$$
$$\Rightarrow x + 4y - 14 = 0$$

10. Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point (–1,2). **[May - 2022]**

Sol. $m = \frac{-5}{4}$, point $= (-1, 2) = (x_1, y_1)$

$$\Rightarrow y - 2 = \frac{-5}{4}(x-(-1))$$
$[\because y - y_1 = m(x - x_1)]$
$$\Rightarrow y - 2 = \frac{-5}{4}(x+1)$$
$$\Rightarrow 4(y-2) = -5(x+1)$$
$$\Rightarrow 4y - 8 = -5x - 5$$
$$\Rightarrow 5x + 4y = 3 \Rightarrow 5x + 4y - 3 = 0$$

11. You are downloading a song. The percent y (in decimal form) of mega bytes remaining to get downloaded in x seconds is given by $y = -0.1x + 1$.

(i) find the total MB of the song.

(ii) after how many seconds will 75% of the song gets downloaded?

(iii) after how many seconds the song will be downloaded completely?

Sol. (i) Given equation is $y = -0.1x + 1$

Total MB of song can be obtained when time = 0

$\therefore x = 0$

$\Rightarrow y = 1$ MB

(ii) time when 75% of song is downloaded

$\Rightarrow$ remaining % = 25% $\Rightarrow y = 0.25$

$0.25 = -0.1x + 1$

$\Rightarrow 0.1x = 0.75$

$\Rightarrow x = 7.5$ ⇒ 7.5 Seconds

(iii) song will downloaded completely when remaining % = 0 ⇒ $y = 0$

$\Rightarrow 0 = -0.1x + 1$

$\Rightarrow x = 10$

$\therefore$ 10 seconds

12. Find the equation of a line whose intercepts on the x and y axes are given below.

(i) 4, –6 (ii) –5, $\frac{3}{4}$

Sol. (i) Equation of the line in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$
$$\Rightarrow \frac{x}{4} + \frac{y}{-6} = 1 \quad [\because a = 4, b = -6]$$
$$\Rightarrow \frac{3x - 2y}{12} = 1 \Rightarrow 3x - 2y = 12$$
$$\Rightarrow 3x - 2y - 12 = 0$$

(ii) $\frac{x}{-5} + \frac{y}{\frac{3}{4}} = 1$

Which is the equation of the line in intercept form

$$\Rightarrow \frac{-x}{5} + \frac{4y}{3} = 1$$
$$\Rightarrow \frac{-3x + 20y}{15} = 1$$
$$\Rightarrow -3x + 20y = 15$$
$$3x - 20y + 15 = 0$$

13. Find the intercepts made by the following lines on the coordinate axes.

(i) $3x - 2y - 6 = 0$ **[Sep. - 2021]**

(ii) $4x + 3y + 12 = 0$

Sol. (i)
$$\begin{aligned} 3x-2y-6 &= 0 \\ \text{To find } x \text{ intercept put } y &= 0 \\ \Rightarrow \quad 3x-6 &= 0 \\ \Rightarrow \quad x &= 2 \\ \text{To find } y \text{ intercept put } x &= 0 \\ \Rightarrow \quad 0-2y-6 &= 0 \\ \Rightarrow \quad y &= -3 \end{aligned}$$

Hence the intercepts are 2, –3

(ii)
$$\begin{aligned} 4x+3y+12 &= 0 \\ \text{To find } x \text{ intercept put } y &= 0 \\ \Rightarrow \quad 4x+0+12 &= 0 \\ \Rightarrow \quad x &= -3 \\ \text{To find } y \text{ intercept put } x &= 0 \\ \Rightarrow \quad 0+3y+12 &= 0 \\ \Rightarrow \quad y &= -4 \end{aligned}$$

Hence the intercepts are –3, –4.

14. Find the equation of a straight line

(i) passing through (1,–4) and has intercepts which are in the ratio 2:5

(ii) passing through (–8, 4) and making equal intercepts on the coordinate axes

Sol. (i)
$$\begin{aligned} \text{ratio of intercept} &= 2:5 \\ \text{Slope} &= \frac{y\text{ - intercept}}{x\text{ - intercept}} \\ \therefore \text{ Slope of line} &= \frac{-5}{2} \Rightarrow m = \frac{-5}{2} \end{aligned}$$

$\therefore$ The required equation is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ \Rightarrow \quad y-(-4) &= \frac{-5}{2}(x-1) \\ \Rightarrow \quad 2(y+4) &= -5(x-1) \\ \Rightarrow \quad 2y+8 &= -5x+5 \\ \Rightarrow \quad 5x+2y+3 &= 0 \end{aligned}$$

(ii)
$$\begin{aligned} \text{Slope of line} &= \frac{y \text{ intercept}}{x \text{ intercept}} \times -1 \\ \Rightarrow \quad m &= \frac{a \times -1}{a} = -1 \end{aligned}$$

$\therefore$ The required equation is

$$\begin{aligned} y-y_1 &= m(x-x_1) \\ \Rightarrow \quad y-4 &= -1(x-(-8)) \\ y-4 &= -x-8 \\ \Rightarrow \quad x+y+4 &= 0 \end{aligned}$$

EXERCISE 5.4

1. Find the slope of the following straight lines

(i) $5y - 3 = 0$ **(ii) $7x - \frac{3}{17} = 0$**

Sol. (i)
$$\begin{aligned} 5y-3 &= 0 \\ 5y &= 3 \\ y &= \frac{3}{5} \quad \text{... (1)} \\ \Rightarrow \quad y &= mx+c \quad \text{... (2)} \end{aligned}$$

Comparing (1) and (2)

$$y = 0x + \frac{3}{5} \qquad \therefore \text{ Slope } m = 0$$

(ii) $7x - \frac{3}{17} = 0 \Rightarrow 7x + 0y - \frac{3}{17} = 0$

$$\begin{aligned} \text{Slope} &= \frac{-\text{Co-efficient of } x}{\text{Co-efficient of } y} \\ m &= \frac{-7}{0} = \infty \text{ (undefined)} \end{aligned}$$

2. Find the slope of the line which is

(i) parallel to $y = 0.7x - 11$

(ii) perpendicular to the line $x = -11$

Sol. (i) Given line is $y = 0.7x - 11$

line parallel to $y = 0.7x - 11$ is $m = 0.7$

$y = 0.7x + C$

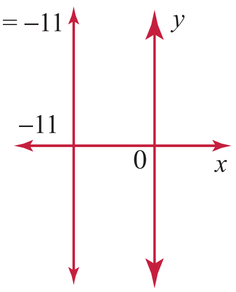
If the lines are parallel, slopes are equal

$\therefore$ The slope of the required line is $m = 0.7$.

(ii) Given line is $x = -11$

$$\begin{aligned} m &= \frac{\text{Co-efficient of } x}{\text{Co-efficient of } y} \\ \Rightarrow \quad m &= \frac{-1}{0} = \infty \end{aligned}$$

$\therefore$ Slope is ∞



3. Check whether the given lines are parallel or perpendicular

(i) $\frac{x}{3} + \frac{y}{4} + \frac{1}{7} = 0$ and $\frac{2x}{3} + \frac{y}{2} + \frac{1}{10} = 0$

(ii) $5x + 23y + 14 = 0$ and $23x - 5y + 9 = 0$

Sol. (i) Given line is $\frac{x}{3}+\frac{y}{4}+\frac{1}{7}=0$ ... (1)

and $\frac{2x}{3}+\frac{y}{2}+\frac{1}{10} = 0$ ... (2)

Let m_1 and m_2 be the slopes of the given lines

$$\therefore m_1 = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } y}$$

$$= \frac{-\frac{1}{3}}{\frac{1}{4}} \quad -\frac{1}{3}\times\frac{4}{1} = -\frac{4}{3} \qquad \text{[From (1)]}$$

$$m_2 = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } y}$$

$$= \frac{\frac{-2}{3}}{\frac{1}{2}} = \frac{-2}{3}\times\frac{2}{1} = -\frac{4}{3} \qquad \text{[From (2)]}$$

Since $m_1 = m_2$, the given lines are parallel.

(ii) The given lines are

$5x + 23y + 14 = 0$ ... (1)

and $23x - 5y + 9 = 0$

Let m_1 and m_2 be the slopes of lines (1) and (2)

$$\therefore m_1 = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } y} = \frac{-5}{23} \text{ and}$$

$$m_2 = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } y} = \frac{-23}{-5} = \frac{23}{5}$$

Consider $m_1 \times m_2 = \left(\frac{-\cancel{5}}{\cancel{23}}\right)\times\left(\frac{\cancel{23}}{\cancel{5}}\right) = -1$

Since $m_1 m_2 = -1$, the given lines are perpendicular

4. If the straight lines $12y = -(p + 3)x + 12$, $12x - 7y = 16$ are perpendicular then find 'p'. **[April - 2023]**

Sol. Given lines are $12y = -(p + 3)x + 12$ ... (1)

and $12x - 7y = 16$ ... (2)

Let m_1 and m_2 be the slopes of the lines (1) and (2)

$(1) \Rightarrow 12y + (p + 3)x = 12$

$$\therefore m_1 = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } y}$$

$$= -\frac{(p+3)}{12} \text{ and}$$

$$m_2 = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } y}$$

$$= \frac{-12}{-7} = \frac{12}{7}$$

Given that the lines (1) and (2) are perpendicular

$\Rightarrow \quad m_1 m_2 = -1$

$\Rightarrow \quad \cancel{-}\left(\frac{p+3}{\cancel{12}}\right)\times\left(\frac{\cancel{12}}{7}\right) = \cancel{-}1 \qquad \Rightarrow \frac{p+3}{7} = 1$

$\Rightarrow \quad p + 3 = 7$

$\Rightarrow \quad p = 7 - 3 = 4$

$\therefore p = 4$

5. Find the equation of a straight line passing through the point P (–5, 2) and parallel to the line joining the points Q(3,–2) and R(–5,4).

Sol. Slope of QR $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{-5-3}$

$$= \frac{4+2}{-8} = \frac{6}{-8} \Rightarrow m = \frac{-3}{4}$$

P is (–5, 2)

Required equation is

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-3}{4}(x + 5)$

$4y - 8 = -3x - 15$

$3x + 4y + 7 = 0$

6. Find the equation of a line passing through (6,–2) and perpendicular to the line joining the points (6,7) and (2,–3).

Sol. Slope of line joining (6, 7) and (2, –3) is

$$= \frac{-3-7}{2-6} = \frac{-10}{-4} = \frac{5}{2} \qquad \left[\because \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}\right]$$

Slope of the perpendicular line $= \frac{-2}{5}$

$$\left[m_1 m_2 = -1 \Rightarrow m_2 = \frac{-1}{m_1}\right]$$

Required equation is

$y + 2 = \frac{-2}{5}(x - 6)$

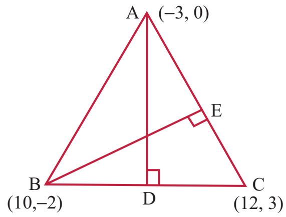
$5y + 10 = -2x + 12$

$2x + 5y - 2 = 0$

7. A(–3, 0) B(10, – 2) and C(12,3) are the vertices of ΔABC . Find the equation of the altitude through A and B. **[Hy. - 2019]**

Sol. A(–3, 0), B(10, –2), C(12, 3)

Since AD ⊥ BC



$$\text{Slope AD} = \frac{-1}{\text{Slope of BC}}$$

$$\therefore \text{Slope of BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-2)}{12-10} = \frac{5}{2}$$

$$\therefore \text{Slope of AD} = \frac{-1}{5/2} = \frac{-2}{5}$$

$\therefore$ Equation of AD $\Rightarrow y - y_1 = m(x - x_1)$

$$y - 0 = \frac{-2}{5}(x-(-3))$$

$$5y = -2x - 6$$

$$2x + 5y + 6 = 0 \quad \text{...(1)}$$

Since BE $\perp$ AC

$$\text{Slope of BE} = \frac{-1}{\text{Slope of AC}}$$

$$\text{Slope of AC} = \frac{3-0}{12-(-3)} = \frac{\cancel{3}}{\cancel{15}_{5}} = \frac{1}{5}$$

B(10, –2), slope of BE $= \dfrac{-1}{1/5} = -5$

$\therefore$ Equation of BE $\Rightarrow y-(-2) = -5\,(x-10)$

$$y + 2 = -5x + 50$$

$$5x + y + 2 - 50 = 0$$

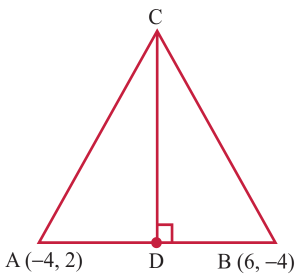
$$5x + y - 48 = 0 \quad \text{...(2)}$$

(1), (2) are the required equations of the altitudes through A and B.

8. Find the equation of the perpendicular bisector of the line joining the points A(–4, 2) and B(6, –4).

Sol. Mid Point AB is DC **[June & Qy.- 2023]**

$$\Rightarrow \text{D is}\left(\frac{-4+6}{2}, \frac{2+(-4)}{2}\right) = \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$$



$$\text{Slope of AB} = \frac{-4-2}{6-(-4)} = \frac{-6}{10} = \frac{-3}{5}$$

$$\therefore \text{Slope of CD} = \frac{-1}{-3/5} = \frac{5}{3} \; [\because \text{CD} \perp \text{AB}]$$

$\therefore$ Equation of CD is

$$y-(-1) = \frac{5}{3}(x-1)$$

$$3(y+1) = 5x - 5 \Rightarrow 3y + 3 = 5x - 5$$

$5x - 3y - 8 = 0$ is the required equation of the line.

9. Find the equation of a straight line through the intersection of lines $7x + 3y = 10$, $5x - 4y = 1$ and parallel to the line $13x + 5y + 12 = 0$

Sol. The given lines are

$$7x + 3y = 10 \quad \text{... (1)}$$

and

$$5x - 4y = 1 \quad \text{... (2)}$$

Let us solve (1) and (2)

$$(1) \times 5 \Rightarrow 35x + 15y = 50$$

$$(2) \times 7 \Rightarrow \underset{(-)}{35x} \underset{(+)}{- 28y} = \underset{(-)}{7}$$

Subtracting, $43y = 43$

$$\Rightarrow y = \frac{43}{43}$$

$$\Rightarrow y = 1$$

Substitute $y = 1$ in (2) we get,

$$5x - 4(1) = 1$$

$$\Rightarrow 5x - 4 = 1$$

$$\Rightarrow 5x = 1 + 4$$

$$\Rightarrow 5x = 5$$

$$x = \frac{5}{5} = 1$$

$\therefore$ The intersecting point (x_1, y_1) is (1, 1).

The required line is parallel to the line

$$13x + 5y + 12 = 0$$

$\Rightarrow$ Slope of the required line = Slope of the given line

$$\Rightarrow m = -\frac{\text{Co-efficient of } x}{\text{Co-efficient of } y}$$

$$\Rightarrow m = \frac{-13}{5}$$

$\therefore$ Equation of the required line is $y - y_1 = m(x - x_1)$

$$\Rightarrow y - 1 = \frac{-13}{5}(x-1)$$

$$\Rightarrow \quad 5(y-1) = -13(x-1)$$
$$\Rightarrow \quad 15y-15 = -13x+13$$
$$\Rightarrow \quad 5y-5+13x-13 = 0$$
$$\Rightarrow \quad 13x+5y-18 = 0$$

10. Find the equation of a straight line through the intersection of lines $5x - 6y = 2$, $3x + 2y = 10$ and perpendicular to the line $4x - 7y + 13 = 0$

Sol. Given lines are

$$5x-6y = 2 \quad \dots(1)$$
and $$3x+2y = 10 \quad \dots(2)$$

Let us solve (1) and (2)

$$(1) \Rightarrow \quad 5x-6y = 2$$
$$(2)\times 3 \Rightarrow \quad 9x+6y = 30$$
Adding, $$14x = 32$$
$$\Rightarrow \quad x = \frac{32}{14} \Rightarrow x = \frac{16}{7}$$

Substituting $x = \frac{16}{7}$ in (1) we get,

$$\Rightarrow \quad 5\left(\frac{16}{7}\right) - 6y = 2 \Rightarrow 5\left(\frac{16}{7}\right) - 2 = 6y$$
$$\Rightarrow \quad 6y = \frac{80}{7} - 2$$
$$\Rightarrow \quad 6y = \frac{80-14}{7} \Rightarrow 6y = \frac{66}{7}$$
$$\Rightarrow \quad y = \frac{\overset{11}{\cancel{66}}}{\cancel{6}(7)} = \frac{11}{7}$$
$$\therefore y = \frac{11}{7}$$

Hence, the intersecting point (x_1, y_1) is $\left(\frac{16}{7}, \frac{11}{7}\right)$

Slope of the given line $4x - 7y + 13 = 0$ is

$$\frac{-4}{-7} = \frac{4}{7}$$

$\therefore$ Slope of the required line $m = \frac{-7}{4}$

(since they are perpendicular)

$\therefore$ Equation of the required line is $y - y_1 = m(x - x_1)$

$$\Rightarrow \quad y - \frac{11}{7} = \frac{-7}{4}\left(x - \frac{16}{7}\right)$$
$$\Rightarrow \quad \frac{7y-11}{\cancel{7}} = \frac{-7}{4}\left(\frac{7x-16}{\cancel{7}}\right)$$
$$\Rightarrow \quad 4(7y-11) = -7(7x-16)$$
$$\Rightarrow \quad 28y-44 = -49x+112$$
$$\Rightarrow \quad 28y-44+49x-112 = 0$$
$$\Rightarrow \quad 49x+28y-156 = 0$$

11. Find the equation of a straight line joining the point of intersection of $3x + y + 2 = 0$ and $x - 2y - 4 = 0$ to the point of intersection of $7x - 3y = -12$ and $2y = x + 3$. **[PTA - 3]**

Sol. Given lines are

$$3x+y+2 = 0 \quad \dots(1)$$
$$x-2y-4 = 0 \quad \dots(2)$$

Solving (1) and (2) we get the point of intersection of the lines (1) and (2).

$$(1) \Rightarrow \quad 3x+y = -2 \quad \dots(1)$$
$$(2)\times 3 \Rightarrow \quad 3x-6y = 12$$
$$(-) \quad (+) \quad (-)$$
$$7y = -14 \Rightarrow \boxed{y = -2}$$

Substitute $y = -2$ in (1), we get

$$3x + (-2) = -2$$
$$3x = -2+2 = 0$$
$$\boxed{x = 0}$$

The intersecting point is $(0, -2) = (x_1, y_1)$

Also, the lines are given as

$$7x-3y = -12 \quad \dots(3)$$
$$x-2y = -3 \quad \dots(4)$$
$$(4)\times 7 \Rightarrow \quad \cancel{7x} - 14y = -21$$
$$(3) \quad \Rightarrow \quad \cancel{7x} - 3y = -12$$
$$(-) \quad (+) \quad (+)$$
$$-11y = -9$$
$$y = \frac{9}{11}$$

Substitute $y = \frac{9}{11}$ in (4), we get

$$x - 2\times\frac{9}{11} = -3 \Rightarrow x - \frac{18}{11} = -3$$
$$\Rightarrow \quad x = -3 + \frac{18}{11} = \frac{-33+18}{11}$$
$$x = \frac{-15}{11}$$

$\therefore$ The intersecting point of (3) and (4) is $\left(\frac{-15}{11}, \frac{9}{11}\right) = (x_2, y_2)$

$\therefore$ The required line passes through $\overset{x_1 \quad y_1}{(0, -2)}$ and $\left(\overset{x_2}{\frac{-15}{11}}, \overset{y_2}{\frac{9}{11}}\right)$

$\therefore$ The required equation is $= \dfrac{y-(-2)}{\frac{9}{11}+2} = \dfrac{x-0}{\frac{-15}{11}-0}$

$$\left[\because \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}\right]$$

$$\Rightarrow \quad \frac{y+2}{\frac{9+22}{11}} = \frac{x}{\frac{-15}{11}} \Rightarrow \frac{y+2}{\frac{31}{\cancel{11}}} = \frac{x}{\frac{-15}{\cancel{11}}}$$

$$\Rightarrow \quad -15(y+2) = 31(x)$$

$$\Rightarrow \quad -15y - 30 = 31x$$

$$\Rightarrow \quad 31x + 15y + 30 = 0$$

$31x + 15y + 30 = 0$ is the required equation of the line.

12. Find the equation of a straight line through the point of intersection of the lines $8x + 3y = 18$, $4x + 5y = 9$ and bisecting the line segment joining the points (5, –4) and (–7,6). [Hy. - 2023]

Sol. Given lines are

$8x + 3y = 18$ ...(1)

$4x + 5y = 9$ ...(2)

solving (1) and (2)

$$(1) \Rightarrow \quad \cancel{8x} + 3y = 18$$
$$(2)\times 2 \Rightarrow \quad \cancel{8x} + 10y = 18$$
$$(-) \quad (-) \quad (+)$$
$$-7y = 0$$
$$y = 0$$

Substitute $y = 0$ in (1), we get

$$8x + 3(0) = 18$$

$$x = \frac{\overset{9}{\cancel{18}}}{\underset{4}{\cancel{4}}} = \frac{9}{4}$$

$\therefore$ The intersecting point is $\left(\frac{9}{4}, 0\right)$

The mid point of the line joining the two points (5, –4) and (–7, 6) is

[$\because$ the line is bisecting the points (5, –4) and (–7, 6)]

$$\left(\frac{5+(-7)}{2}, \frac{-4+6}{2}\right) = \left(\frac{-2}{2}, \frac{2}{2}\right) = (-1, 1)$$

The required line is passing through the points $\begin{pmatrix} \frac{9}{4}, & 0 \\ x_1 & y_1 \end{pmatrix}$ and $\begin{pmatrix} -1, & 1 \\ x_2 & y_2 \end{pmatrix}$

$\therefore$ The required equation is $\dfrac{y-0}{1-0} = \dfrac{x - \frac{9}{4}}{-1 - \frac{9}{4}}$

$$\left[\frac{y_1 - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}\right]$$

$$\Rightarrow \quad \frac{y}{1} = \frac{\frac{4x-9}{\cancel{4}}}{\frac{-4-9}{\cancel{4}}} \Rightarrow y = \frac{4x-9}{-13}$$

$$\Rightarrow \quad -13y = 4x - 9$$

$\therefore 4x + 13y - 9 = 0$ is the required equation.

EXERCISE 5.5

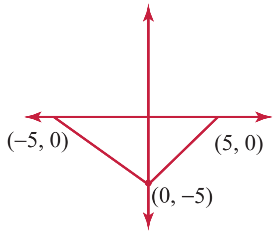
Multiple choice questions.

1. The area of triangle formed by the points (–5,0) , (0, –5) and (5,0) is

[PTA - 2; Qy. & Hy. - 2019; Sep. - 2021; SRT - 2022]

(A) 0 sq.units
(B) 25 sq.units
(C) 5 sq.units
(D) none of these **[Ans. (B) 25 sq. units]**

Hint:



$$\Delta = \frac{1}{2}\begin{vmatrix} -5 & 0 & 5 & -5 \\ 0 & -5 & 0 & 0 \end{vmatrix}$$

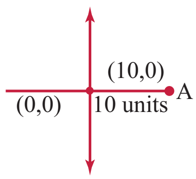
$$= \frac{1}{2}(25 - (-25)) = \frac{1}{2}(50) = 25$$

2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the Y axis. The path travelled by the man is

(A) $x = 10$ (B) $y = 10$
(C) $x = 0$ (D) $y = 0$

[Ans. (A) $x = 10$]

Hint:



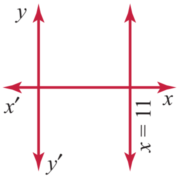
$$\text{Distance} = \sqrt{10^2 + 0^2}$$
$$= \sqrt{100} = 10$$
$$x = 10 \text{ units}$$

3. The straight line given by the equation $x = 11$ is [Sep. - 2020; PTA - 1; June & Qy. - 2023]

(A) parallel to X axis
(B) parallel to Y axis
(C) passing through the origin
(D) passing through the point (0,11)

[Ans. (B) Parallel to y axis]

Hint:



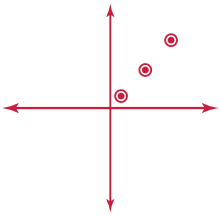
$x = 11$ is parallel to y axis.

4. If (5,7), (3, p) and (6,6) are collinear, then the value of p is [PTA - 5; May - 2022]

(A) 3 (B) 6 (C) 9 (D) 12

[Ans. (C) 9]

Hint: If (5, 7), (3, p) and (6, 6) are collinear $\Delta = 0$



$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} 5 & 3 & 6 & 5 \\ 7 & p & 6 & 7 \end{vmatrix} = 0$$

$$\begin{aligned} (5p + 18 + 42) - (21 + 6p + 30) &= 0 \\ 5p + 60 - (6p + 51) &= 0 \\ 5p - 6p &= -60 + 51 \\ -1p &= -9 \\ p &= +9 \end{aligned}$$

5. The point of intersection of $3x - y = 4$ and $x + y = 8$ is [PTA - 2; Aug. - 2022]

(A) (5,3) (B) (2,4) (C) (3,5) (D) (4,4)

[Ans. (C) (3, 5)]

Hint:

$$\begin{aligned} 3x - \cancel{y} &= 4 \\ x + \cancel{y} &= 8 \\ \hline 4x &= 12 \\ x &= 3 \\ 3 + y &= 8 \\ y &= 5 \end{aligned}$$

$\therefore$ Point of intersection is (3, 5)

6. The slope of the line joining (12, 3), (4, a) is $\frac{1}{8}$. The value of 'a' is [PTA - 3; SRT - 2022; FRT - 2024]

(A) 1 (B) 4 (C) –5 (D) 2

[Ans. (D) 2]

Hint:

$$\begin{aligned} m &= \frac{a-3}{4-12} = \frac{1}{8} \\ \frac{a-3}{-8} &= \frac{1}{8} \\ 8a - 24 &= -8 \\ 8a &= -8 + 24 = 16 \\ a &= 2 \end{aligned}$$

7. The slope of the line which is perpendicular to a line joining the points (0, 0) and (–8, 8) is [May - 2022]

(A) –1 (B) 1 (C) $\frac{1}{3}$ (D) –8

[Ans. (B) 1]

Hint: Slope of the line joining the points (0, 0) and (–8, 8) is

$$m = \frac{8-0}{-8-0} = \frac{8}{-8} = -1$$

Slope of the line $\perp^r$ to the given line is

$$m_2 = \frac{-1}{m} = \frac{-1}{-1} = 1$$

8. If slope of the line PQ is $\frac{1}{\sqrt{3}}$ then slope of the perpendicular bisector of PQ is [PTA - 6; SRT & Aug. - 2022; June - 2023]

(A) $\sqrt{3}$ (B) $-\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 0

[Ans. (B) $-\sqrt{3}$]

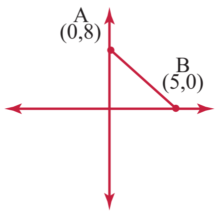
9. If A is a point on the Y axis whose ordinate is 8 and B is a point on the X axis whose abscissa is 5 then the equation of the line AB is

(A) $8x + 5y = 40$ (B) $8x - 5y = 40$

(C) $x = 8$ (D) $y = 5$

[Ans. (A) $8x + 5y = 40$]

Hint:



$$\begin{aligned} \frac{y-8}{0-8} &= \frac{x-0}{5-0} \\ \frac{y-8}{-8} &= \frac{x}{5} \\ -8x &= 5y - 40 \\ 8x + 5y - 40 &= 0 \end{aligned}$$

10. The equation of a line passing through the origin and perpendicular to the line $7x - 3y + 4 = 0$ is [PTA - 4]

(A) $7x - 3y + 4 = 0$ (B) $3x - 7y + 4 = 0$

(C) $3x + 7y = 0$ (D) $7x - 3y = 0$

[Ans. (C) $3x + 7y = 0$]

Hint: Slope of $7x - 3y + 4 = 0$ is $\frac{-7}{-3} = \frac{7}{3}$

Slope of the line that is $\perp^r$ to $7x - 3y + 4 = 0$ is

$$m = \frac{-1}{\frac{7}{3}} = \frac{-3}{7}$$

Passing through origin i.e. (0, 0)

Required equation is $y - 0 = \frac{-3}{7}(x - 0)$

$$3x + 7y = 0$$

11. Consider four straight lines
(i) l_1 ; $3y = 4x + 5$ (ii) l_2 ; $4y = 3x - 1$
(iii) l_3 ; $4y + 3x = 7$ (iv) l_4 ; $4x + 3y = 2$
Which of the following statement is true ?

(A) l_1 and l_2 are perpendicular
(B) l_1 and l_4 are parallel
(C) l_2 and l_4 are perpendicular
(D) l_2 and l_3 are parallel

[Ans.(C) l_2 and l_4 are perpendicular]

Hint: l_1: $4x - 3y + 5 = 0$ $\quad m_1 = \frac{\cancel{-}4}{\cancel{-}3}$

l_2: $3x - 4y - 1 = 0$ $\quad m_2 = \frac{\cancel{-}3}{\cancel{-}4}$

l_3: $3x + 4y - 7 = 0$ $\quad m_3 = \frac{-3}{4}$

l_4: $4x + 3y - 2 = 0$ $\quad m_4 = \frac{-4}{3}$

12. A straight line has equation $8y = 4x + 21$. Which of the following is true **[PTA - 3]**

(A) The slope is 0.5 and the y intercept is 2.6
(B) The slope is 5 and the y intercept is 1.6
(C) The slope is 0.5 and the y intercept is 1.6
(D) The slope is 5 and the y intercept is 2.6

[Ans.(A) The slope is 0.5 and the y intercept is 2.6.]

Hint:
$$8y = 4x + 21 \Rightarrow 4x - 8y + 21 = 0$$
$$m = \frac{-4}{-8} = \frac{1}{2} = 0$$
y intercept is $\frac{21}{8} = 2.6$

13. When proving that a quadrilateral is a trapezium, it is necessary to show **[PTA - 4]**

(A) Two sides are parallel.
(B) Two parallel and two non-parallel sides.
(C) Opposite sides are parallel.
(D) All sides are of equal length.

[Ans.(B) Two parallel and two non-parallel sides]

14. When proving that a quadrilateral is a parallelogram by using slopes you must find

(A) The slopes of two sides
(B) The slopes of two pair of opposite sides
(C) The lengths of all sides
(D) Both the lengths and slopes of two sides

[Ans.(B) The slopes of two pair of opposite sides]

15. (2, 1) is the point of intersection of two lines. **[PTA - 3; Hy. - 2019; Qy.- 2023]**

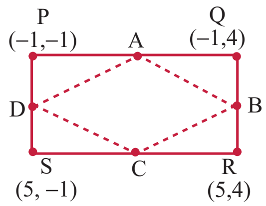
(A) $x - y - 3 = 0$; $3x - y - 7 = 0$
(B) $x + y = 3$; $3x + y = 7$
(C) $3x + y = 3$; $x + y = 7$
(D) $x + 3y - 3 = 0$; $x - y - 7 = 0$

[Ans.(B) $x + y = 3$; $3x + y = 7$]

Unit Exercise - 5

1. PQRS is a rectangle formed by joining the points P(–1, –1), Q(–1, 4) ,R(5, 4) and S(5,–1). A, B, C and D are the mid-points of PQ, QR, RS and SP respectively. Is the quadrilateral ABCD a square, a rectangle or a rhombus? Justify your answer.

Sol. Given A, B, C, D are mid points of PQ, QR, RS and SP respectively.



$\therefore$ A is the mid point of PQ

$$\Rightarrow \text{A is } \left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$$

B is the mid point of QR

$$\Rightarrow \quad \text{B is } = \left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = (2, 4)$$

C is the mid point of SR

$$\Rightarrow \quad \text{C is } = \left(\frac{5+5}{2}, \frac{4+(-1)}{2}\right) = \left(5, \frac{3}{2}\right)$$

D is the mid point of SP

$$\Rightarrow \quad \text{D is } = \left(\frac{5+(-1)}{2}, \frac{-1-1}{2}\right) = (2, -1)$$

$$AB = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2}$$
$$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2} = \sqrt{3^2 + \frac{25}{2}}$$
$$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$
$$BC = \sqrt{(5-2)^2 - \left(\frac{3}{2} - 4\right)^2}$$

$$= \sqrt{3^2 + \left(-\frac{5}{2}\right)^2} = \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{36+25}{4}} = \sqrt{\frac{61}{4}}$$

$$CD = \sqrt{(2-5)^2 + \left(-1-\frac{3}{2}\right)^2}$$

$$= \sqrt{(-3)^2 + \left(-\frac{5}{2}\right)^2} = \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{61}{4}}$$

$$DA = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2}$$

$$= \sqrt{3^2 + \left(-\frac{5}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

Since $AB = BC = CD = DA = \sqrt{\frac{61}{4}}$, ABCD may be a square or rhombus.

$$\text{Slope of AB} = \frac{4-\frac{3}{2}}{2-(-1)} = \frac{\frac{8-3}{2}}{3} = \frac{5}{2(3)} = \frac{5}{6}$$

$$\text{Slope of BC} = \frac{\frac{3}{2}-4}{5-2} = \frac{\frac{3-8}{2}}{3} = \frac{-5}{2(3)} = \frac{-5}{6}$$

$$\text{(Slope of AB) (Slope of BC)} = \left(\frac{5}{6}\right)\left(\frac{-5}{6}\right) = -\frac{25}{36} \neq 1$$

$\therefore$ The sides are not perpendicular.

$\therefore$ ABCD is not a square, but it is a rhombus.

2. The area of a triangle is 5 sq.units. Two of its vertices are (2,1) and (3, –2). The third vertex is (x, y) where $y = x + 3$. Find the coordinates of the third vertex. **[PTA - 1]**

Sol. Area of triangle formed by points (x_1, y_1), (x_2, y_2), and $(x_3, y_3) = \frac{1}{2}\{(x_1y_2 + x_2y_3 + x_3y_1) - (x_2\,y_1) + x_3\,y_2 + x_1\,y_3)\}$

$$\therefore \text{Area} = \frac{1}{2}\{(-4+3x+9+x)-(3-\cancel{2x}+\cancel{2x}+6)\}$$

$$\Rightarrow \quad \text{Area} = 5 \text{ (given)}$$

$$\Rightarrow \quad 5 = \frac{1}{2}\{(4x+5\} - (9)\}$$

$$\Rightarrow \quad 4x - 4 = 10$$

$$\boxed{\begin{array}{cccc} 2 & 3 & x & 2 \\ 1 & -2 & x+3 & 1 \end{array}}$$

$$\Rightarrow \quad 4x = 14$$

$$\Rightarrow \quad x = \frac{14}{4} = \frac{7}{2}$$

Substituting $x = \frac{7}{2}$ in $y = x + 3$ we get

$$\Rightarrow \quad y = \frac{7}{2} + 3 \Rightarrow y = \frac{13}{2}$$

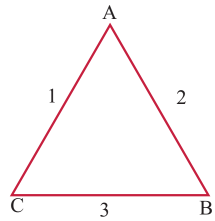
$$\Rightarrow \quad (x, y) = \left(\frac{7}{2}, \frac{13}{2}\right)$$

Hence the co-ordinates of third vertex is $\left(\frac{7}{2}, \frac{13}{2}\right)$

3. Find the area of a triangle formed by the lines $3x + y - 2 = 0$, $5x + 2y - 3 = 0$ and $2x - y - 3 = 0$

Sol. Given lines are

$$3x + y - 2 = 0 \quad \text{... (1)}$$
$$5x + 2y - 3 = 0 \quad \text{... (2)}$$
$$2x - y - 3 = 0 \quad \text{... (3)}$$



Let A $(x_1\,y_1)$ be the point of intersection of lines (1) and (2)

$(1) \times 2 \Rightarrow 6x + 2\cancel{y} = 4$

$(2) \times 3 \Rightarrow 5x + \cancel{2y} = 3$

$\quad (-) \quad (-) \quad (-)$

Subtracting $x = 1$

$\Rightarrow \quad x = 1$

Put $x = 1$ in (1) we get,

$$3(1) + y - 2 = 0 \Rightarrow 1 + y = 0$$
$$y = -1$$

$\therefore$ A is (1, –1)

Let B (x_2, y_2) be the point of intersection of (2) and (3)

$(2) \Rightarrow \quad 5x + 2y - 3 = 0$

$(3) \times 2 \Rightarrow 4x - 2y - 6 = 0$

Adding, $\quad 9x - 9 = 0 \Rightarrow 9x = 9 \Rightarrow x = \frac{9}{9}$

$\Rightarrow \quad x = 1$

Putting $x = 1$ in (3) we get

$$2(1) - y - 3 = 0$$
$$\Rightarrow \quad 2 - y - 3 = 0$$
$$\Rightarrow \quad -1 - y = 0$$
$$\Rightarrow \quad y = -1$$

$\therefore$ B is (1, –1)

Let C (x_3, y_3) be the point of intersection of lines (1) and (3).

$(1) \Rightarrow \quad 3x + y - 2 = 0$

$(3) \times 2 \Rightarrow \quad 2x - y - 3 = 0$

Adding, $\quad 5x - 5 = 0$

$$\Rightarrow \quad 5x = 5$$
$$\Rightarrow \quad x = 1$$

Putting $x = 1$ in (3) we get,

$$2(1) - y - 3 = 0$$
$$\Rightarrow \quad 2 - y - 3 = 0$$
$$\Rightarrow \quad -1 - y = 0$$
$$y = -1$$

$\therefore$ C is $(1, -1)$

Since all the three A, B, C are $(1, -1)$, they do not form the vertices of the triangle.

$\Rightarrow$ Area of ΔABC $= 0$

4. If vertices of a quadrilateral are at A(–5,7) , B(–4, k) , C(–1, –6) and D(4, 5) and its area is 72 sq.units. Find the value of k.

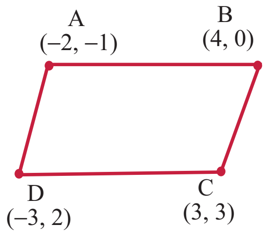
Area (quadrilateral ABCD)

$$= \frac{1}{2}\begin{vmatrix} -5 & -4 & -1 & 4 & -5 \\ 7 & k & -6 & 5 & 7 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}[(-5k+24-5+28)-(-28-k-24-25)] = 72$$
$$\Rightarrow (47 - 5k) - (-77 - k) = 144$$
$$\Rightarrow 47 - 5k + 77 + k = 144$$
$$\Rightarrow 124 - 4k = 144$$
$$\Rightarrow -4k = 20$$
$$\boxed{k = -5}$$

5. Without using distance formula, show that the points (–2, –1) , (4, 0) , (3,3) and (–3,2) are vertices of a parallelogram.

Sol.



$$\text{Mid point of AC} = \left(\frac{-2+3}{2}, \frac{-1+3}{2}\right) = \left(\frac{1}{2}, 1\right)$$

$$\text{Mid point of BD} = \left(\frac{-3+4}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$$

$$\text{Slope of AB} = \frac{0-(-1)}{4-(-2)} = \frac{+1}{6}$$

$$\text{Slope of BC} = \frac{3-0}{3-4} = -3$$

$$\text{Slope of CD} = \frac{2-3}{-3-3} = \frac{+1}{6}$$

$$\text{Slope of DA} = \frac{-1-2}{-2-(-3)} = -3$$

Slope of AB = Slope of CD

Slope of BC = Slope of DA

and the diagonals bisect each other

Hence ABCD forms a parallelogram.

6. Find the equations of the lines, whose sum and product of intercepts are 1 and –6 respectively.

Sol. Let the intercepts of a line are a, b respectively.

$$\text{Given } a + b = 1 \quad \text{...(1)}$$
$$ab = -6 \quad \text{...(2)}$$

By assuming a and b as roots of the equation,

We get $x^2 - x$ (Sum of the roots) + Product of the roots = 0

$$\Rightarrow x^2 - x(1) - 6 = 0 \quad \text{[Using (1) and (2)]}$$
$$\Rightarrow x^2 - x - 6 = 0$$

By factorizing we get,

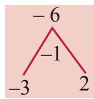
$$\Rightarrow (x-3)(x+2) = 0$$
$$\Rightarrow x = 3 \text{ or } x = -2$$

Hence the intercepts are 3 or –2.

Case (i) :

When $a = 3$, $b = -2$, the equation of the line is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{(Intercept form)}$$



$$\Rightarrow \frac{x}{3} + \frac{y}{-2} = 1$$
$$\Rightarrow \frac{x}{3} - \frac{y}{2} = 1$$
$$\Rightarrow \frac{2x - 3y}{6} = 1 \quad \text{[Taking 6 as LCM]}$$
$$\Rightarrow 2x - 3y = 6$$
$$\Rightarrow 2x - 3y - 6 = 0$$

Case (ii) :

When $a = -2$, $b = 3$, the equation of the line is

$$\frac{x}{a} + \frac{y}{b} = 1$$
$$\Rightarrow \frac{x}{-2} + \frac{y}{3} = 1$$
$$\Rightarrow \frac{x}{-2} + \frac{y}{3} = 1$$
$$\Rightarrow \frac{-3x + 2y}{6} = 1$$
$$\Rightarrow -3x + 2y = 6$$
$$\Rightarrow -3x + 2y - 6 = 0$$
$$\Rightarrow 3x - 2y + 6 = 0$$

[Multiplying by (–1) through out]

7. The owner of a milk store finds that, he can sell 980 litres of milk each week at ₹14/litre and 1220 litres of milk each week at ₹16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at ₹ 17/litre?

Sol. Let x_1, x_2 represent the quantity of milk and y_1, y_2 represent the cost of milk per litre.

Given datas are

980 l	(x_1)	₹14	(y_1)
1220 l	(x_2)	₹16	(y_2)

The linear relationship between selling price and demand is

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

[Two point form]

$$\Rightarrow \quad \frac{y-14}{16-14} = \frac{x-980}{1220-980}$$

$$\Rightarrow \quad \frac{y-14}{\cancel{2}_{1}} = \frac{x-980}{\cancel{240}_{120}}$$

$$\Rightarrow \quad 120\,(y-14) = x - 980$$

$$\Rightarrow \quad 120\,(y-14) + 980 = x$$

$$\Rightarrow \quad x = 120\,(y-14) + 980$$

When $\quad y = ₹17/l,$

$$\therefore \quad x = 120\,(17-14) + 980$$

$$\Rightarrow \quad x = 120\,(3) + 980$$

$$\Rightarrow \quad x = 360 + 980$$

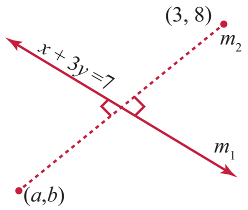
$$\Rightarrow \quad x = 1340$$

Hence the owner of a milk store can sell 1340 l per week at ₹17/ litre.

8. Find the image of the point (3, 8) with respect to the line $x + 3y = 7$ assuming the line to be a plane mirror.

Sol. Let (a, b) the required image.

Let m_1 be the slope of the given line $x + 3y = 7$ and m_2 be the slope of the line joining (3,8) and (a, b)



Slope of line joining $\overset{x_2\ \ y_2}{(3,\ 8)}$ and $\overset{x_1\ \ y_1}{(a,\ b)}$ is

$$m_2 = \frac{8-b}{3-a} \Rightarrow m_1 = \frac{-1}{3}$$

$$m_1 \times m_2 = -1$$

$$\frac{-1}{3} \times \frac{8-b}{3-a} = -1$$

$$8 - b = (3-a) \times 3$$

$$8 - b = 9 - 3a$$

$$3a - b = 1 \Rightarrow b = 3a - 1 \quad ...(1)$$

Mid point of line joining (3, 8) and (a, b) lies on $x + 3y = 7$.

$$\text{mid point} = \left(\frac{a+3}{2}, \frac{b+8}{2}\right)$$

$$\therefore \quad \frac{a+3}{2} + 3\left(\frac{b+8}{2}\right) = 7 \quad ...(2)$$

$\therefore$ Solving (1) and (2)

$$\frac{a+3}{2} + \frac{3}{2}(3a-1+8) = 7$$

as $b = 3a - 1$ from (1).

$$a + 3 + 9a + 21 = 14$$

$$10a = -10$$

$$a = -1$$

$$\therefore b = 3(-1) - 1 = -4$$

$$(a, b) = (-1, -4)$$

9. Find the equation of a line passing through the point of intersection of the lines $4x + 7y - 3 = 0$ and $2x - 3y + 1 = 0$ that has equal intercepts on the axes.

Sol. Given lines are $\quad 4x + 7y - 3 = 0 \quad ... (1)$

and $2x - 3y + 1 = 0 \quad ... (2)$

Let us solve (1) and (2) to find the point of intersection.

$$(1) \Rightarrow \quad 4x + 7y - 3 = 0$$

$$\quad (-) \quad (+) \quad (-)$$

$$(2) \times 2 \Rightarrow 4x - 6y + 2 = 0$$

Subtracting, $\quad 13y - 5 = 0$

$$\Rightarrow \quad 13y = 5 \Rightarrow y = \frac{5}{13}$$

Substituting $\quad y = \frac{5}{13}$ in (2) we get,

$$\Rightarrow \quad 2x - 3\left(\frac{5}{13}\right) + 1 = 0 \Rightarrow 2x - \frac{15}{13} + 1 = 0$$

$$\Rightarrow \quad 2x = \frac{15}{13} - 1 = \frac{15-13}{13} = \frac{2}{13}$$

$$\Rightarrow \quad x = \frac{2}{13(2)} = \frac{1}{13}$$

$\therefore$ The point of intersection of (1) and (2) $(x_1\ y_1)$ is $\left(\frac{1}{13}, \frac{5}{13}\right)$

Given that the intercepts are equal.

Change in surface area = $4\pi(1.5625)r^2 - 4\pi r^2$ [(2) – (1)]

$= 4\pi r^2 (1.5625 - 1)$ [Taking $4\pi r^2$ as common]

$= 4\pi r^2 (0.5625)$

$\therefore$ Percentage increase in surface area

$$= \frac{\text{Change in surface area} \times 100}{\text{old area}}$$

$$= \frac{\cancel{4\pi r^2}\,(.5625) \times 100}{\cancel{4\pi r^2}} = 0.5625 \times 100 = 56.25\%$$

Hence, the percentage increase in surface area = 56.25%

9. The internal and external diameters of a hollow hemispherical vessel are 20 cm and 28 cm respectively. Find the cost to paint the vessel all over at ₹ 0.14 per cm^2 .

Sol. External diameter D = 28 cm

Internal diameter d = 20 cm

$\therefore$ R = $\frac{28}{2}$ = 14 cm, $r = \frac{20}{2}$ = 10 cm.

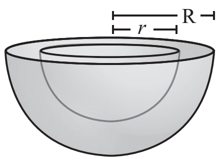
T.S.A of the hemispherical vessel = $\pi(3R^2 + r^2)$

$= \frac{22}{7}(3 \times 14^2 + 10^2)$

$= \frac{22}{7}(3 \times 196 + 100)$

$= \frac{22}{7} \times 688$

$= \frac{15136}{7} = 2162.28 \text{ cm}^2$



Cost of painting for 1 cm^2 = ₹ 0.14

Cost of painting for 2162.28 cm^2

= 2162.28 × 0.14=₹ 302.72

10. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2.

Sol. Here given that R = 12 cm

r = 6 cm

h = 8 cm

$l = \sqrt{h^2 + (R-r)^2}$

$= \sqrt{8^2 + (12-6)^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$

$l = \sqrt{100} = 10$ cm

$\therefore$ CSA of the frustum

$= \pi(R + r)l$

$= \frac{22}{7}(12 + 6)10 = \frac{220}{7} \times 18$

$= \frac{3960}{7} = 565.71 \text{ cm}^2$ ... (1)

Area of the top part = πr^2

$= \frac{22}{7} \times 6 \times 6$

$= \frac{792}{7} = 113.14 \text{ cm}^2$ ... (2)

$\therefore$ The total area to be painted

= 565.71 + 113.14 [(1) + (2)]

= 678.85 cm^2

$\therefore$ The cost of painting for 1 cm^2 = ₹ 2

$\therefore$ Cost of painting for 678.85 cm^2 = 2 × 678.85

= ₹ 1357.72

Hence, the cost of painting the lamp is = ₹ 1357.72

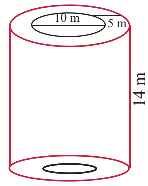
EXERCISE 7.2

1. A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.

Sol. Given inner diameter = 10 m

$\Rightarrow$ Inner radius = $\frac{10}{2}$ = 5 m

and height (h) = 14 m



Volume of the cylinder = Volume of the earth taken out

$= \pi r^2 h = \frac{22}{\cancel{7}} \times 5 \times 5 \times \cancel{14}^{\,2}$

$= 1100 \text{ m}^3$ ... (1)

The earth spread all around the well

= Volume of hollow cylinder

$= \pi(R^2 - r^2)$

Outer radius (R) = inner radius + width

= 5 + 5 = 10 m

r = 5m

and h = ? [not given]

$\therefore$ The earth spread all around the well

$$= \frac{22}{7}(10^2 - 5^2)$$

$$= \frac{22}{7}(100 - 25)\,h$$

$$= \frac{22}{7} \times 75 \times h \qquad \ldots (2)$$

It is also given that the earth taken out = The earth spread all around the well.

Equating (1) and (2) we get,

$$1100 = \frac{22}{7} \times 75 \times h$$

$$\Rightarrow \frac{\overset{2}{\cancel{1100}} \times 7}{\cancel{22} \times \underset{3}{\cancel{75}}} = h \quad \Rightarrow h = \frac{14}{3}$$

$$\Rightarrow h = 4.666 \text{ m}$$

Hence, height of the embankment = 4.67 m

2. A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass? **[Sep. - 2020]**

Sol.

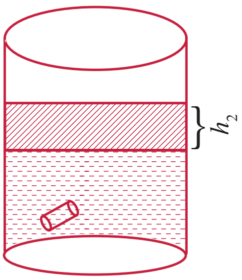
For the cylindrical metal	For the water raised which is in cylindrical form
$r_1 = 5$	diameter = 20 cm
$h_1 = 4$	$r_2 = 10$ cm
	$h_2 = ?$

By Archimedes's principle,

Volume of the water raised

= Volume of the cylindrical metal

$$\Rightarrow \cancel{\pi} r_2^2 h_2 = \cancel{\pi} r_1^2 h_1$$



$$\Rightarrow (10)^2 h_2 = (5)^2 \times 4$$

$$\Rightarrow 100 h_2 = (25)\,4$$

$$\Rightarrow h_2 = \frac{(25)(4)}{100} = \frac{100}{100} = 1 \text{ cm}$$

Hence, height of the raised water in the glass = 1 cm

3. If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm. **[Aug. - 2022]**

Sol. Circumference of the base of the cone = 484 cm

Height = 105 cm

$$\therefore 2\pi r = 484 \Rightarrow 2 \times \frac{22}{7} \times r = 484$$

$$\Rightarrow r = \overset{11}{\cancel{\overset{121}{\cancel{\overset{242}{\cancel{484}}}}}} \times \frac{1}{\cancel{2}} \times \frac{7}{\underset{1}{\cancel{\underset{11}{\cancel{22}}}}} = 77 \text{ cm}$$

$$\Rightarrow r = 77 \text{ cm}$$

$$\therefore \text{Its volume} = \frac{1}{3}\pi r^2 h \text{ cubic units}$$

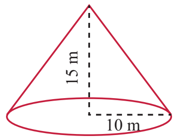
$$= \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \overset{11}{\cancel{77}} \times 77 \times \overset{35}{\cancel{105}}$$

$$= 652190 \text{ cm}^3.$$

$\therefore$ The required volume of the cone = 652190 cm^3.

4. A conical container is fully filled with petrol. The radius is 10 m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.

Sol.



Given $r = 10$ m

$h = 15$m

Volume of the cone $= \frac{1}{3}\pi r^2 h$ cu. units.

Volume of the given conical

Container $= \frac{1}{3} \times \pi \times 10 \times 10 \times 15 = 500\pi \text{ m}^3$

To empty 25 m^3, the time taken = 1 minutes

To empty 500π m^3 the time taken $= \dfrac{500 \times \frac{22}{7} \times 1}{25}$

$$\Rightarrow \frac{\overset{20}{\cancel{500}} \times 22}{7 \times \cancel{25}} \Rightarrow \frac{440}{7} = 62.857 \text{ minutes}$$

$$\cong 63 \text{ minutes.(approx.)}$$

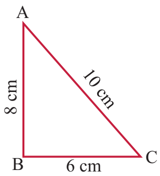
Hence, the conical container will be emptied in 63 minutes.

5. A right angled triangle whose sides are 6 cm, 8 cm and 10 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two solids so formed.

Sol. When the triangle ABC is rotated about AB, the volume of the cone formed $= \frac{1}{3}\pi r^2 h$

Here $r = 6$ cm

$h = 8$ cm



$$V_1 = \frac{1}{3}\times\frac{22}{7}\times 6\times 6\times 8$$

$$= \frac{2112}{7} = 301.71 \text{ cm}^3$$

When the Δ ABC, rotated about BC,

$r = 8$ cm, $h = 6$ cm

$$V_2 = \frac{1}{3}\times\frac{22}{7}\times 6\times 8\times 8 = \frac{2816}{7} = 402.29 \text{ cm}^3$$

$\therefore$ Difference in volume $= V_2 - V_1$

$= 402.29 - 301.71$

$= 100.58 \text{ cm}^3$

6. The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights. **[PTA - 4; May - 2022]**

Sol. Let r_1, r_2 be the radius of two cones and V_1, V_2 be their volume.

Given $V_1 = 3600 \text{ cm}^3$, $r_1 = r_2$

$V_2 = 5040 \text{ cm}^3$

Consider $$\frac{\cancel{\frac{1}{3}}\cancel{\pi} r_1^2 h_1}{\cancel{\frac{1}{3}}\cancel{\pi} r_2^2 h_2} = \frac{3600}{5040} \Rightarrow \frac{r_1^2 h_1}{r_1^2 h_2} = \frac{3600}{5040} \; [\because r_1 = r_2]$$

$$\frac{h_1}{h_2} = \frac{90}{126} = \frac{45}{63} = \frac{15}{21} = \frac{5}{7}$$

$\therefore h_1 : h_2 = 5 : 7$

$\therefore$ Ratio of their heights $= 5 : 7$

7. If the ratio of radii of two spheres is 4 : 7, find the ratio of their volumes. **[April - 2023]**

Sol. Let r_1, r_2 be the radii of the two given spheres

Given $\frac{r_1}{r_2} = \frac{4}{7}$ ... (1)

Ratio of their volumes $$= \frac{\cancel{\frac{4}{3}}\pi\times r_1^3}{\cancel{\frac{4}{3}}\pi\times r_2^3} = \frac{r_1^3}{r_2^3}$$

$$\Rightarrow \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{4}{7}\right)^3 = \frac{4\times4\times4}{7\times7\times7} = \frac{64}{343}$$

Hence, ratio of their volumes = 64 : 343

8. A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}$: 4. **[PTA - 6]**

Sol. Let r_1, r_2 be the radii of the sphere and a solid hemisphere

Given TSA of sphere = TSA of solid hemisphere

$$\Rightarrow 4\cancel{\pi} r_1^2 = 3\cancel{\pi} r_2^2 \Rightarrow 4r_1^2 = 3r_2^2$$

$$\Rightarrow \frac{r_1^2}{r_2^2} = \frac{3}{4} \Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{3}}{2} \quad \text{... (1)}$$

[Taking square root]

Ratio of their volumes $= \frac{\text{Volume of sphere}}{\text{Volume of hemisphere}}$

$$= \frac{\frac{4}{\cancel{3}}\cancel{\pi}\times r_1^3}{\frac{2}{\cancel{3}}\cancel{\pi}\times r_2^3} = \frac{4r_1^3}{2r_2^3} = 2\left(\frac{r_1}{r_2}\right)^3 = 2\left(\frac{\sqrt{3}}{2}\right)^3$$

(using (1))

$$= \frac{\cancel{2}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}}{\cancel{2}\times2\times2} = \frac{3\sqrt{3}}{4}$$

Hence, ratio of their volumes $= 3\sqrt{3}$: 4

9. The outer and the inner surface areas of a spherical copper shell are 576π cm² and 324π cm² respectively. Find the volume of the material required to make the shell.

Sol. Given $4\pi R^2 = 576\pi \text{ cm}^2$

Given $4\pi r^2 = 324\pi \text{ cm}^2$

$$4\cancel{\pi} R^2 = 576\cancel{\pi} \Rightarrow R^2 = \frac{576}{4} = 144$$

$\Rightarrow R^2 = 144 \text{ cm}^2$

$\Rightarrow R = 12$ cm

Also $4\cancel{\pi} r^2 = 324\cancel{\pi}$

$$\Rightarrow r^2 = \frac{324}{4} = 81$$

$$r^2 = 81 \text{ cm}^2 \Rightarrow r = 9 \text{ cm}$$

∴ Volume of the hollow sphere

$$= \frac{4}{3}\pi(R^3 - r^3) \text{ cu. units}$$

$$= \frac{4}{3}\times\frac{22}{7}\times(12^3 - 9^3) = \frac{4}{3}\times\frac{22}{7}\times(1728 - 729)$$

$$= \frac{4}{3}\times\frac{22}{7}\times 999 = \frac{29304}{7} = 4186.285 = 4186.29 \text{ cm}^3$$

∴ The volume of the material needed = 4186.29 cm^3.

10. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹40 per litre. **[May - 2022]**

Sol. Given h = 16 cm

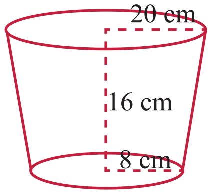
R = 20 cm

r = 8 cm

Volume of the frustum

$$\Rightarrow V = \frac{1}{3}\pi(R^2 + Rr + r^2)h \text{ cubic units}$$

$$= \frac{1}{3}\times\frac{22}{7}(20^2 + 20\times 8 + 8^2)\times 16$$



$$= \frac{1}{3}\times\frac{22}{7}(400 + 160 + 64)\times 16$$

$$= \frac{1}{3}\times\frac{22}{7}\times 624\times 16$$

$$= \frac{73216}{7} = 10459.428 \text{ cm}^3$$

$$1000 \text{ cm}^3 = 1 \text{ litre}$$

$$\therefore 10459.428 \text{ cm}^3 = \frac{10459.428}{1000} = 10.459\ l.$$

Cost of 1l milk = ₹ 40

∴Cost of 10.459 l of milk = 40 × 10.459 = ₹ 418.36.

EXERCISE 7.3

1. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel.

Sol. For cylinder

∴ Given Diameter = 14 cm

Radius = 7 cm

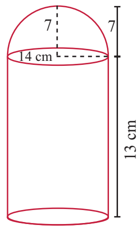
Total height = 13 cm

radius of sphere = 7 cm

Height of the cylindrical part = 13 – 7

= 6 cm

∴ Capacity of the vessel = Capacity of the cylinder + Capacity of the hemisphere.



Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{\cancel{7}}\times\cancel{7}\times 7\times 6$$

$$= 22 \times 42 = 924 \quad \text{... (1)}$$

Volume of the hemisphere

$$= \frac{2}{3}\pi r^3 = \frac{2}{3}\times\frac{22}{\cancel{7}}\times\cancel{7}\times 7\times 7$$

$$= \frac{44\times 49}{3} = \frac{2156}{3}$$

$$= 718.67 \quad \text{... (2)}$$

∴ The total volume = 924 + 718.67 [(1) + (2)]

The capacity of the vessel = 1642.67 cm^3.

2. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made. **[May - 2022]**

Sol. Volume of the model = Volume of the cylinder + Volume of 2 cones.

For cylinder

h = 12 – 2 – 2 = 8 cm

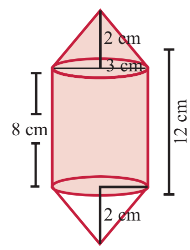
diameter = 3 cm

⇒ $r = \frac{3}{2}$ cm

For cone

h = 2 cm

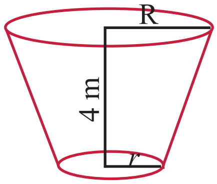
$r = \frac{3}{2}$ cm



Volume of the cylinder part = $\pi r^2 h$

Given l = 4 m

C.S.A of frustum $= \pi l(R + r)$



$$= \frac{22}{7} \times 4\,(2.86 + 2.54)$$

$$= \frac{22}{7} \times 4 \times 5.4$$

$$= 67.88\text{m}^2 \cong 68\text{m}^2$$

Given that cost of painting for 1 sq. m = ₹100

$\therefore$ Cost of painting for 68 m^2 = 68 × 100

= ₹6800

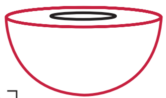
8. A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is 14 cm. Find its thickness.

Sol. Volume of hollow hemisphere

$$= \frac{2}{3}\pi(R^3 - r^3)$$

Given D = 14 cm, R = $\frac{14}{2}$ = 7 cm

$$\Rightarrow \frac{2}{\cancel{3}} \times \frac{\cancel{22}}{7} \times (7^3 - r^3) = \frac{436\cancel{\pi}}{\cancel{3}} \quad \left[\because \pi = \frac{22}{7}\right]$$



$$(343 - r^3) = \frac{436}{2}$$

$$-r^3 = 218 - 343$$

$$\cancel{-}r^3 = \cancel{-}125$$

$$r = 5 \text{ cm}$$

Thickness of the bowl = R – r = 7 – 5 = 2 cm

9. The volume of a cone is $1005\frac{5}{7}$ cu. cm. The area of its base is $201\frac{1}{7}$ sq. cm. Find the slant height of the cone. **[Hy. - 2023]**

Sol. Given that volume of the cone = $1005\frac{5}{7}$ cu.cm

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{7040}{7}$$

$$\Rightarrow \pi r^2 = \frac{7040}{7} \times \frac{3}{h} \quad \ldots (1)$$

Also, it is given that area of the base = $201\frac{1}{7}$ cm^2

$$\Rightarrow \pi r^2 = \frac{1408}{7} \quad \ldots (2)$$

$$\Rightarrow \frac{22}{7} \times r^2 = \frac{1408}{7}$$

$$\Rightarrow r^2 = \frac{\overset{128}{\cancel{1408}}}{\cancel{7}} \times \frac{\cancel{7}}{\underset{2}{\cancel{28}}} = 64$$

$$\Rightarrow r^2 = 64 \Rightarrow r = 8 \text{ cm}$$

Equating (1) and (2) we get,

$$\frac{7040}{\cancel{7}} \times \frac{3}{h} = \frac{1408}{\cancel{7}}$$

$$\Rightarrow 7040 \times \frac{3}{h} = 1408$$

$$\Rightarrow \frac{\overset{5}{\cancel{7040}} \times 3}{\cancel{1408}} = h \quad \Rightarrow 5 \times 3 = h$$

$$\Rightarrow h = 15 \text{ cm}$$

Slant height $l = \sqrt{h^2 + r^2}$

$$= \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$$

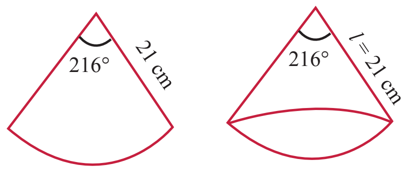
$$= \sqrt{289} = \sqrt{17^2}$$

$$\Rightarrow l = 17 \text{ cm}$$

Hence, slant height = 17 cm

10. A metallic sheet in the form of a sector of a circle of radius 21 cm has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed. **[PTA - 2]**

Sol. Given radius of sector = l = 21 cm



$$\theta = 216$$

Let r be the radius of the cone

When the sector is made into a cone, we get base perimeter of the cone = length of arc.

$$\Rightarrow \cancel{2\pi}r = \frac{\theta}{360} \times \cancel{2\pi} \quad \text{(radius of sector)}$$

$$\Rightarrow r = \frac{\theta}{360} \times \text{radius of sector}$$

$$\Rightarrow r = \frac{\overset{18}{\cancel{54}}\ \cancel{108}\ \cancel{216}}{\cancel{360}\ \cancel{120}\ \cancel{60}\ \cancel{30}\ 10} \times \overset{7}{\cancel{21}} = \frac{18 \times 7}{10}$$

	16.8
1	2,82.24
	1
26	182
	156
328	2624
	2624
	0

$= 12.6$ cm

Height of cone $(h) = \sqrt{l^2 - r^2}$

$\Rightarrow \quad h = \sqrt{21^2 - 12.6^2}$

$\Rightarrow \quad h = \sqrt{441 - 158.76} = \sqrt{282.24}$

$h = 16.8$ cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 16.8$

Hence, volume of the cone = 2794.18 cm^3

PTA EXAM QUESTION & ANSWERS

1 MARK

1. If the volume of sphere is 36π cm^2, then its radius is equal to **[PTA - 3]**

(A) 3 cm (B) 2 cm (C) 5 cm (D) 10 cm

[Ans. (A) 3 cm]

Hint:

$$V = 36 \text{ cm}^2$$

$$\frac{4}{3}\cancel{\pi} r^3 = \cancel{36\pi}$$

$$r^3 = \overset{9}{\cancel{36}} \times \frac{3}{\cancel{4}}$$

$$r^3 = 27$$

$$r = 3$$

2. C.S.A of solid sphere is equal to **[PTA - 5]**

(A) T.S.A of solid sphere

(B) T.S.A of hemisphere

(C) C.S.A of hemisphere

(D) none of these

[Ans. (A) T.S.A of solid sphere]

2 MARKS

1. The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone. **[PTA - 1; Hy. - 2019; June - 2023]**

Sol. Let r and h be the radius and height of the cone respectively.

Given that, volume of the cone = 11088 cm^3

$$\frac{1}{3}\pi r^2 h = 11088$$

$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$$

$$r^2 = 441$$

Therefore, radius of the cone $r = 21$ cm

2. Find the number of spherical lead shots, each of diameter 6cm that can be made from a solid cuboids of lead having dimensions 24cm × 22cm × 12cm. **[PTA - 2]**

Sol. Number of lead shots $= \dfrac{\text{Volume of cuboid}}{\text{Volume of a lead shot}}$

$$= \frac{l \times b \times h}{\frac{4}{3}\pi r^3} = \frac{24 \times 22 \times 12}{\frac{4}{3} \times \frac{22}{7} \times 3^3}$$

$$= \frac{\overset{8}{\cancel{24}} \times \cancel{22} \times \cancel{12} \times \cancel{3} \times \cancel{7}}{\cancel{4} \times \cancel{22} \times \cancel{3} \times \cancel{3} \times \cancel{3}} = 56$$

3. A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder. **[PTA - 3]**

Sol. Let h_1 and h_2 be the heights of a cone and cylinder respectively.

Also, let r be the radius of the cone.

Given that, height of the cone $h_1 = 24$ cm; radius of the cone and cylinder $r = 6$ cm

Since, Volume of cylinder = Volume of cone

$$\pi r^2 h_2 = \frac{1}{3}\pi r^2 h_1$$

$$\Rightarrow \quad h_2 = \frac{1}{3} \times h_1 \text{ gives}$$

$$h_2 = \frac{1}{3} \times 24 + 8$$

Therefore, height of cylinder is 8 cm

4. If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum. **[PTA - 5; Sep. - 2021; Hy. - 2023]**

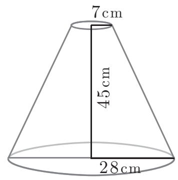
Sol. Let h, r and R be the height, top and bottom radii of the frustum.

Given that, $h = 45$ cm, R = 28 cm, $r = 7$ cm

Volume $\Rightarrow \frac{1}{3} \times \pi[R^2 + Rr + r^2]h$ cu. units

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times [28^2 + (28 \times 7) + 7^2] \times 45$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 1029 \times 45 = 48510$



Therefore, volume of the frustum is 48510 cm^3

5 MARKS

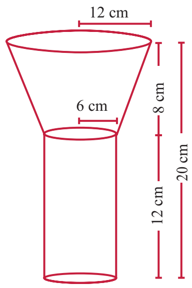
1. A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface area of the funnel. **[PTA - 1]**

Sol. Let R, r be the top and bottom radii of the frustum.

Let h_1, h_2 be the heights of the frustum and cylinder respectively.

Given that, R = 12 cm, r = 6 cm , h_2 = 12 cm

Now $h_1 = 20 - 12 = 8$ cm



Here, Slant height of the frustum

$$l = \sqrt{(R-r)^2 + h_1^2} \text{ units}$$

$$= \sqrt{36+64}$$

$$l = 10 \text{ cm}$$

$$\begin{aligned}\text{Outer surface area} &= 2\pi r h_2 + (R+r)l \text{ sq. units}\\ &= \pi[2rh_2 + (R+r)l]\\ &= \pi[(2\times 6\times 12) + (18\times 10)]\\ &= \pi[144+180]\\ &= \frac{22}{7}\times 324 = 1018.28\end{aligned}$$

Therefore, outer surface area of the funnel is 1018.28 cm^2

2. The radius of a conical tent is 7 m and the height is 24 m. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m? **[PTA - 3; April - 2023]**

Sol. Let r and h be the radius and height of the cone respectively.

Given that, radius r = 7 m and height h = 24 m

$$\text{Hence, } l = \sqrt{r^2+h^2} = \sqrt{49+576}$$

$$l = \sqrt{625} = 25 \text{ m}$$

C.S.A. of the conical tent = πrl sq. units

$$\text{Area of the canvas} = \frac{22}{7}\times 7\times 25 = 550 \text{ m}^2$$

Now, length of the canvas

$$= \frac{\text{Area of the canvas}}{\text{width}}$$

$$= \frac{550}{4} = 137.5 \text{ m}$$

Therefore, the length of the canvas is137.5 m

3. A cylindrical bucket of 32 cm high and with radius of base 18 cm, is filled with sand completely. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap. **[PTA - 4; Govt. MQP - 2019]**

Sol. Volume of Cylinder = $\pi r^2 h = \pi(18^2)\,32$

$$\text{Volume of Cone} = \frac{1}{3}\pi r_1^2 h = \frac{1}{3}\pi r_1^2(24)$$

$$\pi(18^2)\,32 = \frac{1}{3}\pi r_1^2(24)$$

$$r_1^2 = \frac{\cancel{3}\times 18^2\times \cancel{32}^{4}}{\cancel{24}_{\cancel{8}}} = 18^2\times 4$$

$$r_1^2 = 18^2\times 2^2 = 36^2$$

$$r_1 = 36$$

$$\begin{aligned}\text{Slant height } (l) &= \sqrt{r_1^2+h^2} = \sqrt{36^2+24^2}\\ &= \sqrt{1296+576} = \sqrt{1872}\\ &= \sqrt{12\times 12\times 13} = 12\sqrt{13}\end{aligned}$$

4. A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container. **[PTA - 5 & 6]**

Sol. Let h and r be the height and radius of the cylinder respectively.

Given that, h =15 cm, r = 6 cm

Volume of the container V = $\pi r^2 h$ cubic units.

$$= \frac{22}{7}\times 6\times 6\times 15$$

Let, r_1 = 3 cm, h_1 = 9 cm be the radius and height of the cone.

Also, r_1 = 3 cm is the radius of the hemispherical cap.

Volume of one ice cream cone = (Volume of the cone + Volume of the hemispherical cap)

$$= \frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3$$

$$= \frac{1}{3}\times\frac{22}{7}\times 3\times 3\times 9 + \frac{2}{3}\times\frac{22}{7}\times 3\times 3\times 3$$

$$= \frac{22}{7} \times 9\,(3+2) = \frac{22}{7} \times 45$$

$$\text{Number of cones} = \frac{\text{volume of the cylinder}}{\text{volume of one ice cream cone}}$$

Number of ice cream cones needed

$$= \frac{\frac{27}{7} \times 6 \times 6 \times 15}{\frac{22}{7} \times 45} = 12$$

Thus 12 ice cream cones are required to empty the cylindrical container.

5. A well of diameter 3m is dug 14m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4m to form an embankment. Find the height of the embankment. **[PTA - 5]**

Sol. Volume of sand $= \pi r^2 h = \pi\left(\frac{3}{2}\right)^2 14$

Volume of sand = Volume of ring

$$\cancel{\pi}\left(\frac{3}{2}\right)^2 14 = \cancel{\pi}\left(\left(\frac{3}{2}+4\right)^2 - \left(\frac{3}{2}\right)^2\right)h$$

$$\frac{9}{4} \times 14 = \left(\frac{121}{4} - \frac{9}{4}\right)h$$

$$\frac{9}{\cancel{4}} \times 14 = \frac{112}{\cancel{4}}\,4 \Rightarrow h = \frac{9 \times 14}{\cancel{112}_{8}} = 1.125 \text{ m}$$

$$\begin{array}{r} 1.125 \\ 8\,\overline{)\,9} \\ 8 \\ \hline 10 \\ 8 \\ \hline 20 \\ 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

6. If the slant height of the frustum cone is 10cm and perimeters of its circular base are 18cm and 28cm respectively. What is the curved surface area of the frustum? **[PTA - 6]**

Sol. $l = 10$ cm, $2\pi R = 28$ cm, $2\pi r = 18$ cm, $\pi R = 14$ cm, $\pi r = 9$ cm,

$$\begin{aligned} \text{CSA of frustum} &= (\pi R + \pi r)l \\ &= (14 + 9)\,10 = 230 \text{ cm}^2 \end{aligned}$$

GOVT. EXAM QUESTION & ANSWERS

1 MARK

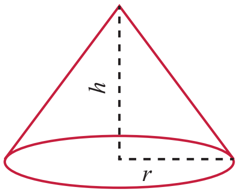
Multiple choice questions.

1. A child reshapes a cone made up of clay of height 24 cm and radius 6 cm into a sphere, then the radius of sphere is : **[Sep. - 2020]**

(A) 24 cm (B) 12 cm
(C) 6 cm (D) 48 cm

[Ans. (C) 6 cm]

Hint:



$h = 24$ cm
$r = 6$ cm

Let R be the radius of the sphere

Given, Volume of the Sphere = Volume of the cone.

$$\Rightarrow \frac{4}{\cancel{3}}\cancel{\pi}R^3 = \frac{1}{\cancel{3}}\cancel{\pi}r^2h$$

$$\Rightarrow 4R^3 = (6)^2\,(24)$$

$$\Rightarrow R^3 = \frac{6 \times 6 \times \cancel{24}^{\,6}}{\cancel{4}}$$

$$\Rightarrow R^3 = 6 \times 6 \times 6 \Rightarrow R^3 = 6^3$$

$$\Rightarrow R = 6 \text{ cm}$$

2. The height of a right circular cone whose radius is 3 cm and slant height is 5 cm will be **[Hy. - 2019]**

(A) 12 cm (B) 4 cm
(C) 13 cm (D) 5 cm **[Ans. (B) 4 cm]**

Hint: height $= \sqrt{l^2 - r^2} = \sqrt{25 - 9}$
$= \sqrt{16} = 4$ cm

3. If the radius of the cylinder is doubled, the new volume of the cylinder will be _______ times the original volume. **[May - 2022]**

(A) same (B) 3 (C) 4 (D) 2

[Ans. (C) 4]

Hint: Original Volume $= \pi r^2 h$
New Volume $= \pi\,(2r)^2\,h = 4\,\pi r^2\,h$
$\Rightarrow$ 4 times the Original Volume

2 MARKS

1. Find the diameter of a sphere whose surface area is 154 m². **[Sep.- 2020]**

Sol. Let r be the radius of the sphere.

Given that, surface area of sphere = 154 m^2

$$4\pi r^2 = 154$$

$$4 \times \frac{22}{7} \times r^2 = 154$$

$$\Rightarrow r^2 = 154 \times \frac{1}{4} \times \frac{7}{22}$$

$$r^2 = \frac{49}{4}$$

$$\text{We get } r = \frac{7}{2}$$

Therefore, diameter is 7 m

2. If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area. **[Sep.- 2020; FRT - 2024]**

Sol. Let r be the radius of the hemisphere.

Given that, base area = πr^2 = 1386 sq. m

T.S.A. = $3\pi r^2$ sq.m = 3 × 1386 = 4158

Therefore, T.S.A. of the hemispherical solid is 4158 m^2.

3. Find the volume of the iron used to make a hollow cylinder of height 9 cm and whose internal and external radii are 3 cm and 5 cm respectively. **[Sep.- 2020]**

Sol. Let r, R and h be the internal radius, external radius and height of the hollow cylinder respectively.

Given that, r = 3cm, R = 5 cm, h = 9 cm

Now, volume of hollow cylinder

$$= \pi(R^2 - r^2)h \text{ cu. units}$$

$$= \frac{22}{7}(5^2 - 3^2) \times 9$$

$$= \frac{22}{7}(25 - 9)$$

$$= \frac{22}{7} \times 16 \times 9$$

$$= \frac{22}{7} \times 144 = 452.16 \text{ cm}^3$$

Therefore, volume of iron used = 452.16 cm^3

4. The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases. **[Hy. - 2019 & 2023; May - 2022]**

Sol. Given that, $\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$

Now, ratio of C.S.A. of balloons

$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Therefore, ratio of C.S.A. of balloons is 9:16.

5. An aluminium sphere of radius 15 cm is melted to make a cylinder of radius 10 cm. Find the height of the cylinder. **[Hy. - 2019]**

Sol. Volume of sphere = Volume of cylinder

$$\frac{4\cancel{\pi}}{3} r_1^3 = \cancel{\pi} r_2^2 h$$

$$\frac{\cancel{4}}{\cancel{3}} \times \cancel{15} \times \cancel{15} \times 15 = \cancel{10} \times \cancel{10} \times h$$

$$\boxed{h = 45 \text{ cm}}$$

6. Find the volume of a cylinder whose height is 2 m and base area is 250 m². **[Sep. - 2021]**

Sol. Let r and h be the radius and height of the cylinder respectively.

Given that, height h = 2 m, base area = 250 m^2

Now, volume of a cylinder = $\pi r^2 h$ *cu.* units

= base area × h

= 250 × 2 = 500 m^3

Therefore, volume of the cylinder = 500 m^3

7. The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes. **[Sep. - 2021; May - 2022]**

Sol. Let the height and radius of the cone 1 be h_1 and r_1 respectively.

Let the height and radius of the cone 2 be h_2 and r_2 respectively.

Given : $\frac{h_1}{h_2} = \frac{1}{2} \Rightarrow h_1 = \frac{1}{2}h_2$.

Similarly $\frac{r_1}{r_2} = \frac{3}{4} \Rightarrow r_1 = \frac{3}{4}r_2$

Ratio of volume of two cones $= \dfrac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \dfrac{r_1^2 h_1}{r_2^2 h_2}$

$$= \frac{\left(\frac{3}{4}\right)^2 r_2^2 \cdot \frac{1}{2} h_2}{r_2^2 h_2} = \left(\frac{3}{4}\right)^2 \left(\frac{1}{2}\right) = \frac{9}{32}$$

∴ Required ratio = 9 : 32

8. If the total surface area of a cone of radius 7 cm is 704 cm², then, find its slant height.

Sol. Refer Text Book Example 7.6 **[Aug. - 2022]**

9. **The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the cylinder.** **[June - 2023]**

Sol. Refer Text Book Example 7.2

5 MARKS

1. **Calculate the weight of a hollow brass sphere if the inner diameter is 14cm and thickness is 1mm, and whose density is 17.3 g/cm^3.** **[Govt. MQP - 2019]**

Sol. Let r and R be the inner and outer radii of the hollow sphere.

Given that, inner diameter d = 14 cm; inner radius r = 7 cm; thickness = 1 mm = $\frac{1}{10}$ cm

Outer radius R $= 7 + \frac{1}{10} = \frac{71}{10} = 7.1$ cm

Volume of hollow sphere = $\frac{4}{3}\pi(R^3 - r^3)$ cu. cm

$= \frac{4}{3} \times \frac{22}{7}(357.91 - 343) = 62.48$ cm^3

But, weight of brass in 1 cm^3 = 17.3 gm

Total weight = 17.3×62.48 = 1080.90 gm

∴ Total weight is 1080.90 grams.

2. **A doll is made by surmounting a cone on a hemisphere of equal radius. The radius of the hemisphere is 7 cm and slant height of the cone is 11 cm. Find the surface area of the doll.** **[Hy. - 2019]**

Sol. Surface Area $= 2\pi r^2 + \pi rl$

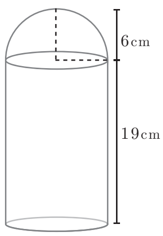
$= \pi r\,[2r + l] = \frac{22}{\cancel{7}} \times \cancel{7}\,[2 \times 7 + 11]$

$= 22\,[25] = 550$ cm^2

$\boxed{A = 18}$

3. **A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm. Find the total surface area of the toy if its common diameter is 12 cm.** **[Sep. - 2021]**

Sol. Let r and h be the radius and height of the cylinder respectively.



Given that, diameter

d = 12 cm, radius r = 6 cm

Total height of the toy is 25 cm

Therefore, height of the cylindrical portion

$h = 25 - 6 = 19$ cm

T.S.A. of the toy = C.S.A. of the cylinder + C.S.A. of the hemisphere + Base Area of the cylinder

$= 2\pi rh + 2\pi r^2 + \pi r^2 = \pi r\,(2h + 3r)$ sq. units

$= \frac{22}{7} \times 6 \times (38 + 18) = \frac{22}{7} \times 6 \times 56 = 1056$

Therefore, T.S.A. of the toy is 1056 cm^2

4. **A cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area.** **[Aug. - 2022]**

Sol. Refer Text Book Example 7.1

5. **Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person requires 4 sq.m. of the space on ground and 40 cu. meter of air to breathe. Find the height of the conical part of the tent if the height of cylindrical part is 8 m.** **[April - 2023]**

Sol. Refer Text Book Example 7.26

6. **A metallic sphere of radius 16 cm is melted and recast into small spheres each of radius 2 cm. How many small spheres can be obtained?** **[June - 2023]**

Sol. Refer Text Book Example 7.29

ADDITIONAL QUESTION & ANSWERS

1. **Find the depth of a cylindrical tank of radius 28 m, if its capacity is equal to that of a rectangular tank of size 28 m × 16 m × 11 m.**

Sol. Volume of the cylindrical tank =

Volume of the rectangle tank $\pi r^2 h = 28 \times 16 \times 11$ m^3

$\frac{22}{\cancel{7}} \times \cancel{28}^{4} \times \cancel{28} \times h = \cancel{28} \times 16 \times 11$

$h = \frac{16 \times 11}{88} = 2$ m

2. **What is the ratio of the volume of a cylinder, a cone, and a sphere. If each has the same diameter and same height?**

Sol. Volume of a cylinder $= \pi r^2 h$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Their ratio $V_1 : V_2 : V_3$

$\pi \cancel{r^2} h : \frac{1}{3}\pi \cancel{r^2} h : \frac{4}{3}\pi \cancel{r^3}$

$h : \frac{h}{3} : \frac{4r}{3}$

$\Rightarrow 3h : h : 4r$

$\Rightarrow 3h : h : 2(2r)$ (where $2r = h$)

$\therefore V_1 : V_2 : V_3 = 3 : 1 : 2$

3. Find the number of coins, 1.5 cm is diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Sol. No. of coins required $= \frac{\text{Volume of the cylinder}}{\text{Volume of 1 coin}}$

$$= \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$

$$= \frac{\pi \times \frac{45}{20} \times \frac{45}{20} \times 10}{\pi \times \frac{15}{20} \times \frac{15}{20} \times \frac{2}{10}}$$

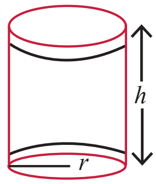
$$= \frac{\cancel{45} \times \cancel{45} \times \cancel{10}}{\cancel{20} \times \cancel{20}} \times \frac{\cancel{20}^{10}}{\cancel{15}} \times \frac{\cancel{20}^{10}}{\cancel{15}} \times \frac{\cancel{10}^{5}}{\cancel{2}} = 450$$

4. A spherical ball of iron has been melted and made into small balls. If the radius of each smaller ball is one-fourth of the radius of the original one, how many such balls can be made?

Sol. No. of balls required $= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi \times \left(\frac{r}{4}\right)^3}$

$$= r^3 \times \frac{4}{r} \times \frac{4}{r} \times \frac{4}{r} = 64$$

5. A wooden article was made by scooping out a hemisphere from each end of a cylinder as shown in figure. If the height of the cylinder is 10cm and its base is of radius 3.5 cm find the total surface area of the article.



Radius of the cylinder be r

Height of the cylinder be h

Total surface area of the article

= CSA of cylinder + CSA of 2 hemispheres

$= 2\pi rh + 2\pi r^2 = 2\pi r\,(h + 2r)$

$$= 2 \times \frac{22}{7} \times 3.5 \times (10 + 2 \times 3.5)$$

$= 22 \times 17 = 374$ cm^2

Unit Test

Time : 45 Minutes **Marks : 25**

SECTION - A (5 × 1 = 5)

1. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is
   (A) $4\pi r^2$ sq. units (B) $6\pi r^2$ sq. units
   (C) $3\pi r^2$ sq. units (D) $8\pi r^2$ sq. units
2. The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is
   (A) $\frac{9\pi h^2}{8}$ sq.units (B) $24\pi h^2$ sq.units
   (C) $\frac{8\pi h^2}{9}$ sq.units (D) $\frac{56\pi h^2}{9}$ sq.units
3. The total surface area of a hemi-sphere is how much times the square of its radius.
   (A) π (B) 4π (C) 3π (D) 2π
4. A shuttle cock used for playing badminton has the shape of the combination of
   (A) a cylinder and a sphere
   (B) a hemisphere and a cone
   (C) a sphere and a cone
   (D) frustum of a cone and a hemisphere
5. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 = 1 : 2$ then $r_2 : r_1$ is
   (A) 1 : 3 (B) 1 : 2 (C) 2 : 1 (D) 3 : 1

SECTION - B (5 × 2 = 10)

1. The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.
2. A conical container is fully filled with petrol. The radius is 10 m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.
3. An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.
4. A hemi-spherical tank of radius 1.75 m is full of water. It is connected with a pipe which empties the tank at the rate of 7 litre per second. How much time will it take to empty the tank completely?
5. Find the number of coins, 1.5 cm is diameter and 0.2 cm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

SECTION - C (2 × 5 = 10)

1. A right circular cylinder just enclose a sphere of radius r units.
   Calculate
   (i) the surface area of the sphere
   (ii) the curved surface area of the cylinder
   (iii) the ratio of the areas obtained in (i) and (ii).
2. A metallic sheet in the form of a sector of a circle of radius 21 cm has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.

Answers

SECTION - A

1. (A) $4\pi r^2$ sq. units
2. (C) $\frac{8\pi h^2}{9}$ sq.units
3. (C) 3π
4. (D) frustum of a cone and a hemisphere
5. (B) 1 : 2

SECTION - B

1. Refer Sura's Guide Exercise 7.1, Q.No.3
2. Refer Sura's Guide Exercise 7.2, Q.No.4
3. Refer Sura's Guide Exercise 7.4, Q.No.1
4. Refer Sura's Guide Unit Exercise Q.No.2
5. Refer Sura's Guide Additional Q.No.4

SECTION - C

1. Refer Sura's Guide Exercise No.7.3, Q.No.7
2. Refer Sura's Guide Unit Exercise Q.No.10

Chapter 8

STATISTICS AND PROBABILITY

FORMULAE TO REMEMBER

- Range R = L –S.
- Coefficient of range = $\frac{L-S}{L+S}$, L - Largest value, S-smallest value standard deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$.
- Variance $\sigma^2 = \frac{\sum_{i=1}^{n}(x_1-\bar{x})^2}{n}$
- Assumed mean method $\sigma = c \times \sqrt{\frac{\sum d_i^2}{n} - \left(\frac{\sum d_i}{n}\right)^2}$

EXERCISE 8.1

1. Find the range and coefficient of range of the following data.
(i) 63, 89, 98, 125, 79, 108, 117, 68

[Sep.- 2020; April - 2023]

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Sol. Range R = L – S.

Co-efficient of range $= \dfrac{L-S}{L+S}$

L – Largest value, S – Smallest value.

(i) 63, 89, 98, 125, 79, 108, 117, 68.

Here L = 125

S = 63

$\therefore$ R = L – S = 125 – 63 = 62

Co-efficient of range $= \dfrac{L-S}{L+S} = \dfrac{125-63}{125+63}$

$= \dfrac{62}{188} = 0.33$

(ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

L = 61.4

S = 13.6

R = L – S = 61.4 – 13.6 = 47.8

Co-efficient of range $= \dfrac{L-S}{L+S} = \dfrac{47.8}{75} = 0.64$

2. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value. **[Hy. - 2019]**

Sol. Given range = 36.8 and

the smallest value = 13.4 (S)

the largest value = L = R + S [$\because$ Range = L – S]

= 36.8 + 13.4 = 50.2

3. Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Sol. Here the largest value = 650

The smallest value = 400

$\therefore$ Range = L – S = 650 – 400 = 250

4. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32, 35, 37, 30, 33, 36, 35 and 37 pages. Find the standard deviation of the pages completed by them.

Sol. $\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{275}{8} = 34.3$

x_i	x_i^2
32	1020
35	1225
37	1369
30	900
33	1089
36	1296
35	1225
37	1369
$\Sigma x_i = 275$	$\Sigma x_i^2 = 9497$

Standard deviation: $\sigma = \sqrt{\dfrac{\Sigma x_i^2}{n} - \left(\dfrac{\Sigma x_i}{n}\right)^2}$

$= \sqrt{\dfrac{9497}{8} - \left(\dfrac{275}{8}\right)^2} = \sqrt{\dfrac{9497\times 8 - 275\times 275}{8\times 8}}$

$= \sqrt{\dfrac{75976-75625}{64}} = \sqrt{\dfrac{351}{64}}$ $\qquad \sigma = \sqrt{5.48}$

$\therefore$ Standard deviation σ = 2.34

5. Find the variance and standard deviation of the wages of 9 workers given below:
₹ 310, ₹ 290, ₹ 320, ₹ 280, ₹ 300, ₹ 290, ₹ 320, ₹ 310, ₹ 280.

Sol. Arrange in ascending order we get,
280, 280, 290, 290, 300, 310, 310, 320 and 320

$\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{2700}{9} = 300$

x	$d = x - \bar{x}$	d^2
280	–20	400
280	–20	400
290	–10	100
290	–10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
$\Sigma x = 2700$	0	$\Sigma d^2 = 2000$

Variance $= \dfrac{\Sigma d^2}{n} = \dfrac{2000}{9} = 222.22$

Standard deviation σ $= \sqrt{\dfrac{\Sigma d^2}{n}} = \sqrt{222.22} = 14.91$

6. A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

Sol.

x	$d = x - \bar{x}$	d^2
$(1 + 1) = 2$	–11	121
$(2 + 2) = 4$	–9	81
$(3 + 3) = 6$	–7	49
$(4 + 4) = 8$	–5	25
$(5 + 5) = 10$	–3	9
$(6 + 6) = 12$	–1	1
$(7 + 7) = 14$	1	1
$(8 + 8) = 16$	3	9
$(9 + 9) = 18$	5	25
$(10 + 10) = 20$	7	49
$(11 + 11) = 22$	9	81
$(12 + 12) = 24$	11	121
156		572

$$\bar{x} = \frac{\Sigma x}{n} = 13$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{572}{12}} = \sqrt{47.66} \simeq 6.9$$

7. Find the standard deviation of first 21 natural numbers. **[PTA - 6; June - 2023]**

Sol. Standard deviation of first n natural number is

$$\sigma = \sqrt{\frac{n^2 - 1}{12}}$$

∴ Standard deviation of first 21 natural numbers

$$\sigma = \sqrt{\frac{21^2 - 1}{12}} = \sqrt{\frac{441 - 1}{12}} = \sqrt{\frac{440}{12}} = \sqrt{36.67}$$

$$\sigma = 6.06$$

∴ Standard deviation of first 21 natural numbers $\sigma = 6.06$

8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Sol. If the standard deviation of a data is 4.5 and each value of the data decreased by 5, the new standard deviation does not change and it is also 4.5.

9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation. **[PTA - 1]**

Sol. If the standard deviation of a data is 3.6, and each of the data is divided by 3 then the new standard deviation is also divided by 3.

∴ The new standard deviation $= \frac{3.6}{3} = 1.2$

The new variance = (standard deviation)^2

$= \sigma^2 = 1.2^2 = 1.44$

10. The rainfall recorded in various places of five districts in a week are given below. Find its standard deviation.

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

Sol.

Rainfall x_i (mm)	No. of places f_i	$f_i x_i$	$d = x - \bar{x}$	d_i^2	$f_i d_i^2$
45	5	225	–11	121	605
50	13	650	–6	36	468
55	4	220	–1	1	4
60	9	540	4	16	144
65	5	325	9	81	405
70	4	280	14	196	784
	N = 40	$\Sigma x_i f_i = 2240$			$\Sigma f_i d_i^2 = 2410$

mean, $\bar{x} = \frac{\Sigma x_i f_i}{N} = \frac{2240}{40} = 56$

∴ Standard deviation

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}} = \sqrt{\frac{2410}{40}} = \sqrt{60.25} = 7.76$$

11. In a study about viral fever, the number of people affected in a town were noted as. Find its standard deviation.

Age in years	0 - 10	10-20	20-30	30-40	40-50	50-60	60-70
Number of people affected	3	5	16	18	12	7	4

Sol. Let the assumed mean A = 35, C = 10

Age (X)	No. of people affected f_i	Mid (value) x_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i d_i$	d_i^2	$f_i d_i^2$
0–10	3	5	–30	–3	–9	9	27
10–20	5	15	–20	–2	–10	4	20
20–30	16	25	–10	–1	–16	1	16
30–40	18	35	0	0	0	0	0
40–50	12	45	10	1	12	1	12
50–60	7	55	20	2	14	4	28
60–70	4	65	30	3	12	9	36
	N = 65				$\Sigma x_i f_i = 3$	139	

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$= 10 \times \sqrt{\frac{139}{65} - \left(\frac{3}{65}\right)^2} = 10 \times \sqrt{2.138 - (0.046)^2}$

$= 10 \times \sqrt{2.138 - 0.002116} = 10 \times \sqrt{2.136}$

$= 10 \times 1.46 = 14.6$ $\therefore$ Standard deviation = 14.6

12. The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter (cm)	21-24	25-28	29-32	33-36	37-40	41-44
Number of plates	15	18	20	16	8	7

Sol. Assumed mean A = 30.5, C = 4

Diameter class interval X	Mid value x_i	f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i d_i$	d_i^2	$f_i d_i^2$
20.5–24.5	22.5	15	–8	–2	–30	4	60
24.5–28.5	26.5	18	–4	–1	–18	1	18
28.5–32.5	30.5	20	0	0	0	0	0
32.5–36.5	34.5	16	4	1	16	1	16
36.5–40.5	38.5	8	8	2	16	4	32
40.5–44.5	42.5	7	12	3	21	9	63
		N = 84			5		189

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$= 4 \times \sqrt{\frac{189}{84} - \left(\frac{5}{84}\right)^2} = 4 \times \sqrt{2.25 - (0.059)^2}$

$= 4 \times \sqrt{2.25 - 0.0035} = 4 \times \sqrt{2.2465} = 4 \times 1.498$

$\cong 5.99 = 6$

13. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation. **[PTA - 5]**

Time taken (sec)	8.5-9.5	9.5-10.5	10.5-11.5	11.5-12.5	12.5-13.5
Number of students	6	8	17	10	9

Sol. Assumed mean A = 11, C = 1

Time Taken X	Mid value x_i	No. of Students f_i	$d_i = x_i - A$	$d_i = \frac{x_i - A}{C}$	$f_i d_i$	d_i^2	$f_i d_i^2$
8.5–9.5	9	6	–2	–2	–12	4	24
9.5–10.5	10	8	–1	–1	–8	1	8
10.5–11.5	11	17	0	0	0	0	0
11.5 – 12.5	12	10	1	1	10	1	10
12.5 – 13.5	13	9	2	2	18	4	36
		N = 50			8		78

Standard deviation $\sigma = c \times \sqrt{\frac{\Sigma f_i d^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

$= 1 \times \sqrt{\frac{78}{50} - \left(\frac{8}{50}\right)^2} = 1 \times \sqrt{1.56 - (0.16)^2}$

$= 1 \times \sqrt{1.56 - 0.0256} = 1 \times \sqrt{1.534} = 1 \times 1.238$

$= 1.238 = 1.24$

14. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the correct mean and standard deviation.

Sol. Given $n = 100, \bar{x} = 60, \sigma = 15$

$\bar{x} = \frac{\Sigma x}{n} \Rightarrow \Sigma x = \bar{x} \times n$

$\therefore \quad \Sigma x = \bar{x} \times n = 60 \times 100 = 6000$

Correct Σx = Incorrect Σx – Wrong value + Correct value

Correct $\Sigma x = 6000 + 45 + 72 - 40 - 27 = 6117 - 67$

Correct $\Sigma x = 6050$

$n = 100$

Correct $\bar{x}$ = Correct $\frac{\Sigma x}{n} = \frac{6050}{100} = 60.5$

Standard deviation $\sigma = \sqrt{\left(\frac{\Sigma x^2}{n}\right) - \left(\frac{\Sigma x}{n}\right)^2}$

Incorrect value of $\sigma = 15 = \sqrt{\frac{\Sigma x^2}{100} - 60^2}$

Squaring, $225 = \frac{\Sigma x^2}{100} - 3600$ gives,

$$\Rightarrow \frac{\Sigma x^2}{100} = 3600 + 225 \Rightarrow \frac{\Sigma x^2}{100} = 3825$$

$$\Sigma x^2 = 382500$$

Correct Σx^2 = Incorrect Σx^2 + Correct value –Wrong value

Correct value of $\Sigma x^2 = 382500 + 45^2 + 72^2 - 40^2 - 27^2$

$= 382500 + 2025 + 5184 - 1600 - 729$

$= 389709 - 2329 = 387380$

Correct standard deviation

$$\sigma = \sqrt{\frac{387380}{100} - (60.5)^2} = \sqrt{3873.80 - 3660.25} = \sqrt{213.55} = 14.61$$

15. The mean and variance of seven observations are 8 and 16 respectively. If five of these are 2, 4, 10, 12 and 14, then find the remaining two observations.

Sol. Given $\overline{x} = 8$, $\sigma^2 = 16$, $n = 7$. If five of these are 2, 4, 10, 12, 14.

Let the remaining observations be a and b.

$$\Rightarrow \overline{x} = \frac{\Sigma x}{n} = \frac{2+4+10+12+14+a+b}{7}$$

$$\Rightarrow 8 = \frac{42+a+b}{7} \quad [\because \overline{x} = 8, n = 7]$$

$$42 + a + b = 56$$

$$a + b = 56 - 42 = 14$$

The given 5 number are 2, 4, $\underline{a}$, b, 10, 12, 14.

$\Rightarrow$ a, b are 6 and 8.

[$\because$ For other numbers, the common difference is 2. Hence $a = 6$, $b = 8$]

EXERCISE 8.2

1. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation. **[Govt. MQP - 2019; Qy. - 2019]**

Sol. Co-efficient of variation C.V = $\frac{\sigma}{\overline{x}} \times 100$

Given $\sigma = 6.5$, $\overline{x} = 12.5$

$$\therefore \text{C.V} = \frac{6.5}{12.5} \times 100\% = 52\%.$$

2. The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Sol. Given $\sigma = 1.2$, C.V. = 25.6, $\overline{x}$ = ?

$$\text{C.V} = \frac{\sigma}{\overline{x}} \times 100\% \Rightarrow \overline{x} = \frac{\sigma}{\text{C.V.}} \times 100\%$$

$$\overline{x} = \frac{1.2}{25.6} \times 100 = 4.687 = 4.69$$

3. If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Sol. Given $\overline{x} = 15$, C.V. = 48, σ = ?

$$\text{C.V} = \frac{\sigma}{\overline{x}} \times 100$$

$$\Rightarrow \sigma = \frac{\text{C.V.} \times \overline{x}}{100} = \frac{48 \times 15}{100} = 7.2$$

4. If $n = 5$, $\overline{x} = 6$, $\Sigma x^2 = 765$, then calculate the coefficient of variation.

Sol. Given $n = 5$, $\overline{x} = 6$, $\Sigma x^2 = 765$, C.V = ?

$$\sigma = \sqrt{\left(\frac{\Sigma x^2}{n}\right) - \left(\frac{\Sigma x}{n}\right)^2} = \sqrt{\frac{765}{5} - 6^2} \quad \left[\because x = \frac{\Sigma x}{n} = 6\right]$$

$$= \sqrt{153 - 36} = 10.82$$

$$\text{C.V} = \frac{10.82}{6} \times 100\% = 180.28\%$$

5. Find the coefficient of variation of 24, 26, 33, 37, 29, 31. **[Govt. MQP - 2019; June-2023]**

Sol.

x	$d = x - \overline{x}$	d^2
24	–6	36
26	–4	16
33	3	9
37	7	49
29	–1	1
31	1	1
180	$\Sigma d = 0$	112

$$\overline{x} = \frac{\Sigma x}{n} = \frac{180}{6} = 30$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{112}{6}} = \sqrt{18.66} = 4.32$$

$\therefore$ Co-efficient of variation C.V = $\frac{\sigma}{x} \times 100$

$$\text{C.V} = \frac{4.32}{30} \times 100\% = 14.4\%$$

6. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation. **[Qy. - 2019]**

Sol. Mean $= \overline{x} = \frac{\Sigma x}{n}$

$$= \frac{38+40+47+44+46+43+49+53}{8}$$

$$= \frac{360}{8} = 45 \Rightarrow \overline{x} = 45$$

x	$d = x - \bar{x}$	d^2
38	–7	49
40	–5	25
47	2	4
44	–1	1
46	1	1
43	–2	4
49	4	16
53	8	64
360	0	164

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{164}{8}} = \sqrt{20.5} = 4.53$$

Co-efficient of variation

$$\text{C.V.} = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.53}{45} \times 100 = 10.07\%$$

7. The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?

Sol.

Sathya	Vidhya
Given $\Sigma x_1 = 460$	Given $\Sigma x_2 = 480$
$\sigma_1 = 4.6$	$\sigma_2 = 2.4$
$\bar{x}_1 = \frac{\Sigma x_1}{n} = \frac{460}{5}$	$\bar{x}_2 = \frac{\Sigma x_2}{n} = \frac{480}{5}$
$= 92$	$= 96$
$\therefore \text{C.V}_1 = \frac{4.6}{92} \times 100$	$\text{C.V}_2 = \frac{2.4}{96} \times 100$
$= 5\%$	$= 2.5\%$

C.V of Vidhya is less than C.V. of Sathya.

$\therefore$ Vidhya is more consistent

8. The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below.

Subject	Mean	SD
Mathematics	**56**	**12**
Science	**65**	**14**
Social Science	**60**	**10**

Which of the three subjects shows more consistent and which shows less consistent in marks?

Sol. **For Maths**

Given $\bar{x} = 56, \sigma = 12$

$$\therefore \text{C.V for Maths} = \frac{\sigma}{\bar{x}} \times 100 = \frac{12}{56} \times 100$$

$$= 21.43\% \qquad \text{... (1)}$$

For Science

Given that $\bar{x} = 65, \sigma = 14$

$$\therefore \text{C.V for Science} = \frac{\sigma}{\bar{x}} \times 100 = \frac{14}{65} \times 100$$

$$= 21.54\% \qquad \text{... (2)}$$

For Social Science

Given that $\bar{x} = 60, \sigma = 10$

$$\therefore \text{C.V for Social Science} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{10}{6\cancel{0}} \times 10\cancel{0} = \frac{100}{6}$$

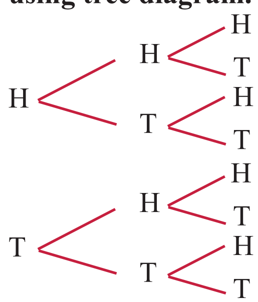
$$= 16.67\% \qquad \text{... (3)}$$

More consistent marks in science and less consistent marks in social.

EXERCISE 8.3

1. Write the sample space for tossing three coins using tree diagram.

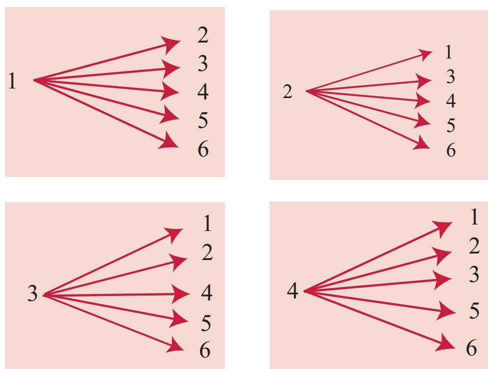
Sol.

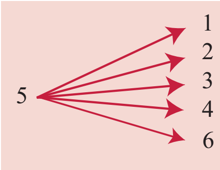


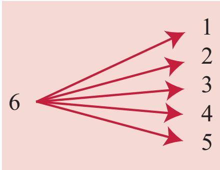
$\therefore$ S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

2. Write the sample space for selecting two balls at a time from a bag containing 6 balls numbered 1 to 6 (using tree diagram). **[PTA - 4]**

Sol.







$\therefore$ S = {(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)}

$n(S) = 30$

3. If A is an event of a random experiment such that P(A) : P($\bar{A}$) =17:15 and n(S) = 640 then find (i) P ($\bar{A}$) [PTA - 3] (ii) n(A).

Sol. $P(A) : P(\bar{A}) = 17:15$

(i) $P(A) = \frac{17}{32}, P(\bar{A}) = \frac{15}{32}$ [$\because$ 17+ 15 = 32]

(ii) $P(A) = \frac{17}{32} = \frac{17\times 20}{32\times 20} = \frac{340}{640}$

[$\because n(s) = 640$]

$\Rightarrow \quad n(A) = 340 \quad [\because P(A) = \frac{n(A)}{n(S)}]$

4. A coin is tossed thrice. What is the probability of getting two consecutive tails?

Sol. Sample space S = {(HHH),(THH),(HTH),(HHT), (HTT), (THT), (TTH), (TTT)}

$\Rightarrow \quad n(S) = 8$

Let A be the event of getting 2 consecutive tails

$\therefore$ A = {HTT, TTH, TTT}

$\Rightarrow \quad n(A) = 3$

$\therefore P = \frac{n(A)}{n(S)} = \frac{3}{8}$

5. At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?

Sol. The cards are bearing numbers from 1 to 1000.

S = { 1, 2, 3, ... 1000} $\Rightarrow n$ (S) = 1000

Let A be the event of getting a square number greater than 500.

$\therefore$ Since $23^2 = 529$, $24 = 576$ ... $31^2 = 964$, $32^2 = 1024 \notin$ S.

$\therefore$ A = {23, 24, 25, 26, 27, 28, 29, 30, 31}

$\Rightarrow n$ (A) = 9

(i) $\therefore$ Probability of first player wins a prize, is

$P(A) = \frac{n(A)}{n(S)} = \frac{9}{1000}$

(ii) The first player has already won a prize, with a particular card and the card is not replaced

$\therefore n(B) = 9 - 1 = 8$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{8}{999}$

[$\because n$ (S) = 1000 – 1 = 999]

6. A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x.

Sol. Since the bag contains 12 blue balls and x red balls

Sample space S = {12 blue balls, x red balls]

$\Rightarrow \quad n(S) = x + 12$

(i) Let A be the event of getting a red ball

$\Rightarrow \quad n(A) = x$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{x}{x+12}$ ... (1)

(ii) Since 8 red balls are added to the bag,

$n(S) = x + 12 + 8 = x + 20$

Let B be the event of getting red ball

$\Rightarrow \quad n(B) = x + 8$

Probability of red ball

$P(B) = \frac{n(B)}{n(S)} = \frac{x+8}{x+20}$ ... (2)

Given that $P(B) = 2\,P(A)$

$\Rightarrow \quad \frac{x+8}{x+20} = 2\times\frac{x}{x+12} = \frac{2x}{x+12}$

$\Rightarrow \quad (x + 8)(x + 12) = 2x(x + 20)$

[By cross multiplication]

$\Rightarrow x^2 + 12x + 8x + 96 = 2x^2 + 40x$

$\Rightarrow \quad x^2 + 20x + 96 = 2x^2 + 40x$

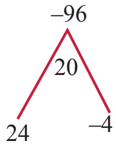
$\Rightarrow \quad 2x^2 + 40x - x^2 - 20x - 96 = 0$

$\Rightarrow \quad x^2 + 20x - 96 = 0$

$\Rightarrow \quad (x+24)(x-4) = 0$

$\Rightarrow \quad x = -24$ or

$x = 4$



Since x cannot be negative,

$\boxed{x = 4}$

Substituting $x = 4$ in (1) we get,

$$\text{P (A)} = \frac{4}{4+12} = \frac{4}{16} = \frac{1}{4}$$

7. Two unbiased dice are rolled once. Find the probability of getting

[Sep. - 2020; Aug. - 2022; April - 2023]

(i) a doublet (equal numbers on both dice)
(ii) the product as a prime number
(iii) the sum as a prime number
(iv) the sum as 1

Sol. Two unbiased dice are rolled

$\Rightarrow$ Sample Space (S) = {(1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) .... (6,6)}

$\Rightarrow \quad n(\text{S}) = 36$

(i) Let A be the event of getting a doublet

$\therefore$ A = {(1,1), (2,2) (3,3) (4,4) (5,5) (6,6)}

$\Rightarrow \quad n(\text{A}) = 6$

$$\therefore \text{P (A)} = \frac{n(\text{A})}{n(\text{S})} = \frac{6}{36} = \frac{1}{6}$$

(ii) Let B be the event of getting the product as a prime number.

$\therefore$ B = {(1, 2) (2, 1) (1, 3)(3, 1)(1, 5)(5, 1)}

$\Rightarrow \quad n(\text{B}) = 6$

$$\therefore \text{P (B)} = \frac{n(\text{B})}{n(\text{S})} = \frac{6}{36} = \frac{1}{6}$$

(iii) Let C be the event of getting the sum as a prime number.

$\therefore$ C = {(1, 1) (1, 2) (1, 4), (1, 6) (2,1) (2,3) (2,5) (3,2) (3, 4) (4,1) (4, 3), (5, 2) (5, 6) (6, 1) (6,5)}

$\Rightarrow \quad n(\text{C}) = 15$

$$\therefore \text{P (C)} = \frac{n(\text{C})}{n(\text{S})} = \frac{15}{36} = \frac{5}{12}$$

(iv) Let D be the event of getting the sum as 1

$\therefore$ D = { }

[No pair is having summation as 1]

$\Rightarrow \quad n(\text{D}) = 0$

$$\therefore \text{P (D)} = \frac{n(\text{D})}{n(\text{S})} = \frac{0}{36} = 0$$

8. Three fair coins are tossed together. Find the probability of getting

(i) all heads
(ii) atleast one tail **[PTA - 5]**
(iii) atmost one head **[PTA - 5]**
(iv) atmost two tails

Sol. When 3 fair coins are tossed,

Sample space S = {HHH, HHT, HTH, THH, HTT, TTH, THT, TTT}

$\Rightarrow \quad n(\text{S}) = 8$

(i) Let A be the event of getting all heads

$\therefore$ A = {HHH}

$\Rightarrow \quad n(\text{A}) = 1$

$$\therefore \text{P (A)} = \frac{n(\text{A})}{n(\text{S})} = \frac{1}{8}$$

(ii) Let B be the event of getting atleast one tail

$\therefore$ B = {THH, HTH, HHT, TTH, THT, HTT, TTT}

$\Rightarrow \quad n(\text{B}) = 7$ [atleast one tail $\Rightarrow$ 1 tail, or 2 tails or 3 tails]

$$\therefore \text{P (B)} = \frac{n(\text{B})}{n(\text{S})} = \frac{7}{8}$$

(iii) Let C be the event of getting atmost one head

$\therefore$ C = {HTT, THT, TTH, TTT}

[atmost one head means one head or no head]

$\Rightarrow \quad n(\text{C}) = 4$

$$\therefore \text{P (C)} = \frac{n(\text{C})}{n(\text{S})} = \frac{4}{8} = \frac{1}{2}$$

(iv) Let D be the event of getting atmost two tails

$\therefore$ D = {HHH, THH, HTH, HHT, TTH, THT, HTT}

[atmost 2 tails means 2 tails or 1 tail or no tail]

$\Rightarrow \quad n(\text{D}) = 7$

$$\therefore \text{P (D)} = \frac{n(\text{D})}{n(\text{S})} = \frac{7}{8}$$

9. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black

Sol. Sample space = { 5 red, 6 white, 7 green, 8 black balls}

$\therefore n(\text{S}) = 26$

Answers

SECTION - A

1. (C) Arithmetic mean

2. (A) 3.5 3. (B) $\frac{7}{10}$

4. (C) 15 5. (D) $\frac{4}{5}$

SECTION - B

1. Refer Sura's Guide Exercise 8.1, Q.No.6
2. Refer Sura's Guide Exercise 8.2, Q.No.6
3. Refer Sura's Guide Exercise 8.3, Q.No.4
4. Refer Sura's Guide unit Exercise Q.No.4
5. Refer Sura's Guide unit Exercise Q.No.8

SECTION - C

1. Refer Sura's Guide Exercise No.8.1, Q.No.11
2. Refer Sura's Guide Unit Exercise Q.No.3

PUBLIC EXAMINATION - APRIL 2024

Reg. No.

PART - III

Mathematics (With Answers)

Time Allowed : 3.00 Hours] **[Maximum Marks : 100**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

Note : This question paper contains **four** parts.

Part - I

Note : (i) Answer **all** the questions. **14 × 1 = 14**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$, then $n(B)$ is :

(a) 1 (b) 2 (c) 3 (d) 6

2. If $f : A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to :

(a) 7 (b) 49 (c) 1 (d) 14

3. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is :

(a) 2025 (b) 5220 (c) 5025 (d) 2520

4. An A.P. consists of 31 terms. If its 16^{th} term is m, then the sum of all the terms of this A.P. is :

(a) $16\,m$ (b) $62\,m$ (c) $31\,m$ (d) $\frac{31}{2}m$

5. Which of the following should be added to make $x^4 + 64$ a perfect square?

(a) $4x^2$ (b) $16x^2$ (c) $8x^2$ (d) $-8x^2$

6. Graph of a linear equation is a ________ .

(a) straight line (b) circle

(c) parabola (d) hyperbola

7. If in ΔABC, $DE \parallel BC$, $AB = 3.6$ cm, $AC = 2.4$ cm and $AD = 2.1$ cm then the length of AE is :

(a) 1.4 cm (b) 1.8 cm

(c) 1.2 cm (d) 1.05 cm

8. How many tangents can be drawn to the circle from an exterior point?

(a) One (b) Two

(c) Infinite (d) Zero

9. The area of triangle formed by the points $(-5, 0)$, $(0, -5)$ and $(5, 0)$ is :

(a) 0 sq. units (b) 25 sq. units

(c) 5 sq. units (d) 10 sq. units

10. If $x = a\tan\theta$ and $y = b\sec\theta$, then :

(a) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (d) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

11. The curved surface area of a right circular cylinder of height 4 cm and base diameter 10 cm is :

(a) $40\,\pi$ sq. units (b) $20\,\pi$ sq. units

(c) $14\,\pi$ sq. units (d) $80\,\pi$ sq. units

12. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is :

(a) 1 : 2 : 3 (b) 2 : 1 : 3

(c) 1 : 3 : 2 (d) 3 : 1 : 2

13. Which of the following values cannot be a probability of an event?

(a) 0 (b) 0.5 (c) 1.05 (d) 1

14. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$, then the value of x is :

(a) 2 (b) 1 (c) 3 (d) 1.5

Part - II

Note : Answer **any 10** questions. Question No.**28** is **compulsory**. **10 × 2 = 20**

15. If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$ then find A and B.

16. If $f(x) = 3x - 2$, $g(x) = 2x + k$ and $fog = gof$, then find the value of k.

17. 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

18. Simplify : $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4}$

19. Find the sum and product of the roots for following quadratic equation. $x^2 + 8x - 65 = 0$

20. A man goes 18 m due East and then 24 *m* due North. Find the distance of his current position from the starting point.

21. If the points A (–3, 9), B(a, b) and C(4, –5) are collinear and if $a + b = 1$, then find a and b.

22. Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point (–1, 2).

23. Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$.

24. If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area.

25. Find the volume of cylinder whose height is 2 *m* and base area is 250 sq. m.

26. Find the range and coefficient of range of the following data :
25, 67, 48, 53, 18, 39, 44

27. What is the probability that a leap year selected at random will contain 53 Saturdays?

28. Find the HCF of 23 and 12.

Part - III

Note : Answer **any 10** questions. Question No.**42** is **compulsory**. **10 × 5 = 50**

29. Let A = $\{x \in N \mid 1 < x < 4\}$, B = $\{x \in W \mid 0 \le x < 2\}$ and C = $\{x \in N \mid x < 3\}$. Then verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

30. Let A = {0, 1, 2, 3} and B = {1, 3, 5, 7, 9} be two sets. Let $f : A \to B$ be a function given by $f(x) = 2x + 1$. Represent this function
(i) by arrow diagram (ii) in a table form
(iii) as a set of ordered pairs
(iv) in a graphical form

31. Find the sum of $9^3 + 10^3 + \ldots\ldots\ldots\ldots\ldots 21^3$

32. Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$.

33. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I_2 = 0$.

34. State and prove Thales Theorem.

35. Find the area of quadrilateral whose vertices are at (–9, –2), (–8, –4), (2, 2) and (1, –3).

36. Find the equation of the perpendicular bisector of the line joining the points A(–4, 2) and B(6, –4).

37. Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200m high, find the distance between the two ships. ($\sqrt{3} = 1.732$)

38. If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

39. A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container.

40. Find the coefficient of variation of 24, 26, 33, 37, 29, 31.

41. Two dice are rollled once. Find the probability of getting an even number on the first die or the total of face sum 8.

42. Find the sum to n terms of the series 7 + 77 + 777 + ...

Part - IV

Note : Answer **all** the questions. **2 × 8 = 16**

43. (a) Construct a ΔPQR which the base PQ = 4.5 cm, $\angle R = 35°$ and the median RG from R to PQ is 6 cm. **(OR)**

(b) Draw a circle of diameter 6 cm. from a point P, which is 8 cm. away from its centre. Draw the two tangents. PA and PB to the circle and measure their lengths.

44. (a) Draw the graph of $y = 2x^2 - 3x - 5$ and hence solve $2x^2 - 4x - 6 = 0$. **(OR)**

(b) Draw the graph of $xy = 24$, $x, y > 0$. Using the graph find,
(i) y when $x = 3$ and (ii) x when $y = 6$.

ANSWERS

Part - I

1. (c)	3	**2.** (a)	7
3. (d)	2520	**4.** (c)	31 m
5. (b)	$16x^2$	**6.** (a)	straight line
7. (a)	1.4 cm	**8.** (b)	Two
9. (b)	25 sq. units	**10.** (a)	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
11. (a)	40 π sq. units	**12.** (d)	3 : 1 : 2
13. (c)	1.05	**14.** (b)	1

Part - II

15. $A \times B$ = {(3,2), (3,4), (5,2), (5,4)}
We have A = {set of all first coordinates of elements of $A \times B$}. ∴ A = {3, 5}
B = {set of all second coordinates of elements of $A \times B$}. ∴ B = {2,4}.
Thus A = {3,5} and B = {2,4}.

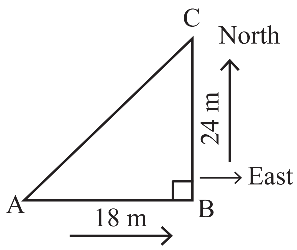
16.
$$\begin{aligned} f(x) &= 3x-2,\ g(x)=2x+k \\ fog(x) &= f(g(x)) = f(2x+k) \\ &= 3(2x+k)-2 = 6x+3k-2 \\ \text{Thus,}\quad fog(x) &= 6x+3k-2 \\ gof(x) &= g(3x-2) = 2(3x-2)+k \\ \text{Thus,}\quad gof(x) &= 6x-4+k \\ \text{Given that } fog &= gof \\ \therefore\quad 6x+3k-2 &= 6x-4+k \\ 6x-6x+3k-k &= -4+2 \Rightarrow k=-1 \end{aligned}$$

17. The number 800 can be factorized as
$800 = 2\times 2\times 2\times 2\times 2\times 5\times 5 = 2^5\times 5^2$
Hence, $a^b\times b^a = 2^5\times 5^2$
This implies that $a = 2$ and $b = 5$ (or) $a = 5$ and $b = 2$

18.
$$\frac{\overset{2}{\cancel{4}}\,x^2y}{\cancel{2}\,z^2}\times\frac{\overset{3}{\cancel{6}}\,xz^3}{\underset{\underset{5}{\cancel{10}}}{\cancel{20}}\,y^4} = \frac{3x^3yz^3}{5y^4z^2} = \frac{3x^3z}{5y^3}$$

19. Let α and β be the roots of the given quadratic equation
$x^2+8x-65=0$
$a = 1,\ b = 8,\ c = -65$
$\alpha+\beta = -\dfrac{b}{a} = -8$ and $\alpha\beta = \dfrac{c}{a} = -65$
$\alpha+\beta = -8;\ \ \alpha\beta = -65$

20. Let A be the position of the man.



Using Pythagoras theorem
$$\begin{aligned} AC^2 &= AB^2+BC^2 \quad [\because AB = 18 \text{ and } BC = 24] \\ &= (18)^2+(24)^2 = 324+576 = 900 \\ AC &= \sqrt{900} = 30 \text{ m} \end{aligned}$$
∴ The distance from the starting point is 30 m.

21.
$$\begin{matrix} A(-3,9), & B(a,b) & C(4,-5), \\ \downarrow & \downarrow & \downarrow \\ (x_1,y_1) & (x_2,y_2) & (x_3,y_3) \end{matrix}$$
are collinear points, $a+b = 1$ (given)
$$\therefore \text{Area of the } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix} \text{ sq. units}$$
$$= \frac{1}{2}\begin{vmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{vmatrix} = 0 (\because \text{ points are collinear})$$
$$\begin{aligned} (-3b-5a+36)-(9a+4b+15) &= 0 \\ (-3b-4b)+(-5a-9a)+(36-15) &= 0 \\ -7b-14a &= -21 \\ -7(b+2a) &= -21 \\ b+2a &= 3 \\ (b+a)+a &= 3 \\ 1+a &= 3 \\ =a=2 \Rightarrow b = 1-2 &= -1 \\ a &= 2 \\ b &= -1 \end{aligned}$$

22. $m = \dfrac{-5}{4}$, point $= (-1, 2) = (x_1, y_1)$
$$\begin{aligned} \Rightarrow\quad y-2 &= \frac{-5}{4}(x-(-1)) \\ & \qquad [\because y-y_1 = m(x-x_1)] \\ \Rightarrow\quad y-2 &= \frac{-5}{4}(x+1) \\ \Rightarrow\quad 4(y-2) &= -5(x+1) \\ \Rightarrow\quad 4y-8 &= -5x-5 \\ \Rightarrow\quad 5x+4y &= 3 \Rightarrow 5x+4y-3 = 0 \end{aligned}$$

23.
$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$$
[multiply numerator and denominator by the conjugate of $1-\cos\theta$]
$$\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \frac{1+\cos\theta}{\sqrt{\sin^2\theta}} \quad [\because \sin^2\theta+\cos^2\theta = 1]$$
$$= \frac{1+\cos\theta}{\sin\theta} = \text{cose}\,\theta + \cot\theta$$

24. Let r be the radius of the hemisphere.
Given that, base area $= \pi r^2 = 1386$ sq. m
T.S.A. $= 3\pi r^2$ sq.m $= 3\times 1386 = 4158$
Therefore, T.S.A. of the hemispherical solid is 4158 m^2.

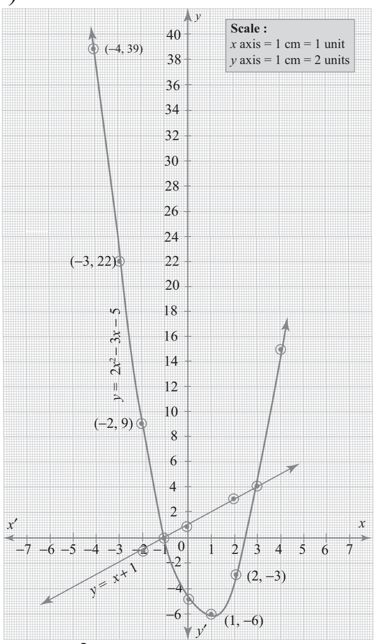
25. Let r and h be the radius and height of the cylinder respectively
Given that, height $h = 2$ m, base area $= 250$ m^2
Now, volume of a cylinder $= \pi r^2h$ cu. units
$= \textit{base area}\times h$
$= 250\times 2 = 500$ m^3
Therefore, volume of the cylinder $= 500$ m^3

26. Largest value $L = 67$; Smallest value $S = 18$
Range $R = L-S = 67-18 = 49$
$$\text{Coefficient of range} = \frac{L-S}{L+S}$$
$$\text{Coefficient of range} = \frac{67-18}{67+18} = \frac{49}{85} = 0.576$$

44. (a)

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
$-3x$	12	9	6	3	0	–3	–6	–9	–12
-5	–5	–5	–5	–5	–5	–5	–5	–5	–5
$y = 2x^2 - 3x - 5$	39	22	9	0	–5	–6	–3	4	15

Draw the parabola using the points (–4, 39), (–3, 22), (–2, 9), (–1, 0), (0, –5), (1, –6), (2, –3), (3, 4), (4, 15).



To solve $2x^2 - 4x - 6 = 0$, subtract it from $y = 2x^2 - 3x - 5$

$$\begin{aligned} y &= \cancel{2x^2} - 3x - 5 \\ 0 &= \cancel{2x^2} - 4x - 6 \\ & \quad (-) \quad (+) \qquad (+) \\ \hline y &= x + 1 \text{ is a straight line} \end{aligned}$$

x	–2	0	2
1	1	1	1
$y = x + 1$	–1	1	3

Draw a straight line using the points (–2, –1), (0, 1), (2, 3). The points of intersection of the parabola and the straight line forms the roots of the equation.

The x-coordinates of the points of intersection forms the solution set.

$\therefore$ Solution {–1, 3}

(OR)

(b)

x	1	2	3	4	6
y	24	12	8	6	4

From the table we observe that as x increases y decreases. This type of variation is called indirect variation.

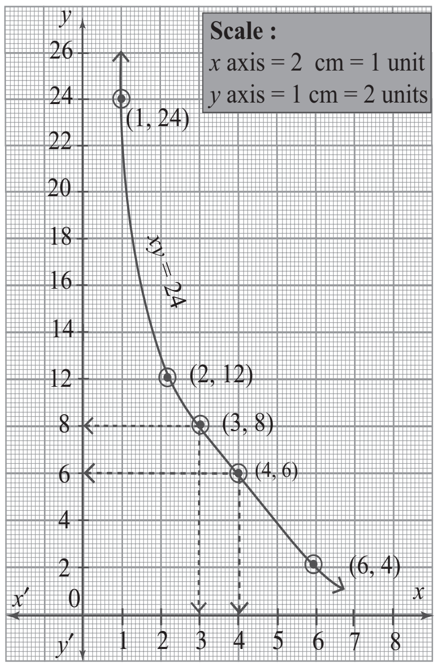
$y \alpha \frac{1}{x}$ or $xy = k$ where k is a constant of proportionality.

Also from the table we find that,

$1 \times 24 = 2 \times 12 = 3 \times 8 = 4 \times 6 = 6 \times 4 = 24 = k$.

$\therefore$ We get $k = 24$

Plot the points (1, 24), (2, 12), (3, 8), (4, 6) and (6, 4) and join them.



$\therefore$ The relation $xy = 24$ is a rectangular hyperbola as exhibited in the graph. From the graph, we find

(i) when $x = 3$, $y = 8$ (ii) when $y = 6$, $x = 4$

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