



MATHEMATICS

- **NUMBERS**
- **INTRODUCTION TO ALGEBRA**
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MATHEMATICS

NUMBERS

IMPORTANT POINTS

- When 1 is added to a number we get its **Successor**.
- When 1 is subtracted from a number we get its **Predecessor**.

Indian Number System			International Number System		
Period	Name	Natural	Period	Name	Natural
Ones	One	1	Ones	One	1
	Ten	10		Ten	10
	Hundred	100		Hundred	100
Thousands	Thousand	1000	Thousands	Thousand	1000
	Ten Thousand	10,000		Ten Thousand	10,000
Lakhs	Lakh	1,00,000		Hundred Thousand	100,000
	Ten Lakh	10,00,000	Millions	Million	1,000,000
Crores	Crore	1,00,00,000		Ten Million	10,000,000
	Ten Crore	10,00,00,000		Hundred Million	100,000,000
	Hundred Crore	100,00,00,000	Billions	Billion	1,000,000,000
	Thousand Crore	1000,00,00,000		Ten Billion	10,000,000,000

- The number 10100 is called **googol** (This is, 10 multiplied 100 times)
- The number 10googol = $10^{(10^{100})}$ is called **googolplex**.

Thomas Harriot (1560 - 1621)

- A famous mathematician was the first to use "<" (less than) and ">" (greater than) symbols.

Expansion of BIDMAS

- The collection of {1, 2, 3, ...} is called **Natural numbers** denoted by N.

• B	-	Bracket
• I	-	Indices
• D	-	Division
• M	-	Multiplication
• A	-	Addition
• S	-	Subtraction

- The collection of {0, 1, 2, 3, ...} is called **Whole numbers**, denoted by W.
- The smallest natural number is 1.
- The smallest whole number is 0.

- When two numbers are added or multiplied, the order of the numbers does not affect the sum or the product. This is called **Commutativity of addition or multiplication**.

Example

$$43 + 57 = 57 + 43 ; \quad 12 \times 15 = 15 \times 12$$

- Subtraction and division are not Commutative.

Example

$$7 - 3 \neq 3 - 7 ; \quad 12 \div 6 \neq 6 \div 12$$

- When several numbers are added, the order in which the numbers are added does not matter. This is called "**Associativity of addition**".

Example

$$(43+57) + 25 = 43 + (57+25)$$

- When several numbers are to be multiplied, the order in which the numbers are multiplied does not matter. This is called "**Associativity of multiplication**".

Example

$$12 \times (15 \times 17) = (12 \times 15) \times 17$$

- Subtraction and division are **NOT** associative.

- Natural Number + Natural Number = Natural number
- Natural Number $\times$ Natural Number = Natural number
- Whole Number + Whole Number = Whole Number
- Whole Number $\times$ Whole Number = Whole Number
- Whole Number + Natural Number = Natural Number
- Whole Number $\times$ Natural Number = Whole Number

- ➡ When zero is added to any number, we get the same number. Hence zero is called the "**Additive identity**".
- ➡ Similarly, When we multiply any number by 1, we get the same number. Hence 1 is called the "**Multiplicative identity**".
 - Any number multiplied by zero gives zero.
 - Division by zero is not defined. It is indeterminant.
 - Estimation is approximating a quantity to a reasonable accuracy.
- ➡ A natural number greater than 1, having only two factors namely 1 and the number itself, is called a '**Prime Number**'.
 - (e.g.) $13 = 1 \times 13$; $7 = 1 \times 7$
- ➡ A natural number having more than two factors is called a '**Composite Number**'.
 - (e.g.) $30 = 1 \times 2 \times 3 \times 5$
- ➡ A Pair of prime numbers whose difference is 2; it is called as '**Twin Primes**'.
 - (e.g.) (5, 7) and (17, 19)
- ➡ Every composite number can be expressed as a product of prime numbers in a '**Unique Way**'.
- ➡ The sum of any two odd numbers is always **even number**.

Sieve of Eratosthenes Method:

This method is a simple method of elimination by which we can easily find the prime numbers upto a given number. This method given by a Greek Mathematician, Eratosthenes of Alexandria.

- ➡ The sum of an odd number and an even number is always an **odd number**
- ➡ The product of any two odd numbers is always an **odd number**.
- ➡ The product of an odd and an even number is always an **even number**.
- ➡ The product of three odd numbers is always an **odd number**.
- ➡ 1 is the first odd **natural number**
- ➡ 0 is the first even **whole number**
- ➡ **Divisibility by 2**
A number is divisible by 2, if its ones place is any one of the even numbers 0, 2, 4, 6 or 8.
- ➡ **Divisibility by 3**
A number is divisible by 3, if the sum of its digits is divisible by 3.

➡ Divisibility by 4

A number is divisible by 4, if the last two digits of the given number is divisible by 4. If the last two digits of a numbers are zeros, then also it is divisible by 4.

➡ Divisibility by 5

A number is divisible by 5, if its ones place is either 0 or 5.

➡ Divisibility by 6

A number is divisible by 6, if it is divisible by both 2 and 3.

➡ Divisibility by 7

Though a rule for divisibility of a number by 7 exists, it is a bit tricky and dividing directly by 7 will be easier.

➡ Divisibility by 8

A number is divisible by 8, if the last three digits of the given number is divisible by 8. If the last three digits of a number are zeros, then also it is divisible by 8.

Since multiplication Satisfies commutativity the order of the factors in the product does not affect the value of the number.

➡ Divisibility by 9

A number is divisible by 9, if the sum of its digits is divisible by 9. The numbers divisible by 9 are also divisible by 3.

➡ Divisibility by 10

A number is divisible by 10, if its ones place is only zero. The numbers divisible by 10 are also divisible by 5.

➡ Divisibility by 11

A number is divisible by 11, if the difference between the sum of alternative digits of the number is either 0 or divisible by 11.

Example 256795.

Here, the difference between the sum of alternative digits = $(2 + 6 + 9) - (5 + 7 + 5) = 17 - 17 = 0$

➡ Perfect number:

A number is a **Perfect Number** if the sum of its factors other than the given number gives the same number.

Example : 6 is a perfect number, since adding the factors of 6 (other than 6) namely 1, 2 and 3 gives the given number 6.

i.e., $1 + 2 + 3 = 6$ is the given number.

➡ The total number of primes upto 100 is 25.

➡ The only prime number that ends with 5 is 5.

➡ 1 is neither prime nor composite.

➡ The Highest Common Factor (HCF) is also called as the Greatest Common Divisor (GCD) or the Greatest Common Factor (GCF).

All the prime numbers, except 2 and 3 can be expressed as 1 more or 1 less to a multiple of 6.

example: $37 = (6 \times 6) + 1$
 $11 = (6 \times 2) - 1$
 $13 = (6 \times 2) + 1$
 $17 = (6 \times 3) - 1$
 $19 = (6 \times 3) + 1$

- ➡ The Highest Common Factor of any two non-zero whole numbers is the largest common factor of both the numbers.
- ➡ The Least Common Multiple of any two non-zero whole numbers is the smallest common multiple of both the numbers.
- ➡ Two numbers having 1 as their only common factor are said to be **Co-Primes** or **Relatively Prime**.
- ➡ The product of two given numbers is equal to the product of their HCF and LCM.

Example Problems

1. **The predecessor of the smallest 5 digit number is**
Solution :
 The predecessor = $10,000 - 1 = 9999$
2. **How many hundreds are there in 10 lakh ?**
Solution :

$$\frac{10,00,000}{100} = 10,000$$
 10 lakh = 10,000 Hundreds
3. **How many ten thousands are there in the smallest 6 digit number ?**
Solution :

$$\frac{\text{One lakh}}{\text{Ten thousand}} = \frac{1,00,000}{10,000} = 10$$
 1 lakh = 10 Ten thousands
4. **Using the digits 5, 2, 0, 7, 3 form the largest 5 digit number and the smallest 5 digit number**
Solution :
 Largest number = 75320
 Smallest number = 20357
 (Reverse order of 75320 is 02357. But as it becomes a four digit number 2357, we have to take 20357 as the smallest number.)
5. **Rajan writes a 3-digit number using the digits 4, 7 and 9. What are the possible numbers that he can write ?**
Solution :
 Rajan can write 6 numbers.
 479, 497, 794, 749, 974, 947
6. **The password to access my ATM card includes the digits 9, 4, 6 and 8. It is the smallest 4 digit even number. Find the password of my ATM card.**
Solution :
 Given digits are 9, 4, 6 and 8
 Smallest number with these digits is 4689
 Given that it is an even number.
 $\therefore$ It may be 4698
 So password of ATM card is 4698.
7. **Simplify : $24 + 2 \times 8 \div 2 - 1$**
Solution :
 $24 + 2 \times 8 \div 2 - 1$ (Given question)
- $\Rightarrow 24 + 2 \times 4 - 1$ ($\div$ operation completed first)
 $\Rightarrow 24 + 8 - 1$ ($\times$ operation completed second)
 $\Rightarrow 32 - 1$ ($+$ operation completed third)
 $\Rightarrow 31$ ($-$ operation completed third)
8. **Simplify : $20 + [8 \times 2 + \{6 \times 3 - 10 \div 5\}]$**
Solution :
 $\Rightarrow 20 + [8 \times 2 + \{18 - 10 \div 5\}]$ (bar completed first)
 $\Rightarrow 20 + [8 \times 2 + \{18 - 2\}]$ ($\div$ completed second)
 $\Rightarrow 20 + [8 \times 2 + 16]$ ($\{ \}$ completed third)
 $\Rightarrow 20 + [16 + 16]$ ($\times$ completed fourth)
 $\Rightarrow 20 + 32$ ($[]$ operation completed fifth)
 $= 52$ ($+$ completed last)
9. **Simplify : $(10 + 17) \div 3$**
Solution :
 $(10 + 17) \div 3 \Rightarrow 27 \div 3 = 9$
10. **Simplify : $12 - [3 - \{6 - (5 - 1)\}]$**
Solution :
 $\Rightarrow 12 - [3 - \{6 - 4\}]$
 $\Rightarrow 12 - [3 - 2] \Rightarrow 12 - 1 = 11$
11. **Simplify : $100 + 8 \div 2 + \{(3 \times 2) - 6 \div 2\}$**
Solution :
 $\Rightarrow 100 + 8 \div 2 + \{6 - 6 \div 2\}$
 $\Rightarrow 100 + 8 \div 2 + \{6 - 3\}$
 $\Rightarrow 100 + 8 \div 2 + 3$
 $\Rightarrow 100 + 4 + 3 = 107$
12. **If the cost of a copy of a Thirukkural book is ₹188, then find the estimated cost of 31 copies of such book. (Note : Find the rounded values of 188 and 31 and then find the result)**
Solution :
 188 is nearer to 200 and 31 is nearer to 30.
 The estimated cost of 31 copies
 $= 200 \times 30 = ₹ 6000$
13. **Find the estimated value of $5598 \div 689$.**
Solution :
 5600 is nearest to 5598.
 700 is nearest to 689.
 $5600 \div 700 = 8$
 Hence, the estimated value is 8.

7th
Standard

Mathematics

- 1. Number System**
- 2. Measurements**
- 3. Algebra**
- 4. Direct and Inverse Proportion**
- 5. Geometry**
- 6. Percentage and Simple Interest**
- 7. Statistics**

MATHEMATICS

1. NUMBER SYSTEM

INTEGERS :

- ◆ Collection of the natural numbers, zero and the negative numbers gives integers. The collection of integers are denoted by Z .
- ◆ Negative integers are represented on the number line to the left of zero and the positive integers to the right of zero.
- ◆ Every integers on the number line is placed in an increasing order from left to right.

ADDITION OF INTEGERS :

- ◆ The sum of two positive integers is positive.
E.g: $(+5) + (+4) = +9$
- ◆ The sum of two negative integers is negative.
E.g: $(-2) + (-5) = -7$
- ◆ The sum of a positive and a negative integer is the difference of the two numbers in value and has the sign of the greater integer.
E.g: $(-3) + (+5) = +2$
 $(+3) + (-5) = -2$

PROPERTIES OF ADDITION :

- ◆ **Closure Property:** The sum of two integers is always an integer i.e. for any two integers a and b ; $a + b$ is also an integer. This property is known as 'closure property' of integers on addition.
- ◆ **Commutative Property:** For any two integers a and b ; $a + b = b + a$. This property is known as 'commutative property' of integers.

ASSOCIATIVE PROPERTY:

- ◆ For any three integers a , b , and c ; $a + (b + c) = (a + b) + c$. This property is known as **Associative property** of integers under addition.

ADDITIVE IDENTITY:

- ◆ When '0' is added to an integer, we get the same integer.
i.e. For any integer a , $a + 0 = a = 0 + a$
Due to this property zero is called the additive identity.

ADDITIVE INVERSE

- ◆ When opposites are added together always give the value zero.
E.g. $(-5) + (+5) = 0$
In this case either of the pair of opposites is known as the additive inverse of the other.
i.e. For any integer a , $-a$ is the additive inverse.
 $a + (-a) = 0 = (-a) + a$

SUBTRACTION OF INTEGERS:

- ◆ To subtract an integer from another, we add the additive inverse of the integer which is to be subtracted.
E.g. (i) $7 - (-5) = 7 + (+5) = 12$
(ii) $(-7) - (+5) = (-7) + (-5) = -12$

- ◆ Every subtraction statement has a corresponding addition statement.

E.g. $8 - 5 = 3$; Subtraction statement. $3 + 5 = 8$; Addition statement.

PROPERTIES OF SUBTRACTION :

- ◆ The difference of two integers is always an integer. i.e. For any two integers a , b ; $a - b$ is also an integer. Closure property is true for integers on subtraction.
- ◆ For any two integers a , b ; $a - b \neq b - a$.
 $\therefore$ Commutative property does not hold for subtraction of integers.
E.g. $3 - (-1) = 3 + 1 = 4$
 $(-1) - (3) = -1 + (-3) = -4$
 $3 - (-1) \neq (-1) - 3$

Fractions and Decimals

- ◆ A decimal is a fraction written in a special form.
- ◆ Decimal comes from the latin word decinus which means tenth. It comes from the root word "decem".
- ◆ In any number when we move towards right from one digit to the next, the place value of a digit is divided by 10.
- ◆ If the denominator of a fraction is any of 10, 10^2 , 10^3 , ... , we can express them as decimals.
- ◆ If the denominator of a fraction is any number that can be made as powers of 10 using the concepts of equivalent fractions, then it can also be expressed as decimals.
- ◆ For any decimal number, number of zeroes in the denominator and number of decimal digits are equal.

Comparison of Decimals

- ◆ To compare two decimal numbers, we compare the digits from left to right.
- ◆ The decimal number that has the greater whole number part is the greatest number.
- ◆ Zeros added to the right end of decimal digits do not change the value of that decimal number.

Multiplication of Decimal Numbers

- ◆ The number of decimal digits in the product of two decimal numbers is equal to the sum of decimal digits of decimal digits that are multiplied.
- ◆ When a decimal number is multiplied by 10, 100 or 1000, the digits in the product are same as in the decimal number but the decimal point in the product is shifted to the right by as many places as there are zeros followed by 1.

Division of Decimal Numbers

- ◆ When a decimal number is divided by 10, 100 and 1000, the digits of the number (Dividend) and the obtained decimal number after division are the same, but the decimal point in the obtained decimal number after division is shifted to the left by as many places as there are zeros followed by 1.

Example Problems

1. Thenmalar appeared for competitive exam which has negative scoring of 1 mark for each incorrect answers. In paper I she answered 25 question incorrectly and in paper II 13 questions incorrectly. Find the total reduction of marks.

Sol:

For each incorrect question the score = -1

In paper I, score for 25 incorrect questions
 $= 25 \times (-1) = -25$

In paper II, for 13 incorrect question the score
 $= 13 \times (-1) = -13$

The total marks get reduced $= (-25) + (-13) = -38$
 -38 marks will be reduced.

2. In a quiz competition, Team A scored +30, -20, 0 and team B scored -20, 0, +30 in three successive rounds. Which team will win? Can we say that we can add integers in any order?

Sol :

Total score of team A $= [(+30) + (-20)] + 0$
 $= (+10) + 0 = 10$

Total score of team B $= [(-20) + 0] + (+30)$
 $= -20 + 30 = +10$

Score of team A = Score of team B. Yes, we say that we can add integers in any order.

3. Are $(11 + 7) + 10$ and $11 + (7 + 10)$ equal? Mention the property.

Sol :

First we take $(11 + 7) + 10 = 18 + 10 = 28$

Now $11 + (7 + 10) = 11 + 17 = 28$

In both the cases the sum is 28.

$\therefore (11 + 7) + 10 = 11 + (7 + 10)$

This property is known as associative property of integers under addition.

4. Find 5 pairs of integers that added to 2.

Sol :

$$0 + 2 = 2$$

$$1 + 1 = 2$$

$$-1 + 3 = 2$$

$$-2 + 4 = 2$$

$$-3 + 5 = 2 \text{ (and many more)}$$

5. Kabilan was having 10 pencils with him. He gave 2 pencils to Senthil and 3 to Karthick. Next day his father gave him 6 more pencils, from that he gave 8 to his sister. How many pencils are left with him?

Sol :

Total pencils Kabilan had = 10

No. of pencils given to Senthil = 2

No. of pencils given to Karthick = 3

Now number of pencils left with Kabilan $= 10 - 2 - 3$
 $= 8 - 3 = 5$

Number of pencils got from his father = 6

No. total pencils Kabilan had $= 5 + 6 = 11$

Number of pencils given to his sister = 8

Number of pencils left with Kabilan $= 11 - 8 = 3$

6. A lift is on the ground floor. If it goes 5 floors down and then moves up to 10 floors from there, then in which floor will the lift be?

Sol :

Initially the lift will be in the ground floor representing '0'

It goes to 5 floors down $\Rightarrow -5$

Then it moves 10 floors up $\Rightarrow +10$.

Now the lift will be $= 0 - 5 + 10 = -5 + 10$
 $= 5\text{th floor (above the ground floor)}$

7. What number should be added to (-17) to get -19?

Sol :

According to the problem $= -17 + \text{A number} = -19$

The number $= -19 + 17 = -2$

$\therefore -2$ should be added to -17 to get -19

8. A student was asked to subtract (-12) from -47. He got -30. Is he correct? Justify.

Sol :

Subtracting -12 from -47, we get

$$-47 - (-12) = -47 + (\text{Additive inverse of } -12)$$

$$= -47 + (+12) = -35$$

But the students answer is -30.

So he is not correct.

9. One night in Kashmir, the temperature is -5°C . Next day the temperature is 9°C . What is the increase in temperature?

Sol :

Temperature in the first day $= -5^\circ\text{C}$

Temperature in the next day $= 9^\circ\text{C}$

$\therefore$ Increase in temperature $= 9^\circ\text{C} - (-5^\circ\text{C})$
 $= 9^\circ\text{C} + (+5^\circ\text{C}) = 14^\circ\text{C}$

10. Add 2 to me. Then multiply by 5 and subtract 10 and divide now by 4 and I will give you 15! Who am I?

Sol :

According to the problem

$$\{[(I + 2) \times 5] - 10\} \div 4 = 15$$

$$\{[(I + 2) \times 5] - 10\} = 15 \times 4 = 60$$

$$I + 2 = \frac{70}{5} = 14$$

$$(I + 2) \times 5 = 60 + 10 = 70$$

$$I = 14 - 2 ; I = 12$$

11. Kamatchi, a fruit vendor sells 30 apples and 50 pomegranates. If she makes a profit of ₹ 8 per apple and loss ₹ 5 per pomegranate. What will be her overall profit or loss?

Sol :

Number of apples Kamatchi sold = 30

Profit per apple = ₹ 8(+)

$\therefore$ Profit for 30 apples = $30 \times 8 = ₹ 240$
 Number of pomegranates sold 50
 Loss per pomegranate = ₹ 5(-)
 Loss on selling 50 pomegranates = $50 \times (-5)$
 $= ₹ -250$
 Overall loss = $-250 + 240 = ₹ -10$
 i.e. loss ₹ 10.

12. Arrange the following in ascending order.

(i) 2.35, 2.53, 5.32, 3.52, 3.25

(ii) 123.45, 123.54, 125.43, 125.34, 125.3

Sol :

(i) 2.35, 2.53, 5.32, 3.52, 3.25

Comparing the whole number parts of all the numbers 5 is the greatest and $5 > 3 > 2$.

$\therefore$ Greatest number is 5.32 ... (1)

Next 3.52 and 3.25 are equal in their whole number.

So comparing their digits in tenths place, we get $5 > 2$

So $3.52 > 3.25$

Now comparing 2.35 and 2.53 their whole number parts also equal.

$\therefore$ Comparing the digit in tenths place we get

$2.53 > 2.35$... (2)

Ascending order :

$2.35 < 2.53 < 3.25 < 3.52 < 5.32$

(ii) 123.45, 123.54, 125.43, 125.34, 125.3

Comparing the whole number parts we have 123 is the smallest number and two numbers 123.45 and 123.54 have same whole number part.

123.45 and 123.54 comparing their digits in the tenths place we get $4 < 5$

$\therefore$ 123.45 < 123.54 ... (1)

Now comparing the remaining numbers

125.43, 125.34, 125.3 they all have the same whole number part.

Comparing the numbers in the tenth place we have $3 < 4$

$\therefore$ 125.43 is the greatest ... (2)

Also tenth place value $3 = 3$ in 125.34 and 125.3

Again comparing the hundredth place value in 125.34 and 125.3, we get

$125.3 < 125.34$... (3)

From (1), (2) and (3) we have,

$123.45 < 123.54 < 125.3 < 125.34 < 125.43$

13. Which is greater?

(i) 0.888 (or) 0.28 (ii) 23.914 (or) 23.915

Sol :

(i) 0.888 (or) 0.28

The whole number parts is equal for both the numbers.

Comparing the digits in the tenth place we get, $8 > 2$.

$\therefore$ 0.888 > 0.28

0.888 is greater.

(ii) 23.914 or 23.915

The whole number part is equal in both the numbers.

Also the tenth place and hundredth place are also equal.

$\therefore$ Comparing the thousandth place, we get $5 > 4$.

$23.915 > 23.914$

$\therefore$ 23.915 is greater.

14. In a 25 m swimming competition, the time taken by 5 swimmers A, B, C, D and E are 15.7 seconds, 15.68 seconds, 15.6 seconds, 15.74 seconds and 15.67 seconds respectively. Identify the winner.

Sol :

The winner is one who took less time for swimming 25 m.

Comparing the time taken by A, B, C, D, E the whole number part is equal for all participants.

Comparing digit in tenths place we get $6 < 7$.

$\therefore$ Comparing 15.68, 15.6, 15.67, that is comparing the digits in hundredths place we get $15.60 < 15.67 < 15.68$

One who took 15.6 seconds is the winner.

$\therefore$ C is the winner.

15. There are 26 boys and 24 girls in a class. Express the fractions of boys and girls as decimal numbers.

Sol :

Boys = 26

Girls = 24

Total = 50

Fraction of boys = $\frac{26}{50} = \frac{26 \times 2}{50 \times 2} = \frac{52}{100} = 0.52$

Fraction of girls = $\frac{24}{50} = \frac{24 \times 2}{50 \times 2} = \frac{48}{100} = 0.48$

16. What should be subtracted from 7.1 to get 0.713?

Sol :

To get the number to be subtracted

We have $7.1 - 0.713 = 6.387$

$\therefore$ The number to be subtracted

$= 6.387$

	7.100
(-)	0.713
	6.387

17. How much is 35.6 km less than 53.7 km?

Sol :

To get the answer we must subtract

$53.7 \text{ km} - 35.6 \text{ km} = 18.1 \text{ km}$

So 35.6 km is 18.1 km less than 53.7 km.

	53.7
(-)	35.6
	18.1

18. Find the perimeter of an equilateral triangle with a side measuring 3.8 cm.

Sol :

Perimeter of an equilateral triangle } = (Side + Side + Side) Sq. units.

	3.8
(+)	3.8
(+)	3.8
	11.4

Given side = 3.8

$\therefore$ Perimeter = $3.8 + 3.8 + 3.8$

Perimeter of the triangle = 11.4 cm



Mathematics

- 1. Numbers**
- 2. Measurements**
- 3. Algebra**
- 4. Life Mathematics**
- 5. Geometry**
- 6. Statistics**
- 7. Information Processing**

MATHEMATICS

1. NUMBERS

IMPORTANT POINTS

- ◆ A number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$ is called a rational number.
- ◆ All natural numbers, whole numbers, integers and fractions are rational numbers.
- ◆ Every rational number can be represented on a number line.
- ◆ 0 is neither a positive nor a negative rational number.
- ◆ A rational number $\frac{a}{b}$ is said to be in the standard form if its denominator b is a positive integer and HCF (a, b) = 1
- ◆ There are unlimited numbers of rational numbers between two rational numbers.
- ◆ Subtracting two rational numbers is the same as adding the additive inverse of the second number to the first rational number.
- ◆ Multiplying two rational numbers is the same as multiplying their numerators and denominators separately and then writing the product in the standard form.
- ◆ Dividing a rational number by another rational number is the same as multiplying the first rational number by the reciprocal of the second rational number.
- ◆ The following table is about the properties of rational numbers ($\mathbb{Q}$).

$\mathbb{Q}$	Closure	Commutative	Associative	Multiplication is distributive over +/−
+	✓	✓	✓	✓
−	✓	×	×	✓
×	✓	✓	✓	−
÷	×	×	×	−

- ◆ 0 and 1 are respectively the additive and the multiplicative identities of rational numbers.

- ◆ The additive inverse for $\frac{a}{b}$ is $\frac{-a}{b}$ and vice – versa.
- ◆ The reciprocal or the multiplicative inverse of a rational number $\frac{a}{b}$ is $\frac{b}{a}$ since $\frac{a}{b} \times \frac{b}{a} = 1$.
- ◆ A natural number n is called a square number, if we can find another natural number m such that $n = m^2$.
- ◆ The square root of a number n , written as $\sqrt{n}$ (or) $n^{\frac{1}{2}}$, is the number that gives n when multiplied by itself.
- ◆ The number of times a prime factor occurs in the square is equal to twice the number of times it occurs in the prime factorization of the number.
- ◆ For any two positive numbers a and b , we have

$$(i) \sqrt{ab} = \sqrt{a} \times \sqrt{b} \text{ and } (ii) \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \text{ (} b \neq 0 \text{)}$$

- ◆ If you multiply a number by itself and then by itself again, the result is a cube number.
- ◆ The cube root of a number is the value that when cubed gives the original number.
- ◆ An expression that represents repeated multiplication of the same factor is called a power.
- ◆ The exponent corresponds to the number of times the base is used as a factor.
- ◆ Laws of Exponents: (i) $a^m \times a^n = a^{m+n}$

$$(ii) \frac{a^m}{a^n} = a^{m-n}$$

$$(iii) (a^m)^n = a^{mn}$$

- ◆ Other results: (i) $a^0 = 1$ (ii) $a^{-m} = \frac{1}{a^m}$

$$(iii) a^m \times b^m = (ab)^m \text{ (iv) } \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

Example Problems

1. Compare the following pairs of rational numbers.

$$(i) \frac{-11}{5}, \frac{-21}{8} \quad (ii) \frac{3}{-4}, \frac{-1}{2} \quad (iii) \frac{2}{3}, \frac{4}{5}$$

Sol:

$$(i) \frac{-11}{5}, \frac{-21}{8}$$

LCM of 5, 8 is 40

$$\frac{-11}{5} = \frac{-11 \times 8}{5 \times 8} = \frac{-88}{40}$$

$$\frac{-21}{8} = \frac{-21 \times 5}{8 \times 5} = \frac{-105}{40}$$

$$\frac{-105}{40} < \frac{-88}{40} \quad \therefore \frac{-21}{8} < \frac{-11}{5}$$

(ii) $\frac{3}{-4}, \frac{-1}{2}$

LCM of 4 and 2 = 4

$$\frac{3}{-4} = \frac{-3}{4}$$

$$\frac{-1}{2} = \frac{-1 \times 2}{2 \times 2} = \frac{-2}{4}$$

$$\frac{3}{-4} < \frac{-2}{4} \Rightarrow \frac{3}{-4} < \frac{-1}{2}$$

(iii) $\frac{2}{3}, \frac{4}{5}$

LCM of 3 and 5 is 15.

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

$$\frac{10}{15} < \frac{12}{15} \quad \therefore \frac{2}{3} < \frac{4}{5}$$

2. Arrange the following rational numbers in ascending and descending order.

(i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{9}, \frac{12}{36}$ (ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

Sol:

(i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{9}, \frac{12}{36}$

LCM of 12, 8, 24, 9, 36 is $4 \times 3 \times 2 \times 3 = 72$

$$\frac{-5}{12} = \frac{-5 \times 6}{12 \times 6} = \frac{-30}{72}$$

$$\frac{-11}{8} = \frac{-11 \times 9}{8 \times 9} = \frac{-99}{72}$$

$$\frac{-15}{24} = \frac{-15 \times 3}{24 \times 3} = \frac{-45}{72}$$

$$\frac{-7}{9} = \frac{7 \times 8}{9 \times 8} = \frac{56}{72}$$

$$\frac{12}{36} = \frac{12 \times 2}{36 \times 2} = \frac{24}{72}$$

Now comparing the numerators -30, -99, -45, 56, 24 we get $56 > 24 > -30 > -45 > -99$

i.e. $\frac{56}{72} > \frac{24}{72} > \frac{-30}{72} > \frac{-45}{72} > \frac{-99}{72}$

and so $\frac{-7}{-9} > \frac{12}{36} > \frac{-5}{12} > \frac{-15}{24} > \frac{-11}{8}$

$\therefore$ Descending order $\frac{-7}{-9} > \frac{12}{36} > \frac{-5}{12} > \frac{-15}{24} > \frac{-11}{8}$

Ascending order $\frac{-11}{8} < \frac{-15}{24} < \frac{-5}{12} < \frac{12}{36} < \frac{-7}{-9}$

(ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

LCM of 10, 5, 4, 20 is $5 \times 2 \times 2 = 20$

$$\frac{-17}{10} = \frac{-17 \times 2}{10 \times 2} = \frac{-34}{20}$$

$$\frac{-7}{5} = \frac{-7 \times 4}{5 \times 4} = \frac{-28}{20}$$

$$\frac{-2}{4} = \frac{-2 \times 5}{4 \times 5} = \frac{-10}{20}$$

$$\frac{-19}{20} = \frac{-19}{20}$$

Negative numbers are less than zero.

$\therefore$ Arranging the numerators we get

$$-34 < -28 < -19 < -10 < 0$$

$$\therefore \frac{-34}{20} < \frac{-28}{20} < \frac{-19}{20} < \frac{-10}{20} < 0$$

$$\text{Ascending order} = \frac{-17}{10} < \frac{-7}{5} < \frac{-19}{20} < \frac{-2}{4} < 0$$

$$\text{Descending order } 0 > \frac{-2}{4} > \frac{-19}{20} > \frac{-7}{5} > \frac{-17}{10}$$

3. Find the sum :

(i) $\frac{7}{5} + \frac{3}{5}$

(ii) $\frac{7}{5} + \frac{5}{7}$

(iii) $\frac{6}{5} + \left(\frac{-14}{15}\right)$

(iv) $-4\frac{2}{3} + 7\frac{5}{12}$

Sol:

(i) $\frac{7}{5} + \frac{3}{5} = \frac{7+3}{5} = \frac{10}{5} = 2$

(ii) $\frac{7}{5} + \frac{5}{7} = \frac{7 \times 7 + 5 \times 5}{35} = \frac{49+25}{35} = \frac{74}{35}$

(iii) $\frac{6}{5} + \left(\frac{-14}{15}\right) = \frac{6 \times 3 + (-14)}{15} = \frac{18 + (-14)}{5} = \frac{4}{5}$

(iv) $-4\frac{2}{3} + 7\frac{5}{12} = \frac{14}{3} + \frac{18}{12} = \frac{-14 \times 4 + 89}{12}$

$$= \frac{-56 + 89}{12} = \frac{33}{12} = \frac{11}{4}$$

4. Subtract : $\frac{-8}{44}$ from $\frac{-17}{11}$.

Sol:

$$\frac{-17}{11} - \left(\frac{-8}{44}\right) = \frac{-17}{11} + \frac{8}{44} = \frac{-17 \times 4 + 8}{44}$$

$$= \frac{-68 + 8}{44} = \frac{-60}{44} = \frac{-15}{11}$$

5. Evaluate : (i) $\frac{9}{132} \times \frac{-11}{3}$ (ii) $\frac{-7}{27} \times \frac{24}{-35}$

Sol:

(i) $\frac{\cancel{9}}{\cancel{12} \times \cancel{11}} \times \frac{-\cancel{11}}{\cancel{3}} = \frac{-1}{4}$ (ii) $\frac{-\cancel{7}}{\cancel{9} \times \cancel{3}} \times \frac{\cancel{24}^8}{\cancel{35}_5} = \frac{8}{45}$

6. Divide :

(i) $\frac{-21}{5}$ by $\frac{-7}{-10}$ (ii) $\frac{-3}{13}$ by -3 (iii) -2 by $\frac{-6}{15}$

Sol:

(i) $\frac{-21}{5} \div \frac{-7}{-10} = \frac{-\cancel{21}^3}{\cancel{5}} \times \frac{\cancel{10}^2}{\cancel{7}} = -6$

(ii) $\frac{-3}{13} \div -3 = \frac{-3}{13} \times \frac{-1}{3} = \frac{-3 \times -1}{13 \times 3} = \frac{3}{39}$

(iii) $-2 \div \frac{-6}{15} = -2 \times \frac{15}{-6} = \frac{-2 \times 15}{-6} = \frac{-30}{-6} = 5$

7. Find $(a + b) \div (a - b)$ if (i) $a = \frac{1}{2}, b = \frac{2}{3}$ (ii) $a = \frac{-3}{5}, b = \frac{2}{15}$

Sol:

(i) $a + b = \frac{1}{2} + \frac{2}{3} = \frac{1 \times 3 + 2 \times 2}{6} = \frac{3 + 4}{6} = \frac{7}{6}$

$a - b = \frac{1}{2} - \frac{2}{3} = \frac{1 \times 3 - 2 \times 2}{6} = \frac{3 - 4}{6} = \frac{-1}{6}$

$(a + b) \div (a - b) = \frac{7}{6} \div \frac{-1}{6} = \frac{7}{6} \times \frac{\cancel{6}}{\cancel{-1}} = -7$

(ii) $a + b = \frac{-3}{5} + \frac{2}{15} = \frac{-3 \times 3 + 2}{15} = \frac{-9 + 2}{15} = \frac{-7}{15}$

$a - b = \frac{-3}{5} - \frac{2}{15} = \frac{-3 \times 3 - 2}{15} = \frac{-9 - 2}{15} = \frac{-11}{15}$

$(a + b) \div (a - b) = \frac{-7}{15} \div \frac{-11}{15} = \frac{-7}{15} \times \frac{\cancel{15}}{\cancel{-11}} = \frac{7}{11}$

8. Simplify :

(i) $\left[\frac{11}{8} \times \left(\frac{-6}{33} \right) \right] + \left[\frac{1}{3} + \left(\frac{3}{5} \div \frac{9}{20} \right) \right] - \left[\frac{4}{7} \times \frac{-7}{5} \right]$

(ii) $\left[\frac{4}{3} \div \left(\frac{8}{-7} \right) \right] - \left[\frac{3}{4} \times \frac{4}{3} \right] + \left[\frac{4}{3} \times \left(\frac{-1}{4} \right) \right]$

Sol:

(i) $\left[\frac{11}{8} \times \left(\frac{-6}{\cancel{33}} \right) \right] + \left[\frac{1}{3} + \left(\frac{3}{5} \div \frac{9}{20} \right) \right] - \left[\frac{4}{7} \times \frac{-7}{5} \right]$

$= \frac{\cancel{11} \times (-\cancel{6})}{\cancel{8} \times \cancel{33}} + \left[\frac{1}{3} + \left(\frac{\cancel{3}}{\cancel{5}} \times \frac{\cancel{20}^4}{\cancel{9}_3} \right) \right] - \left[\frac{4 \times -\cancel{7}}{\cancel{7} \times 5} \right]$

$= -\frac{1}{4} + \left[\frac{1}{3} + \frac{4}{3} \right] - \left(\frac{-4}{5} \right)$

$= -\frac{1}{4} + \frac{5}{3} + \frac{4}{5} = \frac{-15 + 100 + 48}{60} = \frac{133}{60}$

(ii) $\left[\frac{4}{3} \div \left(\frac{8}{-7} \right) \right] - \left[\frac{3}{4} \times \frac{4}{3} \right] + \left[\frac{4}{3} \times \left(\frac{-1}{4} \right) \right]$

$= \left[\frac{\cancel{4}}{3} \times \frac{-7}{\cancel{8}_2} \right] - \left[\frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{3}} \right] + \left[\frac{\cancel{4} \times (-1)}{3 \times \cancel{4}} \right]$

$= \left(\frac{-7}{6} \right) - 1 + \left(\frac{-1}{3} \right) = \frac{-7 - 6 + (-2)}{6} = \frac{-15}{6} = \frac{-5}{2}$

9. A student had multiplied a number by $\frac{4}{3}$ instead of dividing it by $\frac{4}{3}$ and got 70 more than the correct answer. Find the number.

Sol:

Let the number = a

$a \times \frac{4}{3} - a \div \frac{4}{3} = 70$

$a \times \frac{4}{3} - a \times \frac{3}{4} = 70 \Rightarrow a \left[\frac{4}{3} - \frac{3}{4} \right] = 70$

$a \left[\frac{4 \times 4 - 3 \times 3}{12} \right] = 70$

$a \left[\frac{16 - 9}{12} \right] = 70 \Rightarrow a \left[\frac{7}{12} \right] = 70$

$a = \frac{70 \times 12}{7} = 120$

10. Find the square root by prime factorisation method.

(i) 144 (ii) 256 (iii) 784 (iv) 1156 (v) 4761

(vi) 9025

Sol:

(i) 144

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$\sqrt{144} = 2 \times 2 \times 3 = 12$

(ii) 256

$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\sqrt{256} = 2 \times 2 \times 2 \times 2 = 16$

$$\begin{array}{r} 2 \overline{) 144} \\ 2 \overline{) 72} \\ 2 \overline{) 36} \\ 2 \overline{) 18} \\ 3 \overline{) 9} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \overline{) 256} \\ 2 \overline{) 128} \\ 2 \overline{) 64} \\ 2 \overline{) 32} \\ 2 \overline{) 16} \\ 2 \overline{) 8} \\ 2 \overline{) 4} \\ 2 \end{array}$$

9th
Standard

Mathematics

- 1. Set Language**
- 2. Real Numbers**
- 3. Algebra**
- 4. Geometry**
- 5. Coordinate Geometry**
- 6. Trigonometry**
- 7. Mensuration**
- 8. Statistics**
- 9. Probability**

MATHEMATICS

1. SET LANGUAGE

IMPORTANT POINTS

In our daily life, we often deal with collection of objects like books, stamps, coins, etc. Set language is a mathematical way of representing a collection of objects.

SET :

- A set is a well - defined collection of objects.
- The objects of a set are called its members or elements.

For example,

- The collection of all books in a District Central Library.
- The collection of all colours in a rainbow.

REPRESENTATION OF A SET :

- The collection of odd numbers can be described in many ways:
 - "The set of odd numbers" is a fine description, we understand it well.
 - It can be written as $\{1, 3, 5, \dots\}$ and you know what I mean.
 - Also, it can be said as the collection of all numbers x where x is an odd number.

DESCRIPTIVE FORM :

- In descriptive form, a set is described in words.

For example,

 - The set of all vowels in English alphabets.
 - The set of whole numbers.

SET BUILDER FORM OR RULE FORM :

- In set builder form, all the elements are described by a rule.

For example,

 - $A = \{x : x \text{ is a vowel in English alphabets}\}$
 - $B = \{x \mid x \text{ is a whole number}\}$

ROSTER FORM OR TABULAR FORM

- A set can be described by listing all the elements of the set.

For example,

 - $A = \{a, e, i, o, u\}$
 - $B = \{0, 1, 2, 3, \dots\}$

TYPES OF SETS

EMPTY SET OR NULL SET :

- A set consisting of no element is called the empty set or null set or void set.

For example,

 $A = \{x : x \text{ is an odd integer and divisible by } 2\}$ $\therefore A = \{ \} \text{ or } \emptyset$

SINGLETON SET :

- A set which has only one element is called a singleton set.

For example,

$$A = \{x : 3 < x < 5, x \in \mathbb{N}\}$$

FINITE SET :

- A set with finite number of elements is called a finite set.

For example,

- The set of family members.
- The set of indoor/outdoor games you play.

INFINITE SET :

- A set which is not finite is called an infinite set.

For example,

- $\{5, 10, 15, \dots\}$
- The set of all points on a line.

EQUIVALENT SETS :

- Two finite sets A and B are said to be equivalent if they contain the same number of elements. It is written as $A \approx B$.
If A and B are equivalent sets, then $n(A) = n(B)$.

EQUAL SETS :

- Two sets are said to be equal if they contain exactly the same elements, otherwise they are said to be unequal.
In other words, two sets A and B are said to be equal, if
 - every element of A is also an element of B
 - every element of B is also an element of A .

SUBSET :

- Let A and B be two sets. If every element of A is also an element of B , then A is called a subset of B . We write $A \subseteq B$.

PROPER SUBSET :

- Let A and B be two sets. If A is a subset of B and $A \neq B$, then A is called a proper subset of B and we write $A \subset B$.

For example,

If $A = \{1, 2, 5\}$ and $B = \{1, 2, 3, 4, 5\}$ then A is a proper subset of B i.e. $A \subset B$.

POWER SET :

- The set of all subsets of A is said to be the power set of the set A and is denoted as $P(A)$

For example,

$$\text{Let } A = \{-3, 4\}$$

The subsets of A are $\emptyset, \{-3\}, \{4\}, \{-3, 4\}$

Then the power set of A is $P(A) = \{\emptyset, \{-3\}, \{4\}, \{-3, 4\}\}$

SET OPERATIONS :

COMPLEMENT OF A SET

- The Complement of a set A is the set of all elements of U (the universal set) that are not in A . It is denoted by A' or A^c . In symbols $A' = \{x : x \in U, x \notin A\}$

UNION OF TWO SETS

- ◆ The union of two sets A and B is the set of all elements which are either in A or in B or in both. It is denoted by $A \cup B$ and read as A union B. In symbol, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

INTERSECTION OF TWO SETS

- ◆ The intersection of two sets A and B is the set of all elements common to both A and B. It is denoted by $A \cap B$ and read as A intersection B. In symbol, $A \cap B = \{x : x \in A \text{ and } x \in B\}$

DIFFERENCE OF TWO SETS

- ◆ Let A and B be two sets, the difference of sets A and B is the set of all elements which are in A, but

not in B. It is denoted by $A - B$ or $A \setminus B$ and read as A difference B.

In symbol, $A - B = \{x : x \in A \text{ and } x \notin B\}$;

$B - A = \{y : y \in B \text{ and } y \notin A\}$.

SYMMETRIC DIFFERENCE OF SETS

- ◆ The symmetric difference of two sets A and B is the set $(A - B) \cup (B - A)$. It is denoted by $A \Delta B$. $A \Delta B = \{x : x \in A - B \text{ or } x \in B - A\}$.

PROPERTIES OF SET OPERATIONS

- ◆ It is an interesting investigation to find out if operations among sets (like union, intersection, etc) follow mathematics properties such as commutativity, Associativity, etc.

Example Problems

1. Which of the following are sets?

- (i) The Collection of prime numbers upto 100.
(ii) The Collection of rich people in India.
(iii) The Collection of all rivers in India.
(iv) The Collection of good Hockey players.

Sol:

- (i) $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 \text{ and } 97\}$

As the collection of prime numbers upto 100 is known and can be counted (well defined). Hence this is a set.

- (ii) The collection of rich people in India. Rich people has no definition.

Hence, it is not a set.

- (iii) $A = \{\text{Cauvery, Sindhu, Ganga, ...}\}$

Hence, it is a set.

- (iv) The collection of good hockey players is not a well - defined collection because the criteria for determining a hockey player's talent may vary from person to person.

Hence, this collection is not a set.

2. List the set of letters of the following words in Roster form.

- (i) INDIA (ii) PARALLELOGRAM
(iii) MISSISSIPPI (iv) CZECHOSLOVAKIA

Sol:

- (i) $A = \{I, N, D, A\}$
(ii) $B = \{P, A, R, L, E, O, G, M\}$
(iii) $C = \{M, I, S, P\}$
(iv) $D = \{C, Z, E, H, O, S, L, V, A, K, I\}$.

3. Consider the following sets $A = \{0, 3, 5, 8\}$ $B = \{2, 4, 6, 10\}$ $C = \{12, 14, 18, 20\}$

- (a) State whether True or false.

- (i) $18 \in C$ (ii) $6 \in A$ (iii) $14 \in C$ (iv) $10 \in B$
(v) $5 \in B$ (vi) $0 \in B$

- (b) Fill in the blanks?

- (i) $3 \in \underline{\hspace{1cm}}$ (ii) $14 \in \underline{\hspace{1cm}}$ (iii) $18 \in \underline{\hspace{1cm}} B$
(iv) $4 \in \underline{\hspace{1cm}} B$

Sol:

- (a) (i) True (ii) True (iii) False (iv) True (v) False
(vi) False.

- (b) (i) A (ii) C (iii) $\notin$ (iv) $\in$

4. Represent the following sets in Roster form.

- (i) $A =$ The set of all even natural numbers less than 20.

- (ii) $B = \{y : y = \frac{1}{2n}, n \in \mathbb{N}, n \leq 5\}$

- (iii) $C = \{x : x \text{ is perfect cube, } 27 < x < 216\}$

- (iv) $D = \{x : x \in \mathbb{Z}, -5 < x \leq 2\}$

Sol:

- (i) $A = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

- (ii) $n = \{1, 2, 3, 4, 5\}$

$$\text{if, } n = 1, y = \frac{1}{2n} = \frac{1}{2 \times 1} = \frac{1}{2}$$

$$n = 2, y = \frac{1}{2 \times 2} = \frac{1}{4}$$

$$n = 3, y = \frac{1}{2 \times 3} = \frac{1}{6}$$

$$n = 4, y = \frac{1}{2 \times 4} = \frac{1}{8}$$

$$n = 5, y = \frac{1}{2 \times 5} = \frac{1}{10}$$

$$\therefore B = \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10} \right\}$$

- (iii) $C = \{64, 125\}$

- (iv) $D = \{-4, -3, -2, -1, 0, 1, 2\}$

5. Represent the following sets in set builder form.

- (i) $B =$ The set of all Cricket players in India who scored double centuries in One Day Internationals.

- (ii) $C = \left\{ \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots \right\}$.

- (iii) $D =$ The set of all Tamil months in a year.

- (iv) $E =$ The set of odd Whole numbers less than 9.

Sol:

(i) $B = \{x : x \text{ is an Indian player who scored double centuries in one day internationals}\}$

(ii) $C = \{x : x = \frac{n}{n+1}, n \in \mathbb{N}\}$

(iii) $D = \{x : x \text{ is a tamil month in a year}\}$

(iv) $E = \{x : x \text{ is odd number, } x \in \mathbb{W}, x < 9, \text{ where } \mathbb{W} \text{ is the set of whole numbers}\}.$

6. Represent the following sets in descriptive form.

(i) $P = \{\text{January, June, July}\}$

(ii) $Q = \{7, 11, 13, 17, 19, 23, 29\}$

(iii) $R = \{x : x \in \mathbb{N}, x < 5\}$

(iv) $S = \{x : x \text{ is a consonant in English alphabets}\}$

Sol:

(i) P is the set of English Months beginning with J.

(ii) Q is the set of all prime numbers between 5 and 31.

(iii) R is the set of all natural numbers less than 5.

(iv) S is the set of all English consonants.

7. Find the cardinal number of the following sets.

(i) $M = \{p, q, r, s, t, u\}$

(ii) $P = \{x : x = 3n + 2, n \in \mathbb{W} \text{ and } x < 15\}$

(iii) $Q = \{y : y = \frac{4}{3n}, n \in \mathbb{N} \text{ and } 2 < n \leq 5\}$

(iv) $R = \{x : x \text{ is an integers, } x \in \mathbb{Z} \text{ and } -5 \leq x < 5\}$

(v) $S = \{\text{The set of all leap years between 1882 and 1906}\}.$

Sol:

(i) $n(M) = 6$

(ii) $W = \{0, 1, 2, 3, \dots\}$

if $n = 0, x = 3(0) + 2 = 2$

if $n = 1, x = 3(1) + 2 = 5$

if $n = 2, x = 3(2) + 2 = 8$

if $n = 3, x = 3(3) + 2 = 11$

if $n = 4, x = 3(4) + 2 = 14$

$\therefore P = \{2, 5, 8, 11, 14\}$

$n(P) = 5$

(iii) $N = \{1, 2, 3, 4, \dots\}$

$n \in \{3, 4, 5\}$

if $n = 3, y = \frac{4}{3(3)} = \frac{4}{9}$

if $n = 4, y = \frac{4}{3(4)} = \frac{4}{12}$

if $n = 5, y = \frac{4}{3(5)} = \frac{4}{15}$

$Q = \left\{\frac{4}{9}, \frac{4}{12}, \frac{4}{15}\right\}$

$n(Q) = 3$

(iv) $x \in \mathbb{Z}$

$R = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

$n(R) = 10.$

(v) $S = \{1884, 1888, 1892, 1896, 1904\}$

$n(S) = 5.$

8. Which of the following sets are equivalent or unequal or equal sets?

i) $A = \{\text{The set of vowels in the English alphabets}\}.$

$B = \{\text{The set of all letters in the word "VOWEL"}\}$

ii) $C = \{2, 3, 4, 5\}$

$D = \{x : x \in \mathbb{W}, 1 < x < 5\}$

iii) $X = A = \{x : x \text{ is a letter in the word "LIFE"}\}$

$Y = \{F, I, L, E\}$

iv) $G = \{x : x \text{ is a prime number and } 3 < x < 23\}$

$H = \{x : x \text{ is a divisor of } 18\}$

Sol:

i) $A = \{a, e, i, o, u\}$

$B = \{V, O, W, E, L\}$

The sets A and B contain the same number of elements. $\therefore$ Equivalent sets

ii) $C = \{2, 3, 4, 5\}$

$D = \{2, 3, 4\}$

$\therefore$ Unequal sets

iii) $X = \{L, I, F, E\}$

$Y = \{F, I, L, E\}$

The sets X and Y contain the exactly the same elements. $\therefore$ Equal sets

iv) $G = \{5, 7, 11, 13, 17, 19\}$

$H = \{1, 2, 3, 6, 9, 18\}$

$\therefore$ Equivalent sets.

9. Identify the following sets as null set or singleton set.

i) $A = \{x : x \in \mathbb{N}, 1 < x < 2\}$

ii) $B = \{\text{The set of all even natural numbers which are not divisible by 2}\}.$

iii) $C = \{0\}.$

iv) $D = \{\text{The set of all triangles having four sides}\}.$

Sol:

i) $A = \{ \}$ Q There is no element in between 1 and 2 in Natural numbers. $\therefore$ Null set

ii) $B = \{ \}$ Q All even natural numbers are divisible by 2. $\therefore$ B is Null set

iii) $C = \{0\}$

$\therefore$ Singleton set

iv) $D = \{ \}$ Q No triangle has four sides.

$\therefore$ D is a Null set.

10. State which pairs of sets are disjoint or overlapping?

i) $A = \{f, i, a, s\}$ and $B = \{a, n, f, h, s\}$

ii) $C = \{x : x \text{ is a prime number, } x > 2\}$ and $D = \{x : x \text{ is an even prime number}\}$

iii) $E = \{x : x \text{ is a factor of } 24\}$ and $F = \{x : x \text{ is a multiple of } 3, x < 30\}$

Sol:

i) $A = \{f, i, a, s\}$

$B = \{a, n, f, h, s\}$

$A \cap B = \{f, i, a, s\} \cap \{a, n, f, h, s\} = \{f, a, s\}$

Since $A \cap B \neq \emptyset$, A and B are overlapping sets.

ii) $C = \{3, 5, 7, 11, \dots\}$

$D = \{2\}$

$C \cap D = \{3, 5, 7, 11, \dots\} \cap \{2\} = \{ \}$

Since $C \cap D = \emptyset$, C and D are disjoint sets.

10th
Standard

Mathematics

- 1. Relations and Functions**
- 2. Numbers and Sequences**
- 3. Algebra**
- 4. Geometry**
- 5. Coordinate Geometry**
- 6. Trigonometry**
- 7. Mensuration**
- 8. Statistics and Probability**

MATHEMATICS

1. RELATIONS AND FUNCTIONS

IMPORTANT POINTS

VERTICAL LINE

- ◆ A curve drawn in a graph represents a function, if every vertical line intersects the curve in at most one point.

HORIZONTAL LINE

- ◆ A function represented in a graph is one - one, if every horizontal line intersects the curve in at most one point.
- ◆ Linear functions have applications in Cryptography as well as in several branches of Science and Technology.

CARTESIAN PRODUCT

If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A$, $b \in B$ is called the **Cartesian Product of A and B**, and is denoted by $A \times B$.

$$\text{Thus, } A \times B = \{(a, b) \mid a \in A, b \in B\}$$

STANDARD INFINITE SETS

Natural Numbers $\mathbb{N} = \{1, 2, 3, 4, \dots\}$;

Whole Numbers $\mathbb{W} = \{0, 1, 2, 3, \dots\}$;

Integers $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$;

$$\text{Rational Numbers } \mathbb{Q} = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0 \right\};$$

Real Numbers $\mathbb{R} = \mathbb{Q} \cup \mathbb{Q}'$, where $\mathbb{Q}'$ is the set of all irrational numbers.

The set of all points in the cartesian plane can be viewed as the set of all ordered pairs (x, y) where x, y are real numbers. In fact, $\mathbb{R} \times \mathbb{R}$ is the set of all points which we call as the cartesian plane.

RELATIONS

Let A and B be any two non-empty sets. A '**relation**' R from A to B is a subset of $A \times B$ satisfying some specified conditions. If $x \in A$ is related to $y \in B$ through R, then we write it as xRy . xRy if and only if $(x, y) \in R$.

Null relation

Let us consider the following example. Suppose $A = \{-3, -2, -1\}$ and $B = \{1, 2, 3, 4\}$. A relation from A to B is defined as $a - b = 8$ i.e., there is no pair (a, b) such that $a - b = 8$. Thus R contains no element and so $R = \phi$.

A relation which contains no element is called a "**Null relation**".

If $n(A) = p$, $n(B) = q$, then the total number of relations that exist between A and B is 2^{pq} .

FUNCTIONS

A relation f between two non-empty sets X and Y is

called a function from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x, y) \in f$.

$$\text{That is, } f = \{(x, y) \mid \text{for all } x \in X, y \in Y\}$$

A function f from X to Y is written as $f: X \rightarrow Y$.

- ◆ If $f: X \rightarrow Y$ is a function then, the set X is called the domain of the function f and the set Y is called its co-domain.
- ◆ If $f(a) = b$, then b is called '**image**' of a under f and a is called a '**pre-image**' of b.
- ◆ The set of all images of the elements of X under f is called the '**range**' of f.
- ◆ $f: X \rightarrow Y$ is a function only if,
 - every element in the domain of f has an image.
 - the image is unique.
- ◆ If A and B are finite sets such that $n(A) = p$, $n(B) = q$ then the total number of functions that exist between A and B is q^p .
- ◆ In this chapter we always consider f to be a real valued function.
- ◆ Describing domain of a function

i. Let $f(x) = \frac{1}{x+1}$. If $x = -1$ then $f(-1)$ is not defined. Hence f is defined for all real numbers except at $x = -1$. So domain of f is $\mathbb{R} - \{-1\}$.

ii. Let $f(x) = \frac{1}{x^2 - 5x + 6}$; If $x = 2, 3$ then $f(2)$ and $f(3)$ are not defined. Hence f is defined for all real numbers except at $x = 2$ and 3. So domain of $f = \mathbb{R} - \{2, 3\}$.

The range of function is a subset of its co-domain.

- ◆ The Cartesian Product of A and B is defined as $A \times B = \{(a, b) \mid \text{for all } a \in A, b \in B\}$
- ◆ A relation R from A to B is always a subset of $A \times B$. That is $R \subseteq A \times B$
- ◆ A relation R from X to Y is a function if for every $x \in X$ there exists only one $x \in Y$.
- ◆ A function can be represented by
 - an arrow diagram
 - a tabular form
 - a set of ordered pairs
 - a graphical form
- ◆ Some types of functions
 - One-one function
 - Onto function
 - Many-one function

iv. Into function

- ◆ Identify function $f(x) = x$
- ◆ Reciprocal function $f(x) = \frac{1}{x}$
- ◆ Constant function $f(x) = c$
- ◆ Linear function $f(x) = ax + b, a \neq 0$
- ◆ Quadratic function $f(x) = ax^2 + bx + c, a \neq 0$
- ◆ Cubic function $f(x) = ax^3 + bx^2 + cx, a \neq 0$

- ◆ For three non-empty sets A, B and C, if $f: A \rightarrow B$ and $g: B \rightarrow C$ are two functions, then the composition of f and g is a function $g \circ f: A \rightarrow C$ will be defined as $g \circ f(x) = g(f(x))$ for all $x \in A$.
- ◆ If f and g are any two functions, then in general, $f \circ g \neq g \circ f$
- ◆ If f, g and h are any three functions, then $f \circ (g \circ h) = (f \circ g) \circ h$.

Example Problems

1. Find $A \times B, A \times A$ and $B \times A$

(i) $A = \{2, -2, 3\}$ and $B = \{1, -4\}$

(ii) $A = B = \{p, q\}$

(iii) $A = \{m, n\}; B = \phi$

Sol:

(i) $A = \{2, -2, 3\}, B = \{1, -4\}$

$$A \times B = \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$$

$$A \times A = \{(2, 2), (2, -2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3, 3)\}$$

$$B \times A = \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$$

(ii) $A = B = \{p, q\}$

$$A \times B = \{(p, p), (p, q), (q, p), (q, q)\}$$

$$A \times A = \{(p, p), (p, q), (q, p), (q, q)\}$$

$$B \times A = \{(p, p), (p, q), (q, p), (q, q)\}$$

(iii) $A = \{m, n\}, B = \phi$

$$A \times B = \{\}$$

$$A \times A = \{(m, m), (m, n), (n, m), (n, n)\}$$

$$B \times A = \{\}$$

2. Let $A = \{1, 2, 3\}$ and $B = \{x \mid x \text{ is a prime number less than } 10\}$. Find $A \times B$ and $B \times A$.

Sol:

$$A = \{1, 2, 3\}, B = \{2, 3, 5, 7\}$$

$$A \times B = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$$

$$B \times A = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$$

3. If $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ find A and B.

Sol:

Given

$$B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$$

$$\text{Here } B = \{-2, 0, 3\}$$

[All the first elements of the order pair]

$$\text{and } A = \{3, 4\}$$

[All the second elements of the order pair]

4. Let $A = \{x \in \mathbb{W} \mid x < 2\}$, $B = \{x \in \mathbb{N} \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Sol:

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$$A = \{x \in \mathbb{W} \mid x < 2\} = \{0, 1\}$$

[Whole numbers less than 2]

$$B = \{x \in \mathbb{N} \mid 1 < x \leq 4\} = \{2, 3, 4\}$$

$$C = \{3, 5\} \quad [\text{Natural numbers from 2 to 4}]$$

$$\text{LHS} = A \times (B \cup C)$$

$$B \cup C = \{2, 3, 4\} \cup \{3, 5\}$$

$$= \{2, 3, 4, 5\}$$

$$A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\} \quad \dots(1)$$

$$\text{RHS} = (A \times B) \cup (A \times C)$$

$$(A \times B) = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$$

$$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$$

$$(A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\} \quad \dots(2)$$

(1) = (2), LHS = RHS Hence it is proved.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$\text{LHS} = A \times (B \cap C)$$

$$(B \cap C) = \{3\}$$

$$A \times (B \cap C) = \{(0, 3), (1, 3)\} \quad \dots(1)$$

$$\text{RHS} = (A \times B) \cap (A \times C)$$

$$(A \times B) = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$$

$$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$$

$$(A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\} \quad \dots(2)$$

$$(1) = (2) \Rightarrow \text{LHS} = \text{RHS}$$

Hence it is verified.

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

$$\text{LHS} = (A \cup B) \times C$$

$$A \cup B = \{0, 1, 2, 3, 4\}$$

$$(A \cup B) \times C = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\} \quad \dots(1)$$

$$\text{RHS} = (A \times C) \cup (B \times C)$$

$$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$$

$$(B \times C) = \{(2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$$

$$(A \times C) \cup (B \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\} \quad \dots(2)$$

$$(1) = (2)$$

$\therefore \text{LHS} = \text{RHS}$. Hence it is verified.

5. Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$, which of the following are relation from A to B ?

(i) $R_1 = \{(2, 1), (7, 1)\}$

(ii) $R_2 = \{(-1, 1)\}$

(iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$

(iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$

Sol:

Given $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$

(i) $R_1 = \{(2, 1), (7, 1)\}$

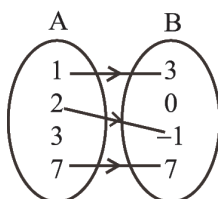
2 and 7 cannot be related to 1 since $1 \notin B$

$\therefore R_1$ is not a relation.

(ii) $R_2 = \{(-1, 1)\}$

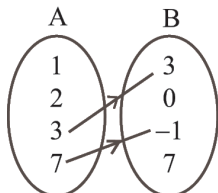
-1 cannot be related to 1 since $-1 \notin A$ and $1 \notin B$
 $\therefore R_2$ is not a relation.

(iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$



R_3 is a relation since 2 is related to -1, 7 is related to 7 and 1 is related to 3.

(iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$



7 is related to -1
 3 is related to 3
 Since $0 \notin A$, 0 cannot be related to 3 and 7.
 $\therefore R_4$ is not a relation.

6. A function f is defined by $f(x) = 2x - 3$

(i) find $\frac{f(0)+f(1)}{2}$

(ii) find x such that $f(x) = 0$.

(iii) find x such that $f(x) = x$.

(iv) find x such that $f(x) = f(1 - x)$.

Sol:

Given $f(x) = 2x - 3$

(i) $\frac{f(0)+f(1)}{2}$

$f(0) = 2(0) - 3 = -3$

$f(1) = 2(1) - 3 = -1$

$\therefore \frac{f(0)+f(1)}{2} = \frac{-3-1}{2} = \frac{-4}{2} = -2$

(ii) $f(x) = 0 \Rightarrow 2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

(iii) $f(x) = x \Rightarrow 2x - 3 = x \Rightarrow 2x - x = 3$

$x = 3$

(iv) $f(x) = f(1 - x)$

$2x - 3 = 2(1 - x) - 3$

$2x - 3 = 2 - 2x - 3$

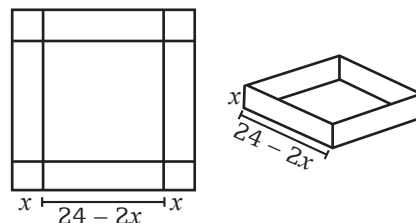
$2x + 2x = 2 - 3 + 3$

$4x = 2$

$x = \frac{2}{4} = \frac{1}{2}$

$x = \frac{1}{2}$

7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown in figure. Express the volume V of the box as a function of x .



Sol:

Volume of the box = Volume of the cuboid

$= l \times b \times h$ cu. units

Here $l = 24 - 2x$

$b = 24 - 2x$

$h = x$

$\therefore V = (24 - 2x)(24 - 2x) \times x$

$= (576 - 48x - 48x + 4x^2)x$

$V = 4x^3 - 96x^2 + 576x$

8. function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = [f(x)]^2$.

Sol:

Given $f(x) = 3 - 2x$

Also, it is given that $f(x^2) = [f(x)]^2$

$f(x^2) = 3 - 2x^2$ [Replacing x by x^2] ... (1)

$[f(x)]^2 = (3 - 2x)^2 = 9 - 12x + 4x^2$... (2)

$[\because (a - b)^2 = a^2 - 2ab + b^2]$

From (1) and (2),

$\Rightarrow 9 - 12x + 4x^2 = 3 - 2x^2$

$\Rightarrow 9 - 12x + 4x^2 - 3 + 2x^2 = 0$

$\Rightarrow 6x^2 - 12x + 6 = 0$

Dividing by 6, we get $x^2 - 2x + 1 = 0$

On factorizing we get, $(x - 1)(x - 1) = 0$

$\Rightarrow x = 1$

9. A plane is flying at a speed of 500 km per hour. Express the distance d travelled by the plane as function of time t in hours.

Sol:

Speed = $\frac{\text{distance covered}}{\text{time taken}}$

$\Rightarrow \text{distance} = \text{Speed} \times \text{time}$

$\Rightarrow d = 500 \times t$

$\Rightarrow d = 500t$

[$\because$ time = t hrs]