

UNION PUBLIC SERVICE COMMISSION

NDA - I – Mathematics

Duration : 2 hours 30 min.

Maximum Marks : 300

Solved Original Question Paper - 2025

Held on : 13.04.2025

1. The sum of the first k terms of a series S is $3k^2 + 5k$. Which one of the following is correct?

- A) The terms of S form an arithmetic progression with common difference 14.
 B) The terms of S form an arithmetic progression with common difference 6.
 C) The terms of S form a geometric progression with common ratio $10/7$.
 D) The terms of S form a geometric progression with common ratio $11/4$.

Explanation

We are given:

The sum of first k terms is

$$S_k = 3k^2 + 5k.$$

Step 1: Find a_k , the k -th term

$$a_k = S_k - S_{k-1}, S_0 = 0.$$

For $k \geq 1$:

$$\begin{aligned} ak &= [3k^2 + 5k] - [3(k-1)^2 + 5(k-1)] \\ &= 3k^2 + 5k - [3(k^2 - 2k + 1) + 5k - 5] \\ &= 3k^2 + 5k - [3k^2 - 6k + 3 + 5k - 5] \\ &= 3k^2 + 5k - [3k^2 - k - 2] \\ &= 3k^2 + 5k - 3k^2 + k + 2 \\ &= 6k + 2 \end{aligned}$$

Step 2: Check if it's an AP

$$a_1 = 6(1) + 2 = 8$$

$$a_2 = 14$$

$$a_3 = 20$$

Common difference $d = 14 - 8 = 6$ (constant).

So AP with $d = 6$.

Step 3: Match options

Option B: "arithmetic progression with common difference 6" $\rightarrow$ matches.

2. The sum of the first 8 terms of a GP is five times the sum of its first 4 terms. If $r \neq 1$ is the common ratio, then what is the number of possible real values of r ?

- A) One B) Two
 C) Three D) More than three

Explanation

Given

- A GP with common ratio $r \neq 1$
- Sum of first 8 terms = 5 times the sum of first 4 terms

Let first term = a

Step 1: Write sum formulas

For a GP ($r \neq 1$):

$$S_n = a \frac{1-r^n}{1-r}$$

So,

$$S_8 = a \frac{1-r^8}{1-r}$$

$$S_4 = a \frac{1-r^4}{1-r}$$

Step 2: Use the given condition

$$S_8 = 5S_4$$

$$a \frac{1-r^8}{1-r} = 5a \frac{1-r^4}{1-r}$$

Cancel a and $(1-r)$:

$$1-r^8 = 5(1-r^4)$$

Step 3: Simplify

$$1-r^8 = 5-5r^4$$

$$r^8-5r^4+4=0$$

Step 4: Substitute

$$\text{Let } x = r^4$$

$$x^2-5x+4=0$$

Factor:

$$(x-4)(x-1)=0$$

$$x=4 \text{ or } x=1$$

Step 5: Find values of r

Case 1: $r^4 = 4$

$$r = \pm\sqrt[4]{4}$$

(two real values)

Case 2: $r^4 = 1$

$$r = \pm 1$$

But $r \neq 1$ (given), so only:

$$r = -1$$

(one real value)

Step 6: Count real values

$$r = \sqrt{2}, -\sqrt{2}, -1$$

Total real values = 3

Final Answer : C) Three

3. If one root of the equation $x^2 - kx + k = 0$ exceeds the other by $2\sqrt{3}$, then which one of the following is a value of k ?

- A) 3 B) 6
 C) 9 D) 12

Explanation

Let roots be α, β with $\alpha - \beta = 2\sqrt{3}$

Given equation: $x^2 - kx + k = 0$.

Sum of roots: $\alpha + \beta = k$

Product: $\alpha\beta = k$.

Ans : (B)

We have:

$$\alpha + \beta = k$$

$$\alpha\beta = k$$

$$\alpha - \beta = 2\sqrt{3}$$

From first and third:

$$\text{Add: } 2\alpha = k + 2\sqrt{3} \Rightarrow \alpha = \frac{k + 2\sqrt{3}}{2}$$

$$\text{Subtract: } 2\beta = k - 2\sqrt{3} \Rightarrow \beta = \frac{k - 2\sqrt{3}}{2}$$

Product:

$$\alpha\beta = \frac{(k + 2\sqrt{3})(k - 2\sqrt{3})}{4} = \frac{k^2 - 12}{4}$$

But product = k.

So:

$$\frac{k^2 - 12}{4} = k$$

$$k^2 - 12 = 4k$$

$$k^2 - 4k - 12 = 0$$

$$(k - 6)(k + 2) = 0$$

$$k = 6 \text{ or } k = -2.$$

Given options: 3, 6, 9, 12 $\rightarrow k = 6$.

4. If $x + \frac{5}{y} = 4$ and $y + \frac{5}{x} = -4$, then what is $(x + y)$

equal to ?

A) 0

B) 1

C) 4

D) 5

Explanation

Given

$$x + \frac{5}{y} = 4 \text{ and } y + \frac{5}{x} = -4$$

We need to find $x + y$.

Step 1: Remove denominators

From the first equation

$$x + \frac{5}{y} = 4$$

Multiply by y:

$$xy + 5 = 4y$$

$$xy - 4y + 5 = 0$$

From the second equation

$$y + \frac{5}{x} = -4$$

Multiply by x:

$$xy + 5 = -4x$$

$$xy + 4x + 5 = 0$$

Step 2: Subtract (1) from (2)

$$(xy + 4x + 5) - (xy - 4y + 5) = 0$$

$$4x + 4y = 0$$

$$x + y = 0$$

Final Answer : A) 0

5. If 5th, 7th and 13th terms of an AP are in GP, then what is the ratio of its first term to its common difference?

A) -3

B) -2

C) 2

D) 3

Explanation

Given

In an AP, the 5th, 7th and 13th terms are in GP.

Let:

• First term = a

• Common difference = d

Step 1: Write the required AP terms

• 5th term:

$$T_5 = a + 4d$$

• 7th term:

$$T_7 = a + 6d$$

• 13th term:

$$T_{13} = a + 12d$$

Step 2: Condition for GP

If three terms x, y, z are in GP, then:

$$y^2 = xz$$

So,

$$(a + 6d)^2 = (a + 4d)(a + 12d)$$

Step 3: Expand both sides

Left side:

$$(a + 6d)^2 = a^2 + 12ad + 36d^2$$

Right side:

$$(a + 4d)(a + 12d) = a^2 + 16ad + 48d^2$$

Step 4: Equate and simplify

$$a^2 + 12ad + 36d^2 = a^2 + 16ad + 48d^2$$

Cancel a²:

$$12ad + 36d^2 = 16ad + 48d^2$$

$$0 = 4ad + 12d^2$$

$$4d(a + 3d) = 0$$

Since AP has a common difference, $d \neq 0$:

$$a + 3d = 0 \Rightarrow a = -3d$$

Step 5: Find the required ratio

$$\frac{a}{d} = -3$$

Final Answer : A) -3

6. If p, 1, q are in AP and p, 2, q are in GP, then which of the following statements is/are correct?

I. p, 4, q are in HP.

II. (1/p), 1/4, (1/q) are in AP.

Select the answer using the code given below.

A) I only

B) II only

C) Both I and II

D) Neither I nor II

Explanation

Given

• p, 1, q are in AP

• p, 2, q are in GP

Ans : (A)

Ans : (A)

...(1)

...(2)

Ans : (C)

Check the two statements.

Step 1: Use the AP condition

For three numbers in AP:

$$2 \times \text{middle} = \text{first} + \text{third}$$

So,

$$2(1) = p + q \Rightarrow p + q = 2$$

Step 2: Use the GP condition

For three numbers in GP:

$$(\text{middle})^2 = (\text{first})(\text{third})$$

So,

$$22 = pq \Rightarrow pq = 4$$

Step 3: Solve for p and q

From (1) and (2), p and q are roots of:

$$x^2 - (p + q)x + pq = 0$$

$$x^2 - 2x + 4 = 0$$

Discriminant:

$$\Delta = 2^2 - 16 = -12 < 0$$

So p and q are complex conjugates, but the relations are enough to test the statements.

Check Statement I

I. p, 4, q are in HP

Three numbers are in HP if their reciprocals are in AP.

So check:

$$\frac{1}{p}, \frac{1}{4}, \frac{1}{q}$$

AP condition:

$$2\left(\frac{1}{4}\right) = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{2} = \frac{p+q}{pq}$$

Substitute $p + q = 2$ and $pq = 4$:

$$\frac{1}{2} = \frac{2}{4} = \frac{1}{2}$$

Statement I is TRUE

Check Statement II

II. (1/p), (1/4), (1/q) are in AP

Statement II is TRUE

Final Answer : C) Both I and II

7. If $x = (1111)_2$, $y = (1001)_2$ and $z = (110)_2$, then what is $x^3 - y^3 - z^3 - 3xyz$ equal to?

- A) $(1111001)_2$ B) $(1001111)_2$
C) $(1)_2$ D) $(0)_2$

Explanation

Given

$$x = (1111)_2, y = (1001)_2, z = (110)_2$$

We need:

$$x^3 - y^3 - z^3 - 3xyz$$

Step 1: Convert to decimal

$$x = (1111)_2 = 8 + 4 + 2 + 1 = 15$$

$$y = (1001)_2 = 8 + 1 = 9$$

...(1)

...(2)

$$z = (110)_2 = 4 + 2 = 6$$

Step 2: Use identity

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

But here:

$$x^3 - y^3 - z^3 - 3xyz$$

Rewrite as:

$$x^3 + (-y)^3 + (-z)^3 - 3x(-y)(-z)$$

Apply identity with $a = x$, $b = -y$, $c = -z$:

$$= (x - y - z)(x^2 + y^2 + z^2 + xy + xz - yz)$$

Step 3: Compute

$$x - y - z = 15 - 9 - 6 = 0$$

So whole expression = 0

Step 4: Convert back to binary

$$0 = (0)_2$$

Correct option: D

8. If

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

and A, B, C, D, G are the cofactors of the elements a, b, c, d, g respectively, then what is $bB + cC - dD - gG$ equal to?

- A) 0 B) 1
C) Δ D) $-\Delta$

Explanation

Ans : (A)

Let:

- A, B, C be cofactors of a, b, c
- D, G be cofactors of d, g

We need:

$$bB + cC - dD - gG$$

Step 1: Use determinant properties

Expansion along first row:

$$aA + bB + cC = \Delta$$

Expansion along first column:

$$aA + dD + gG = \Delta$$

Step 2: Subtract

$$(bB + cC) - (dD + gG) = (aA + bB + cC) - (aA + dD + gG) \\ = \Delta - \Delta = 0$$

Correct option: A

9. Consider the following statements in respect of the determinant

$$\Delta = \begin{vmatrix} k(k+2) & 2k+1 & 1 \\ 2k+1 & k+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

- I. Δ is positive if $k > 0$.
II. Δ is negative if $k < 0$.
III. Δ is zero if $k = 0$.

How many of the statements given above are correct?

- A) None B) One
C) Two D) All three

Explanation

Step 1: Evaluate the determinant

Expand along the third column (easy):

$$\Delta = 1 \begin{vmatrix} k(k+2) & 2k+1 \\ 2k+1 & k+2 \end{vmatrix} - 1 \begin{vmatrix} k(k+2) & 2k+1 \\ 3 & 3 \end{vmatrix} + 1 \begin{vmatrix} 2k+1 & k+2 \\ 3 & 3 \end{vmatrix}$$

Compute each:

$$1. k(k+2)(k+2) - (2k+1)^2 = k(k+2)^2 - (2k+1)^2$$

$$2. 3k(k+2) - 3(2k+1) = 3[k(k+2) - (2k+1)]$$

$$3. 3(2k+1) - 3(k+2) = 3(k-1)$$

Putting together and simplifying:

$$\Delta = k(k+2)^2 - (2k+1)^2 - 3[k(k+2) - (2k+1)] + 3(k-1)$$

After simplification:

$$\Delta = k^3$$

Step 2: Check statements

- I. $\Delta > 0$ if $k > 0 \rightarrow$ true
- II. $\Delta < 0$ if $k < 0 \rightarrow$ true
- III. $\Delta = 0$ if $k = 0 \rightarrow$ true

Answer : D) All three

10. If

$$\begin{vmatrix} 2 & 3+i & -1 \\ 3-i & 0 & i-1 \\ -1 & -1-i & 1 \end{vmatrix} = A + iB$$

where $i = \sqrt{-1}$, then what is $A + B$ equal to?

- A) -10
- B) -6
- C) 0
- D) 6

Explanation

Step 1: Expand along first row

$$D = 2M_1 - (3+i)M_2 + (-1)M_3$$

Minor M_1

$$\begin{vmatrix} 0 & i-1 \\ -1-i & 1 \end{vmatrix} = 0 \cdot 1 - (i-1)(-1-i)$$

$$(i-1)(-1-i) = -(i-1)(1+i)$$

$$(1+i)(i-1) = i-1+i^2-i = -1-1 = -2$$

So,

$$(i-1)(-1-i) = 2$$

Thus,

$$M_1 = -2$$

Contribution:

$$2M_1 = -4$$

Minor M_2

$$\begin{vmatrix} 3-i & i-1 \\ -1 & 1 \end{vmatrix} = (3-i) \cdot 1 - (i-1)(-1) \\ = 3-i + (i-1) = 2$$

Contribution:

$$-(3+i)M_2 = -2(3+i) = -6-2i$$

Minor M_3

$$\begin{vmatrix} 3-i & 0 \\ -1 & -1-i \end{vmatrix} = (3-i)(-1-i)$$

Ans : (D)

Compute:

$$(3-i)(-1-i) = -3-3i+i+i^2 = -4-2i$$

Contribution:

$$(-1)M_3 = 4+2i$$

Step 2: Add contributions

$$D = (-4) + (-6-2i) + (4+2i)$$

Real part:

$$-4-6+4 = -6$$

Imaginary part:

$$-2i+2i = 0$$

$$D = -6$$

Step 3: Identify A,B

$$A = -6, B = 0$$

$$A + B = -6$$

Answer: B) -6

11. If $A^2 + B^2 + C^2 = 0$, then what is the value of the following?

$$\begin{vmatrix} 1 & \cos C & \cos B \\ \cos C & 1 & \cos A \\ \cos B & \cos A & 1 \end{vmatrix}$$

- A) -1
- B) 0
- C) 1
- D) 2

Explanation

If $A^2 + B^2 + C^2 = 0$ for real numbers, then

$$A = B = C = 0$$

Hence,

$$\cos A = \cos B = \cos C = \cos 0 = 1$$

Substitute into the determinant

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

All rows are identical $\Rightarrow$ determinant = 0.

Correct option: B

12. If ω is a non-real cube root of unity, then what is a root of the following equation?

$$\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

- A) $x = 0$
- B) $x = 1$
- C) $x = \omega$
- D) $x = \omega^2$

Explanation

Given

ω is a non-real cube root of unity

$$\omega^3 = 1, 1 + \omega + \omega^2 = 0$$

Solve:

$$\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

Ans : (B)

Ans : (D)

PART - B

51. The focal length of a concave lens is 0.5 m. The power of the lens is

- A) +0.5 D B) -0.5 D
C) +2.0 D D) -2.0 D

Explanation

Ans : (D)

The power of a lens (in diopters, D) is given by $P = \frac{1}{f}$ where f is the focal length in meters. A concave lens has a negative focal length.

Given $f = -0.5$ m,

$$P = \frac{1}{-0.5} = -2.0 \text{ D}$$

52. The rule to determine the direction of a force experienced by a straight current carrying conductor placed in a magnetic field which is perpendicular to it is

- A) Right-hand thumb rule
B) Fleming's left-hand rule
C) Fleming's right-hand rule
D) Hund's rule

Explanation

Ans : (B)

- Fleming's left-hand rule is used for electric motors: it gives the direction of force (motion) on a current-carrying conductor in a magnetic field. The thumb indicates the direction of force, the forefinger indicates the magnetic field, and the middle finger indicates the current.
- The right-hand thumb rule relates to the magnetic field due to a current. Fleming's right-hand rule is for generators (induced current). Hund's rule pertains to electron configuration in atomic physics.

53. One parsec in astronomical units (A.U.) is about

- A) 2×10^3 A.U. B) 2×10^4 A.U.
C) 2×10^5 A.U. D) 2×10^6 A.U.

Explanation

Ans : (C)

One parsec (parallax of one arcsecond) is defined as the distance at which 1 AU subtends an angle of 1 arcsecond.

By definition:

$$1 \text{ parsec} \approx 3.086 \times 10^{16} \text{ m}$$

$$1 \text{ A.U.} \approx 1.496 \times 10^{11} \text{ m}$$

Thus,

$$\frac{1 \text{ parsec}}{1 \text{ A.U.}} \approx \frac{3.086 \times 10^{16}}{1.496 \times 10^{11}} \approx 2.06 \times 10^5$$

Hence, $1 \text{ parsec} \approx 2 \times 10^5 \text{ A.U.}$

54. Which one of the following statements for second law of motion is NOT correct?

- A) The net force on a body is proportional to that body's acceleration
B) The net force on a body is in the same direction as the acceleration

C) The time rate of change of momentum is equal to force

D) The net force on a body is proportional to that body's momentum

Explanation

Ans : (D)

Newton's second law states that the net force acting on a body is directly proportional to the rate of change of momentum of the body, not the momentum itself.

Mathematically, $F = ma$ or $F = \frac{dp}{dt}$. Options A, B, and C

are correct statements of the law. Option D is incorrect; momentum is $p = mv$, while acceleration involves a change in velocity over time.

55. The temperature of a body increases from 310 K to 340 K. The temperature increase in degree Celsius is

- A) 20°C B) 30°C
C) 37°C D) 67°C

Explanation

Ans : (B)

The Kelvin and Celsius scales are related by $T(K) = T(^{\circ}\text{C}) + 273.15$. The difference in temperature is identical in both scales, as 1 K equals 1°C in magnitude. The temperature increase is $340 - 310 = 30$ K, which is precisely a 30°C increase. The initial 310 K (about 37°C) is irrelevant for calculating the incremental change.

56. Which one of the following statements for the work done by gravity on a body is NOT correct ?

- A) Work done is independent of the path followed by the body
B) Work done depends only on the vertical distance separating the initial and the final position of the body
C) Work done depends on the path followed by the body
D) Work done is zero if the body is displaced horizontally to the force of gravity

Explanation

Ans : (C)

Gravity is a conservative force. The work done by gravity on a body depends only on the vertical displacement between its initial and final heights, not on the specific path taken. Therefore, statements A, B, and D are correct properties of gravitational work. Statement C is incorrect, as it contradicts the defining characteristic of a conservative force.

57. Which one among the following statements is correct ?

- A) Pressure has magnitude as well as direction
B) As we go deeper in a liquid, the pressure decreases
C) Action and reaction forces act on the same object
D) Pascal and N/m² represent the same unit

Explanation

Pascal (Pa) is the SI unit of pressure, defined as one newton of force per square meter of area ($1 \text{ Pa} = 1 \text{ N/m}^2$). Pressure is a scalar quantity (magnitude only), not a vector. Liquid pressure increases with depth due to the weight of the fluid column above. Action and reaction forces act on two different interacting objects, not the same one.

58. Which one of the following wavelength radiations may correspond to an X-ray?

- A) 1 nm B) 10 nm
C) 100 nm D) 300 nm

Explanation

X-rays are a form of high-energy electromagnetic radiation with very short wavelengths, typically in the range of **0.01 nm to 10 nm** (or $0.1 \text{ \AA}$ to $100 \text{ \AA}$). A 1 nm ($10 \text{ \AA}$) wavelength falls squarely within the standard X-ray region. Wavelengths of 10 nm, 100 nm, and 300 nm correspond to the extreme ultraviolet, far ultraviolet, and near-ultraviolet/visible light spectrums, respectively.

59. In a Telescope, compared to the eyepiece lens the objective lens has

- A) larger focal length and larger aperture
B) larger focal length and smaller aperture
C) smaller focal length and larger aperture
D) smaller focal length and smaller aperture

Explanation

In an astronomical telescope, the **objective lens** is designed to collect maximum light from distant objects and form a real, inverted image. Therefore, it has a **large aperture** to gather more light and a **large focal length** to produce a highly magnified image. The eyepiece, in contrast, has a shorter focal length and smaller aperture for magnification. Hence, option **A** is correct.

60. The energy, E, of a photon can be expressed as $E = hf$ where f is the frequency and h is Planck's constant. The dimensions of h are the same as that of

- A) linear momentum
B) angular momentum
C) displacement
D) torque

Explanation

Photon energy is given by $E = hf$, where energy has dimensions of ML^2T^{-2} and frequency has T^{-1} . Therefore, Planck's constant h has dimensions ML^2T^{-1} , which are the same as that of angular momentum. Linear momentum has dimensions MLT^{-1} , while torque has ML^2T^{-2} . Hence, the correct answer is **B**.

61. The work done by the force acting on an object is zero if the displacement of the object

- A) is in the opposite direction of the direction of force
B) is in the same direction of the direction of force
C) is in perpendicular direction of the direction of force
D) none of above

Ans : (D)

Explanation

Work done is defined as $W = F s \cos\theta$, where θ is the angle between force and displacement. If displacement is **perpendicular** to the force, $\theta = 90^\circ$, and $\cos 90^\circ = 0$, hence work done becomes zero. If displacement is in the same direction, work is positive; if opposite, work is negative. Therefore, zero work occurs only when displacement is perpendicular to force.

62. The magnetic field inside a current carrying very long solenoid is

- A) Uniform B) Non-uniform
C) Zero D) Highest at mid-point

Explanation

Inside a **very long current-carrying solenoid**, the magnetic field lines are parallel, equally spaced, and directed along the axis, producing a **uniform magnetic field**. The field strength depends on current and number of turns per unit length, not on position inside the solenoid. Near the ends it becomes non-uniform, but deep inside it remains constant. Hence, option **A** is correct.

63. Which one of the following instruments can be used to measure -250°C temperature ?

- A) By using a mercury based thermometer
B) By using an alcohol based thermometer
C) By using a clinical thermometer
D) By using thermocouple based thermometer

Explanation

A temperature of -250°C is extremely low and lies far below the freezing points of common liquids. **Mercury freezes at -39°C and alcohol at about -115°C** , so liquid thermometers cannot operate at such temperatures. A **thermocouple thermometer** works on the principle of thermo-electric effect and can measure **very high and very low temperatures**, including cryogenic ranges. Hence, option **D** is correct.

64. The quantity of heat needed to change unit mass of a substance from liquid to vapour without changing the temperature, is called

- A) specific heat
B) specific latent heat
C) thermal capacity
D) heat energy

Explanation

The heat required to change unit mass of a substance from **liquid to vapour without any change in temperature** is called **specific latent heat of vaporisation**. During this process, heat is used to overcome intermolecular forces rather than raise temperature. Specific heat relates to temperature change, thermal capacity depends on mass, and heat energy is a general term. Therefore, option **B** is correct.

65. The temperature at which "vapour pressure of the liquid in an open vessel becomes equal to the atmospheric pressure" is called

- A) Melting point B) Boiling point
C) Liquid point D) None of the above

Ans : (C)

Ans : (A)

Ans : (A)

Ans : (A)

Ans : (D)

Ans : (B)

Ans : (B)

Explanation

The **boiling point** of a liquid is defined as the temperature at which its **vapour pressure becomes equal to the atmospheric pressure** in an open vessel. At this temperature, bubbles of vapour form throughout the liquid and rise to the surface. Melting point refers to solid–liquid change, while “liquid point” is not a scientific term. Hence, option **B** is correct.

66. Which one of the following statements about surface tension is correct ?

- A) It increases when temperature increases
- B) It decreases when temperature increases
- C) It will remain same with increase or decrease of temperature
- D) It will always decrease with increase or decrease of temperature

Explanation

Surface tension arises due to **cohesive intermolecular forces** at the liquid surface. As temperature increases, the **kinetic energy of molecules increases**, weakening these cohesive forces. Consequently, the surface tension **decreases with rise in temperature**. At sufficiently high temperature, surface tension may become nearly zero. Therefore, the correct statement is option **B**.

67. A body sinks in water when

- A) density of water is greater than density of body
- B) density of water is less than density of body
- C) density of water is equal to density of body
- D) the body is heavy

Explanation

A body sinks if its average density is greater than the density of the fluid (water) it is placed in. This is governed by Archimedes' principle: if the buoyant force (equal to the weight of displaced fluid) is less than the body's weight, the net force is downward, causing it to sink. Option A describes the condition for floating. Weight alone (D) is not the sole factor; density is key.

68. Why a fielder in the ground gradually pulls his hands backward while catching a fast moving cricket ball ?

- A) To reduce the acceleration of the ball
- B) To increase the acceleration of the ball
- C) To increase the velocity of the ball
- D) None of above

Explanation

By pulling his hands backward, the fielder increases the time duration over which the ball's momentum is brought to zero. This reduces the rate of change of momentum (deceleration or retardation) experienced by the ball, as per the impulse-momentum theorem ($F = \Delta p / \Delta t$). A smaller deceleration means a much smaller average force acts on his hands, preventing injury and making the catch easier.

69. Bottom of a tank containing water appears to be raised. It is due to

- A) Reflection of light

Ans : (B)

- B) Refraction of light
- C) Interference of light
- D) Diffraction of light

Explanation

When light rays travel from water (a denser medium) to air (a rarer medium), they bend away from the normal at the interface due to refraction. This bending causes the bottom of the tank to appear shallower or raised to an observer looking from above. This is a common optical phenomenon; reflection, interference, and diffraction do not cause this specific apparent shift in depth.

Ans : (B)

70. SONAR stands for

- A) Sonographic Natural Ranging
- B) Simple Operational Navigation Ranging
- C) Sound Navigation and Ranging
- D) Simple Operational Natural Ranging

Explanation

SONAR stands for **Sound Navigation and Ranging**. It works on the principle of **reflection of ultrasonic sound waves**. A sound pulse is transmitted in water, and the reflected echo from an object is received. By measuring the time taken, the distance and position of underwater objects like submarines or sea beds are determined. Therefore, option **C** is correct.

Ans : (C)

71. Which one of the following radiations has longer wavelength ?

- A) X-rays
- B) Ultraviolet
- C) Micro Wave
- D) Infra-red

Explanation

In the electromagnetic spectrum, wavelength increases in this order: X-rays (shortest), Ultraviolet, Infrared, and then Microwaves. Microwaves have wavelengths ranging from about 1 mm to 1 meter, which is significantly longer than infrared radiation (typically 700 nm to 1 mm). Therefore, among the given options, microwaves have the longest wavelength.

Ans : (C)

72. For a material under static conditions, which one among the following statements is correct ?

- A) An insulator can be charged
- B) A perfect conductor can be charged
- C) Both perfect conductor and insulator can be charged
- D) Both perfect conductor and insulator cannot be charged

Explanation

Under static conditions, an insulator can be charged (e.g., by friction) because the charges are not free to move and remain localized. A perfect conductor cannot hold a static volume charge; any net charge resides only on its surface, and if connected to ground, the charge will neutralize. Therefore, only the insulator can be charged in the context of holding a static charge distribution.

Ans : (A)

73. A person can see nearby objects clearly but cannot see distant object distinctly, the person is suffering with

- A) Hypermetropia
- B) Presbyopia
- C) Myopia
- D) None of above

Explanation

Myopia, or nearsightedness, is a common vision defect where a person can see nearby objects clearly but distant objects appear blurred. This occurs because the eyeball is too long or the cornea is too curved, causing light rays to focus in front of the retina. Hypermetropia (farsightedness) is the opposite condition, and presbyopia is age-related difficulty in focusing on near objects.

74. The energy is always conserved for a system which is

- A) isolated only
- B) non-isolated only
- C) both isolated and non-isolated
- D) none of above

Explanation

The law of conservation of energy states that the total energy of an isolated system (one with no external force or energy exchange) remains constant. For a non-isolated system, external forces can perform work or transfer heat, changing the system's total energy. Therefore, while energy is universally conserved, for a specific system, it is constant only if the system is isolated.

75. Suppose, a ball of mass M is thrown upwards from a point A and it reaches up to the highest point B and returns back to point A, which one among the following is correct ?

- A) Kinetic Energy at A = Potential Energy at B
- B) Kinetic Energy at A = Potential Energy at A
- C) Kinetic Energy at B = Potential Energy at B
- D) Kinetic Energy at B = Kinetic Energy at A

Explanation

Ignoring air resistance, mechanical energy is conserved. At the lowest point A, the ball has maximum kinetic energy ($K E_A$) and zero potential energy (if A is the reference). At the highest point B, its velocity is zero ($K E_B = 0$) and it has maximum potential energy ($P E_B$). By conservation, $K E_A + 0 = 0 + P E_B$, so $K E_A = P E_B$.

76. Who among the following presided over the first session of All India Kisan Sabha in Lucknow in April 1936?

- A) Sahajananda Saraswati
- B) R. G. Ranga
- C) Ram Manohar Lohia
- D) Jayaprakash Narayan

Explanation

The All India Kisan Sabha (AIKS) was founded in the 1936 Lucknow session. Swami Sahajananda Saraswati, a renowned peasant leader, was its first President. N.G. Ranga (also a founding figure) served as the General Secretary. The session marked a critical point in organizing peasant movements on a national scale against zamindari exploitation and high rents.

77. Who among the following founded the 'Self-Respect Movement'?

- A) Jyotiba Phule

Ans : (C)

- B) Swami Vivekananda
- C) E. V. Ramaswami Naicker
- D) Dr. B. R. Ambedkar

Explanation

The Self-Respect Movement was founded in 1925 by E.V. Ramasamy Naicker (popularly known as Periyar) in Tamil Nadu. It was a radical movement aimed at achieving a casteless society, eradicating Brahminical supremacy, and promoting rationalism, women's rights, and self-respect for non-Brahmins and Dravidian identity. It strongly opposed religious dogma and social hierarchies.

78. Chola inscriptions mention several categories of lands. Which one of the following was meant for the maintenance of school?

- A) Tirunamattukkani
- B) Shalabhoga
- C) Vellanvagai
- D) Pallichhandam

Explanation

In the Chola land revenue system, Shalabhoga was the category of land specifically granted for the maintenance of a school. Other categories included Vellanvagai (land of non-Brahmana peasant proprietors), Pallichchhandam (land donated to Jaina institutions), and Tirunamattukkani (land gifted to temples). These grants illustrate the Chola administration's support for village institutions and education.

79. Which of the following statements about the Home Rule League are correct?

1. Lokamanya Tilak set up the Home Rule League at the Bombay Provincial Conference held at Belgaum in April 1916
2. Annie Besant announced the formation of Home Rule League in September 1916
3. To avoid friction, Tilak's League was to work in Maharashtra (excluding Bombay city), Karnataka, Central Provinces and Berar and Besant's League was given charge of the rest of India

Select the answer using the code given below:

- A) 1 and 2 only
- B) 1 and 3 only
- C) 2 and 3 only
- D) 1, 2 and 3

Explanation

The Home Rule Movement (1916) aimed at achieving self-government within the British Empire. Lokamanya Tilak launched his Home Rule League at the Bombay Provincial Conference, Belgaum, in April 1916. Annie Besant announced her Home Rule League in September 1916. To avoid overlap, Tilak's League worked in Maharashtra (except Bombay city), Karnataka, Central Provinces and Berar, while Besant's League covered the rest of India. Thus, all statements are correct.

80. Which one of the following statements about Ishwar Chandra Vidyasagar is NOT correct ?

- A) He was the Principal of Sanskrit College, Calcutta
- B) He was a Government Inspector of Schools
- C) He was a Secretary to the Bethune School
- D) He was the founder of Widow Remarriage Association

Ans : (D)

$$= -\frac{3}{2} + \frac{10}{3} = \frac{11}{6}$$

Total:

$$\frac{7}{6} + \frac{11}{6} = \frac{18}{6} = 3$$

Correct option: B

Direction : Consider the following for the two (02) items that follow :

Let $f(t) = \ln(t + \sqrt{1+t^2})$ and $g(t) = \tan(f(t))$.

93. Consider the following statements :

I. $f(t)$ is an odd function.

II. $g(t)$ is an odd function.

Which of the statements given above is/are correct?

A) I only

B) II only

C) Both I and II

D) Neither I nor II

Explanation

Ans : (C)

$$\text{Let } f(t) = \ln(t + \sqrt{1+t^2})$$

$$f(-t) = \ln(\sqrt{1+t^2} - t)$$

$$= \ln\left(\frac{1}{t + \sqrt{1+t^2}}\right)$$

$$= -\ln(t + \sqrt{1+t^2}) = -f(t)$$

∴ $f(t)$ is an odd function

Now $g(t) = \tan f(t)$

Then, $g(-t) = \tan f(-t)$

$$= -\tan(f(t)) = -g(t)$$

So $g(t)$ is also an odd function

Hence, both statements I and II are correct.

∴ Option (C) is correct

94. What is $\int_{-\pi}^{\pi} g(t) dt$ equal to?

A) -1

B) 0

C) 1/2

D) 1

Explanation

Ans : (B)

$$\text{Let } f(t) = \ln(t + \sqrt{1+t^2})$$

$$f(-t) = \ln(\sqrt{1+t^2} - t)$$

$$= \ln\left(\frac{1}{t + \sqrt{1+t^2}}\right)$$

$$= -\ln(t + \sqrt{1+t^2}) = -f(t)$$

∴ $f(t)$ is an odd function

Now $g(t) = \tan f(t)$

Then, $g(-t) = \tan f(-t)$

$$= -\tan(f(t)) = -g(t)$$

So $g(t)$ is also an odd function

When $g(t)$ is an odd function, then

$$\int_{-\pi}^{\pi} g(t) dt = 0$$

∴ Option (B) is correct

Direction : Consider the following for the two (02) items that follow :

Let $f: (-1, 1) \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $h(x) = f(2f(x) + 2)$ and $g(x) = (h(x))^2$.

95. What is $h'(0)$ equal to?

A) -2

B) -1

C) 0

D) 2

Explanation

Ans : (D)

Let $f: (-1, 1) \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$

$$\Rightarrow h(x) = f(2f(x) + 2)$$

$$\Rightarrow h'(x) = f'(2f(x) + 2) \cdot 2f'(x)$$

$$\Rightarrow h'(0) = f'(2f(0) + 2) \cdot 2f'(0)$$

$$= f'(-2 + 2) \cdot 2(1)$$

$$= f'(0) \cdot 2 = (1) \cdot 2 = 2$$

∴ Option (D) is correct.

96. What is $g'(0)$ equal to?

A) -4

B) -2

C) 0

D) 4

Explanation

Ans : (A)

Given:

$f(0) = -1$ and $f'(0) = 1$ Let $h(x) = f(2f(x) + 2)$ and $g(x) = (h(x))^2$

$$\sim g(x) = (h(x))^2$$

$$\sim g'(x) = 2(h(x)) \cdot h'(x)$$

$$\sim g'(0) = 2(h(0)) \cdot h'(0)$$

$$= 2f(2f(0) + 2) \cdot 2$$

$$= 2f(0) \cdot 2 = (-2) \cdot 2 = -4$$

∴ Option (A) is correct

Direction : Consider the following for the two (02) items that follow :

Let $I = \int_0^{\pi/2} \frac{f(x)}{g(x)} dx$, where $f(x) = \sin x$ and $g(x) = \sin x + \cos x + 1$.

97. What is $\int_0^{\pi/2} \frac{dx}{g(x)}$ equal to?

A) $\frac{\ln 2}{2}$

B) $\frac{\ln 2}{4}$

C) $\ln 2$

D) $2\ln 2$

Explanation

$$\int_0^{\pi/2} \frac{dx}{g(x)} = \int_0^{\pi/2} \frac{dx}{\sin x + \cos x + 1}$$

Step 1: Use the standard property

Let:

$$J = \int_0^{\pi/2} \frac{dx}{1 + \sin x + \cos x}$$

Make substitution $x \rightarrow \frac{\pi}{2} - x$:

$$J = \int_0^{\pi/2} \frac{dx}{1 + \cos x + \sin x}$$

Same integral $\rightarrow$ add both:

$$2J = \int_0^{\pi/2} \frac{2dx}{1 + \sin x + \cos x}$$

Rewrite denominator:

$$1 + \sin x + \cos x = \frac{(1 + \sin x)(1 + \cos x)}{(1 + \sin x + \cos x)}$$

$$2J = \int_0^{\pi/2} \frac{dx}{1 + \sin x}$$

Step 2: Integrate

$$\frac{dx}{1 + \sin x} = \left(\frac{1 - \sin x}{\cos^2 x} dx = \tan x - \sec x \right)$$

Apply limits 0 to $\pi/2$:

$$2J = \ln 2 \Rightarrow J = \frac{\ln 2}{2}$$

Correct option: A

98. What is I equal to?

- A) $\frac{\pi}{4} + \ln 2$ B) $\frac{\pi}{4} - \ln 2$
C) $\frac{\pi}{4} - \frac{\ln 2}{2}$ D) $\frac{\pi}{4} + \frac{\ln 2}{2}$

Explanation

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x + 1} dx$$

Step 1: Use symmetry

Replace $x \rightarrow \frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \frac{\cos x}{\sin x + \cos x + 1} dx$$

Add both forms:

$$2I = 2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x + 1} dx$$

Ans : (A)

Step 2: Split

$$2I = \int_0^{\pi/2} 1 - \frac{1}{\sin x + \cos x + 1} dx$$

$$2I = \int_0^{\pi/2} dx - \int_0^{\pi/2} \frac{dx}{\sin x + \cos x + 1}$$

$$2I = \frac{\pi}{2} - \frac{\ln 2}{2}$$

Step 3: Final value

$$I = \frac{\pi}{4} - \frac{\ln 2}{4}$$

Correct option: C

Direction : Consider the following for the two (02) items that follow :

Let

$$2 \int \frac{x^2 - 1}{\sqrt{x^2 + 1}} dx = U(x)V(x) - 3 \ln \{U(x) + V(x)\} + c$$

99. What is $|U^2(x) - V^2(x)|$ equal to?

- A) 0 B) 1
C) 2 D) 3

Explanation

$$|U^2(x) - V^2(x)| = |x^2 - (x^2 + 1)| = |x^2 - x^2 - 1| = |-1| = 1$$

$\therefore$ Option (B) is correct

100. What is $U(x)V(x)$ equal to?

- A) $\sqrt{x^2 + x^4}$ B) $\sqrt{x + x^3}$
C) $\sqrt{\frac{x^2 + x^4}{2}}$ D) $2\sqrt{x^2 + x^4}$

Explanation

$$U(x) \cdot V(x) = x\sqrt{x^2 + 1} = \sqrt{x^4 + x^2}$$

$\therefore$ Option (A) is correct

101. Let $x - 3y + 4 = 0$ and $2x - 7y + 8 = 0$ be two lines of regression computed from some bivariate data. If b_{yx} and b_{xy} are regression coefficients of lines of regression of y on x and x on y respectively, then what is the value of $b_{xy} + 7b_{yx}$?

- A) -2 B) 1
C) 2 D) 5

Explanation

Lines of regression of x on y

$$\Rightarrow x - 3y + 4 = 0$$

$$\Rightarrow x = 3y - 4$$

$$\Rightarrow b_{xy} = 3$$

Line of regression of y on x

$$\Rightarrow 2x - 7y + 8 = 0$$

Ans : (C)

Ans : (B)

Ans : (A)

Ans : (D)

$$\Rightarrow y = \frac{2}{7}x + \frac{8}{7}$$

$$\Rightarrow b_{yx} = \frac{2}{7}$$

Now

$$\Rightarrow b_{xy} + 7b_{yx} = 3 + \frac{7 \times 2}{7} = 5$$

∴ Option (D) is correct

102. The mean of n observations

1, 4, 9, 16, ..., n^2

is 130. What is the value of n ?

A) 18

B) 19

C) 20

D) 21

Explanation

$$\text{Mean} = \frac{1 + 4 + 9 + \dots + n^2}{n}$$

$$\Rightarrow 130 = \frac{n(n+1)(2n+1)}{6n}$$

$$\Rightarrow 780 = 2n^2 + 3n + 1$$

$$\Rightarrow 2n^2 + 3n - 779 = 0$$

$$\Rightarrow 2n^2 + 41n - 38n - 779 = 0$$

$$\Rightarrow n(2n + 41) - 19(2n + 41) = 0$$

$$\Rightarrow (2n + 41)(n - 19) = 0$$

$$\Rightarrow x = 19 \dots (n = -\frac{41}{2} \text{ is not possible})$$

∴ Option (B) is correct.

103. Three distinct natural numbers are chosen at random from 1 to 10. What is the probability that they are consecutive?

A) 1/12

B) 3/40

C) 1/15

D) 7/120

Explanation

$$n(S) = {}^{10}C_3$$

Favourable outcomes = (1, 2, 3), (2, 3, 4), (3, 4, 5), (4, 5, 6), (5, 6, 7), (6, 7, 8), (7, 8, 9), (8, 9, 10)

$$\Rightarrow n(E) = 8$$

$$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{8}{{}^{10}C_3}$$

$$= \frac{8 \times 3 \times 2 \times 1}{10 \times 9 \times 8} = \frac{1}{15}$$

∴ Option (C) is correct

104. A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. If $3P(B) = 4P(A)$ and $3P(C) = 2P(B)$, then what is $P(A)$ equal to?

A) 7/29

B) 8/29

C) 9/29

D) 10/29

Explanation

Ans : (C)

A, B and C are mutually exclusive and exhaustive events:

$$\Rightarrow P(A) + P(B) + P(C) = 1 \quad \dots (i)$$

Also

$$\Rightarrow 3P(B) = 4P(A)$$

$$\Rightarrow P(A) = \frac{3}{4}P(B)$$

$$\text{and } 3P(C) = 2P(B)$$

$$\Rightarrow P(C) = \frac{2}{3}P(B)$$

From (i)

$$\Rightarrow \frac{3}{4}P(B) + P(B) + \frac{2}{3}P(B) = 1$$

$$\Rightarrow \frac{9P(B) + 12P(B) + 8P(B)}{12} = 1$$

$$\Rightarrow \frac{29P(B)}{12} = 1$$

$$\Rightarrow P(A) = \frac{3}{4} \times \frac{12}{29} = \frac{9}{29}$$

∴ Option (C) is correct.

105. A die has two faces with number 4, three faces with number 5 and one face with number 6. If the die is rolled once, then what is the probability of getting 4 or 5?

A) 1/3

B) 2/3

C) 5/6

D) 1/2

Explanation

Ans : (C)

Number on faces on die are 4, 4, 5, 5, 5 and 6.

$$\Rightarrow n(S) = 6$$

$$P(\text{getting 4 or 5}) = \frac{5}{6}$$

∴ Option (C) is correct

106. A box contains 2 black, 4 yellow and 6 white balls. Three balls are drawn in succession with replacement. What is the probability that all three are of the same colour?

A) 1/6

B) 1/36

C) 1/12

D) 5/12

Explanation

Ans : (A)

Black = 2, yellow = 4 and white = 6

$$\Rightarrow \text{Total Balls} = 2 + 4 + 6 = 12$$

P (all three are of the same colour).

$$= \frac{2^3}{12^3} + \frac{4^3}{12^3} + \frac{6^3}{12^3} = \frac{1}{6}$$

∴ Option (A) is correct

107. A can hit a target 5 times in 6 shots, B can hit 4 times in 5 shots and C can hit 3 times in 4 shots. What is the probability that A and C may hit but B may lose?

A) 1/8

B) 1/6

C) 1/4

D) 1/3

Explanation

Here, $P(A) = \frac{5}{6}$, $P(B) = \frac{4}{5}$, $P(C) = \frac{3}{4}$

Also

$$P(B) = 1 - \frac{4}{5} = \frac{1}{5}$$

Now

$$\Rightarrow P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

$$= \frac{5}{6} \times \frac{1}{5} \times \frac{3}{4} \times \frac{1}{8}$$

$\therefore$ Option (A) is correct

108. The letters of the word ZOOLOGY are arranged in all possible ways. What is the probability that the consonants and vowels occur alternatively?

- A) $\frac{6}{35}$ B) $\frac{3}{35}$
C) $\frac{2}{35}$ D) $\frac{1}{35}$

Explanation

Ans : (D)

The word given is ZOOLOGY. We need to find the probability that the consonants and vowels occur alternatively in all possible arrangements.

- Vowels: O, O, O
- Consonants: Z, L, G, Y

There are 3 vowels and 4 consonants.

The total number of ways the letters of the word "ZOOLOGY" can be arranged (without any restrictions) is:

Formula: Total arrangements = $n! / r_1! r_2! \dots r_k!$

Where:

- n = Total number of letters = 7
- r_1 = Frequency of O = 3

So, the total number of arrangements is:

$$\text{Total arrangements} = \frac{7!}{3!} = \frac{5040}{6} = 840$$

We need to place the consonants and vowels alternately:

- Consonants will occupy the odd positions (1, 3, 5, 7).
- Vowels will occupy the even positions (2, 4, 6).

The number of ways to arrange the consonants (Z, L, G, Y) in the 4 odd positions is:

$$4! = 24$$

The number of ways to arrange the vowels (O, O, O) in the 3 even positions is:

$$\frac{3!}{3!} = 1$$

Thus, the number of favorable arrangements is:

$$\text{Favorable arrangements} = 4! \times \left(\frac{3!}{3!} \right) = 24 \times 1 = 24$$

The probability is given by the ratio of favorable arrangements to the total arrangements:

$$\text{Probability} = \frac{\text{Favorable arrangements}}{\text{Total arrangements}} = \frac{24}{840} = \frac{1}{35}$$

Ans : (A)

The probability that the consonants and vowels occur alternatively is $\frac{1}{35}$.

109. A natural number x is chosen at random from the first 100 natural numbers. What is the probability that $x^2 + x > 50$?

- A) $\frac{93}{100}$ B) $\frac{47}{50}$
C) $\frac{24}{25}$ D) $\frac{23}{25}$

Explanation

Ans : (B)

$$\Rightarrow n(S) = 100$$

Numbers from 7 to 100 satisfy $x^2 + x > 50$

$$\text{Then, } n(E) = 100 - 6 = 94$$

$$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{94}{100} = \frac{47}{50}$$

$\therefore$ Option (B) is correct

110. What is the mean deviation of the first 10 natural numbers?

- A) 2 B) 2.5
C) 3 D) 3.5

Explanation

Ans : (B)

The mean of 10 natural numbers

$$\Rightarrow x = \frac{1 + 2 + 3 + 4 + \dots + 10}{10}$$

$$= \frac{10 \times 11}{2 \times 10} = 5.5$$

$$\text{Mean deviation} = \frac{[1 - 5.5][2 - 5.5][3 - 5.5] \dots [10 - 5.5]}{10} = 2.5$$

$\therefore$ Option (B) is correct

111. Let $\sum_{i=1}^9 x_i^2 = 855$. If M is the mean and σ is the standard deviation of $x_1, x_2, \dots, x_9$, then what is the value of $M^2 + \sigma^2$?

- A) 100 B) 95
C) 90 D) 85

Explanation

Ans : (B)

Given:

$$\sum_{i=1}^9 x_i^2 = 885$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^9 x_i^2 - (\text{mean})^2$$

$$= \frac{855}{9} - (M)^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \sigma^2 + M^2 = \frac{855}{9} = 95$$

$\therefore$ Option (B) is correct

From $a - b = 1 + b$

$$a = 1 + 2b$$

Substitute $b = 1$:

$$a = 3$$

60. What is the value of b ?

- A) -1 B) 1
C) 2 D) 3

Explanation

If a function is continuous at a particular value x .

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a} f(x)$$

Calculation:

$$f(x) = \begin{cases} \frac{x-3}{|x-3|} + a; & x < 3 \\ a - b; & x = 3 \\ \frac{x-3}{|x-3|} + b; & x > 3 \end{cases}$$

And $f(x)$ be continuous at $x = 3$.

For $x < 3$, $|x - 3| = -(x - 3)$

Left-hand Limit,

$$\Rightarrow \text{LHL} = \frac{x-3}{|x-3|} + a$$

$$\Rightarrow \text{LHL} = -\frac{x-3}{x-3} + a$$

$$\Rightarrow \text{LHL} = -1 + a$$

For $x > 3$, $|x - 3| = (x - 3)$

Right-hand Limit,

$$\Rightarrow \text{RHL} = \frac{x-3}{|x-3|} + b$$

$$\Rightarrow \text{RHL} = \frac{x-3}{x-3} + b$$

$$\Rightarrow \text{RHL} = 1 + b$$

For $x = 3$, $f(x) = a - b$,

As the function is continuous at $x = 3$.

So, $\text{LHL} = \text{RHL} = f(x = 3)$

$$\Rightarrow -1 + a = 1 + b = a - b$$

$$\Rightarrow -1 + a = a - b \text{ \& } 1 + b = a - b$$

$$\Rightarrow -1 = -b \text{ \& } 1 + b = a - b$$

$$\Rightarrow b = 1 \text{ \& } 1 + 1 = a - 1$$

$$\Rightarrow b = 1 \text{ \& } a = 3$$

Consider the following for the two (02) items that follow :

$$\text{Let } I = \int_{-2\pi}^{2\pi} \frac{\sin^4 x + \cos^4 x}{1 + 3^x} dx$$

61. What is $\int_0^\pi (\sin^4 x + \cos^4 x) dx$ equal to?

- A) $\frac{3\pi}{8}$ B) $\frac{3\pi}{4}$
C) $\frac{3\pi}{2}$ D) 3π

Explanation

Ans : (B)

$$\bullet \sin^2 x + \cos^2 x = 1$$

$$\bullet \sin 2x = \frac{(1 - \cos 2x)}{2}$$

$$\bullet \int \cos x dx = \sin x + C$$

Calculation:

$$\int_0^\pi (\sin^4 x + \cos^4 x) dx$$

Adding and subtracting $2\sin^2 x \cos^2 x$

$$\Rightarrow \int_0^\pi (\sin^4 x + \cos^4 x) + 2\sin^2 x \cos^2 x - 2\sin^2 x \cos^2 x dx$$

$$\Rightarrow \int_0^\pi (\sin^2 x + \cos^2 x)^2 - \frac{\sin^2 2x}{2} dx$$

$$\Rightarrow \int_0^\pi (1)^2 - \frac{1}{2} \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$\Rightarrow \int_0^\pi 1 - \frac{1}{4} (1 - \cos 2x) dx$$

$$\Rightarrow \int_0^\pi 1 - \frac{1}{4} + \frac{\cos 2x}{4} dx$$

$$\Rightarrow \int_0^\pi \frac{3}{4} + \frac{\cos 2x}{4} dx$$

$$\Rightarrow \left[\frac{3}{4}x + \frac{\sin 2x}{8} \right]_0^\pi$$

$$\Rightarrow \frac{3}{4} [\pi - 0] + \frac{(\sin 2\pi - \sin 0)}{8}$$

$$\therefore \int_0^\pi (\sin^4 x + \cos^4 x) dx = \frac{3\pi}{4}$$

62. What is I equal to?

- A) 0 B) $\frac{3\pi}{4}$
C) $\frac{3\pi}{2}$ D) 3π

Explanation

Ans : (C)

$$\bullet \int_a^b f(x) dx = \int_a^b f(a + b - x) dx$$

$$\bullet \sin^2 x + \cos^2 x = 1$$

$$\bullet \sin 2x = \frac{(1 - \cos 2x)}{2}$$

$$\bullet \int \cos x dx = \sin x + C$$

Calculation:

$$I = \int_{-2\pi}^{2\pi} \frac{\sin^4 x + \cos^4 x}{1 + 3^x} dx \quad \text{-----(1)}$$

Applying the property,

$$I = \int_{-2\pi}^{2\pi} \frac{\sin^4 x + \cos^4 x}{1 + 3^{-x}} dx \quad \text{-----(2)}$$

Adding equation (1) and (2)

$$2I = \int_{-2\pi}^{2\pi} (\sin^4 x + \cos^4 x) dx$$

Adding and subtracting $2\sin^2 x \cos^2 x$

$$\Rightarrow 2I = \int_{-2\pi}^{2\pi} (\sin^4 x + \cos^4 x) + 2\sin^2 x \cos^2 x - 2\sin^2 x \cos^2 x dx$$

$$\Rightarrow 2I = \int_{-2\pi}^{2\pi} (\sin^2 x + \cos^2 x)^2 + \frac{\sin^2 2x}{2} dx$$

$$\Rightarrow 2I = \int_{-2\pi}^{2\pi} (1)^2 + \frac{1}{2} \left(\frac{1 - \cos 2x}{2} \right) dx$$

$$\Rightarrow 2I = \int_{-2\pi}^{2\pi} 1 - \frac{1}{4} (1 - \cos 2x) dx$$

$$\Rightarrow 2I = \int_{-2\pi}^{2\pi} 1 - \frac{1}{4} + \frac{\cos 2x}{4} dx$$

$$\Rightarrow 2I = \int_{-2\pi}^{2\pi} \frac{3}{4} + \frac{\cos 2x}{4} dx$$

$$\Rightarrow 2I = \left[\frac{3}{4} x + \frac{\sin 2x}{8} \right]_{-2\pi}^{2\pi}$$

$$\Rightarrow 2I = \frac{3}{4} [2\pi - (-2\pi)] + \frac{(\sin 4\pi - \sin(-4\pi))}{8}$$

$$\Rightarrow 2I = 3\pi$$

$$\Rightarrow I = \frac{3\pi}{2}$$

Consider the following for the two (02) items that follow :

$$\text{Let } f(x) = \begin{cases} ax(x+1) + b, & x < 1 \\ x - 1, & 1 \leq x \leq 2 \end{cases}$$

63. If the function $f(x)$ is differentiable at $x = 1$, then what is the value of $(a + b)$?

A) $-\frac{1}{3}$

B) -1

C) 0

D) 1

Explanation

Continuity of function at any point:

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a} f(x)$$

A function $f(x)$ is differentiable at $x = a$, if $LHD = RHD$

$$LHD = \lim_{x \rightarrow a^-} f'(x) = \lim_{h \rightarrow 0^-} \frac{f(a-h) - f(a)}{-h}$$

$$RHD = \lim_{x \rightarrow a^+} f'(x) = \lim_{h \rightarrow 0^+} \frac{f(a+h) - f(a)}{h}$$

Ans : (A)

If a function is differentiable at a point x , then it must be continuous at x .

Calculation:

$$f(x) = \begin{cases} ax(x+1) + b, & x < 1 \\ x - 1, & 1 \leq x \leq 2 \end{cases}$$

$$f'(x) = \begin{cases} 2ax + a, & x < 1 \\ 1, & 1 \leq x \leq 2 \end{cases}$$

Left hand derivative = $2ax + a$

At $x = 1$,

Left hand derivative = $2a + a = 3a$

Right hand derivative for $1 \leq x \leq 2$,

Right hand derivative = 1

As, $f(x)$ is differentiable at $x = 1$,

So, Left Hand derivative = Right Hand derivative = $f'(1)$

$$\Rightarrow 3a = 1$$

$$\Rightarrow a = \frac{1}{3}$$

If a function is differentiable at a point x , then it must be continuous at x .

Left Hand Limit,

For $x < 1$,

$$LHL = ax(x+1) + b$$

At $x = 1$,

$$LHL = a(1+1) + b = 2a + b$$

Right Hand Limit,

For $1 \leq x \leq 2$

$$RHL = x - 1$$

At $x = 1$, $RHL = 0$

As, $f(x)$ is continuous at $x = 1$,

Left Hand Limit = Right Hand Limit = $f'(1)$

$$2a + b = 0$$

$$\text{As } a = \frac{1}{3}$$

$$b = -\frac{2}{3}$$

$$a + b = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3}$$

$\therefore$ Correct answer is $-\frac{1}{3}$.

64. What is $\lim_{x \rightarrow 0} f(x)$ equal to?

A) $-\frac{1}{3}$

B) $-\frac{2}{3}$

C) 0

D) 1

Explanation

Ans : (B)

$$f(x) = \begin{cases} ax(x+1) + b, & x < 1 \\ x - 1, & 1 \leq x \leq 2 \end{cases}$$

For $x \rightarrow 0$, the function will be from $x < 1$,

$$\Rightarrow f(x) = ax(x+1) + b,$$

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = ax(x+1) + b$$

Put $x = 0$,

$$\Rightarrow \lim_{x \rightarrow 0} f(x) = 0(0+1) + b = 0 + b = 0$$

As from the above question, $b = -\frac{2}{3}$

$$\text{Therefore, } \lim_{x \rightarrow 0} f(x) = -\frac{2}{3}.$$

65. If $f(x) = |\ln|x||$ where $0 < x < 1$, then what is $f'(0.5)$ equal to?

- A) -2 B) -1
C) 0 D) 2

Explanation

Given:

$$f(x) = |\ln|x|| \text{ where } 0 < x < 1$$

Calculation:

$$\ln|x| = \ln x$$

$$\Rightarrow f(x) = \ln|x|$$

As $0 < x < 1$ so $\ln$ is negative in this range,

$$\Rightarrow f(x) = -\ln x$$

Differentiating with respect to x ,

$$\Rightarrow f'(x) = -\frac{1}{x}$$

Put $x = 0.5$,

$$\Rightarrow f'(0.5) = -\frac{1}{0.5}$$

$$\Rightarrow f'(0.5) = -2$$

66. If $f'(x) = \cos(\ln x)$ and $y = f\left(\frac{2x-3}{x}\right)$, then what is $\frac{dy}{dx}$ equal to?

- A) $\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$
B) $-\frac{3}{x^2}\sin\left(\ln\left(\frac{2x-3}{x}\right)\right)$
C) $\frac{3}{x^2}\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$
D) $-\frac{3}{x^2}\cos\left(\ln\left(\frac{2x-3}{x}\right)\right)$

Explanation

Given:

$$f'(x) = \cos(\ln x)$$

$$y = f\left(\frac{2x-3}{x}\right)$$

Step 1: Use chain rule

Let

$$u = \frac{2x-3}{x}$$

Then,

$$\frac{dy}{dx} = f'(u) \frac{du}{dx}$$

Step 2: Substitute $f'(x)$

$$f'(u) = \cos(\ln u)$$

So,

$$\frac{dy}{dx} = \cos\left(\ln \frac{2x-3}{x}\right) \cdot \frac{du}{dx}$$

Step 3: Differentiate u

$$u = \frac{2x-3}{x}$$

Using quotient rule:

$$\begin{aligned} \frac{du}{dx} &= \frac{x(2) - (2x-3)(1)}{x^2} \\ &= \frac{2x - 2x + 3}{x^2} = \frac{3}{x^2} \end{aligned}$$

Step 4: Final expression

$$\frac{dy}{dx} = \frac{3}{x^2} \cos\left(\ln \frac{2x-3}{x}\right)$$

67. What is $\int_0^{8\pi} |\sin x| dx$ equal to?

- A) 2 B) 4
C) 8 D) 16

Explanation

$$|\sin x| = \begin{cases} \sin(x) & \text{if } 0 \leq x \leq \pi \\ -\sin(x) & \text{if } \pi \leq x \leq 2\pi \end{cases}$$

$$\int \sin x dx = -\cos x$$

Calculation:

Let,

$$I = \int_0^{8\pi} |\sin x| dx$$

As the period of $\sin \theta$ is 2π . Therefore,

$$I = 4 \int_0^{2\pi} |\sin x| dx$$

By using the above concept,

$$\Rightarrow I = 4 \left(\int_0^{\pi} \sin(x) dx + \int_{\pi}^{2\pi} \sin(x) dx \right)$$

$$\Rightarrow I = 4 \left([-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi} \right)$$

$$\Rightarrow I = 4[(-\cos(\pi) - (-\cos(0))) + (\cos(2\pi) - (\cos(\pi)))]$$

$$\Rightarrow I = 4[(-(-1) - (-1)) + (1 - (-1))]$$

$$\Rightarrow I = 4[(1 + 1) + (1 + 1)]$$

$$\Rightarrow I = 4[2 + 2]$$

$$\Rightarrow I = 16$$

$$\therefore \int_0^{8\pi} |\sin x| dx = 16$$

Ans : (A)

Ans : (D)

Ans : (C)

68. What is the area between the curve $f(x) = x|x|$ and x -axis for $x \in [-1, 1]$?

- A) $\frac{2}{3}$ B) $\frac{1}{2}$
C) $\frac{1}{4}$ D) $\frac{1}{3}$

Explanation

Formula Used:

$$\int x^n dx = x^{n+1}/n + 1 + C$$

Calculation:

We have

$$f(x) = x|x|$$

Case: 1

$$\text{When } x > 0 \Rightarrow f(x) = x^2$$

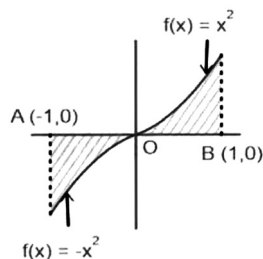
Case: 2

$$\text{When } x < 0 \Rightarrow f(x) = -x^2$$

So, area

$$\begin{aligned} I &= \int_{-1}^1 x|x| dx \\ &= \int_{-1}^0 -x^2 dx + \int_0^1 x^2 dx \\ &= \left[-\frac{x^3}{3} \right]_{-1}^0 + \left[\frac{x^3}{3} \right]_0^1 \\ &= \frac{0 - (-1)^3}{3} + \frac{(1)^3 - 0}{3} \\ &= \frac{1}{3} + \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

Hence, required area is $\frac{2}{3}$ square unit.



Ans : (A)

69. What are the order and the degree respectively of the differential equation $x^2 \left(\frac{d^3 y}{dx^3} \right)^2 + \left(\frac{dy}{dx} \right)^4 + \sin x = 0$?

- A) 3, 4 B) 1, 4
C) 2, 2 D) 3, 2

Explanation

Given:

$$x^2 \left(\frac{d^3 y}{dx^3} \right)^2 + \left(\frac{dy}{dx} \right)^4 + \sin x = 0$$

Concept:

Order:

The order of a differential equation is the order of the highest derivative appearing in it.

Degree:

The degree of a differential equation is the power of the highest derivative occurring in it, after the equation has

been expressed in a form free from radicals as far as the derivatives are concerned.

Calculation:

$$x^2 \left(\frac{d^3 y}{dx^3} \right)^2 + \left(\frac{dy}{dx} \right)^4 + \sin x = 0$$

Order of the highest derivative appearing in it is 3.

$$x^2 \left(\frac{d^3 y}{dx^3} \right)^2 + \left(\frac{dy}{dx} \right)^4 + \sin x = 0$$

The degree of a differential equation is the power of the highest derivative occurring in it is 2.

Hence, the correct answer is option (D).

70. What is the differential equation of all parabolas of the type $y^2 = 4a(x - b)$?

- A) $\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$
B) $\frac{d^2 y}{dx^2} + x^2 \left(\frac{dy}{dx} \right)^2 = 0$
C) $y^2 \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$
D) $y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$

Explanation

Given:

$$\text{Equation of parabola, } y^2 = 4a(x - b)$$

Formula Used:

$$\left(\frac{d}{dx} \right) x^n = nx^{n-1}$$

Calculation:

$$y^2 = 4a(x - b)$$

Differentiating with respect to x ,

$$2y \frac{dy}{dx} = 4a(1 - 0)$$

$$y \frac{dy}{dx} = 2a$$

Differentiating with respect to x ,

$$y \frac{d^2 y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} = 0$$

$$y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$$

∴ The correct answer is option (D).

Consider the following for the two (02) items that follow :

Let $a_1, a_2, a_3, \dots$ be in AP such that

$$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{25} + a_{30} + a_{34} = 300.$$