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PREFACE

The woods are lovely, dark and deep.

But I have promises to keep, and

miles to go before I sleep

- Robert Frost

Respected Principals, Correspondents, Head Masters / Head Mistresses, Teachers, and dear Students.

From the bottom of our heart, we at SURA Publications sincerely thank you for the support and patronage that you have extended to us for more than a decade.

It is in our sincerest effort we take the pride of releasing **SURA'S Mathematics** for 10th Standard. This guide has been authored and edited by qualified teachers having teaching experience for over a decade in their respective subject fields. This Guide has been reviewed by reputed Professors who are currently serving as Head of the Department in esteemed Universities and Colleges.

With due respect to Teachers, I would like to mention that this guide will serve as a teaching companion to qualified teachers. Also, this guide will be an excellent learning companion to students with exhaustive exercises, additional problems and 1 marks as per new model in addition to precise answers for exercise problems.

In complete cognizance of the dedicated role of Teachers, I completely believe that our students will learn the subject effectively with this guide and prove their excellence in Board Examinations.

I once again sincerely thank the Teachers, Parents and Students for supporting and valuing our efforts.

God Bless all.

Subash Raj, B.E., M.S.

- Publisher

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SYLLABUS

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AUGUST	3	Algebra	3.4 - 3.6
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SEPTEMBER	3	Algebra	3.7
	6	Trigonometry	6.1 - 6.2
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OCTOBER	3	Algebra	3.8 - 3.9
	4	Geometry	4.4 - 4.6
NOVEMBER	6	Trigonometry	6.3
	7	Mensuration	7.1 - 7.5
II MID TERM TEST (October, November)			
DECEMBER	8	Statistics and Probability	8.1 - 8.6
HALF YEARLY EXAMINATION (Full Portions)			
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FEBRUARY	SECOND REVISION TEST		
MARCH	THIRD REVISION TEST		

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RELATIONS AND FUNCTIONS

FORMULAE TO REMEMBER

- ☐ **Vertical line test :**
A curve drawn in a graph represents a functions, if every vertical line intersects the curve in at most one point.
- ☐ **Horizontal line test :**
A function represented in a graph is one - one, if every horizontal line intersects the curve in at most one point.
- ☐ Linear functions has applications in Cryptography as well as in several branches of Science and Technology.

EXERCISE 1.1

1. Find $A \times B$, $A \times A$ and $B \times A$ (i) $A = \{2, -2, 3\}$ and $B = \{1, -4\}$ (ii) $A = B = \{p, q\}$ (iii) $A = \{m, n\}$; $B = \phi$ [PTA - 1; FRT - 2022; July - 2025]**Sol.** (i) $A = \{2, -2, 3\}$, $B = \{1, -4\}$

$$A \times B = \{(2, 1), (2, -4), (-2, 1), (-2, -4), (3, 1), (3, -4)\}$$

$$A \times A = \{(2, 2), (2, -2), (2, 3), (-2, 2), (-2, -2), (-2, 3), (3, 2), (3, -2), (3, 3)\}$$

$$B \times A = \{(1, 2), (1, -2), (1, 3), (-4, 2), (-4, -2), (-4, 3)\}$$

(ii) $A = B = \{p, q\}$

$$A \times B = \{(p, p), (p, q), (q, p), (q, q)\}$$

$$A \times A = \{(p, p), (p, q), (q, p), (q, q)\}$$

$$B \times A = \{(p, p), (p, q), (q, p), (q, q)\}$$

(iii) $A = \{m, n\}$, $B = \phi$

$$A \times B = \{\}$$

$$A \times A = \{(m, m), (m, n), (n, m), (n, n)\}$$

$$B \times A = \{\}$$

2. Let $A = \{1, 2, 3\}$ and $B = \{x \mid x \text{ is a prime number less than } 10\}$. Find $A \times B$ and $B \times A$.

[May - 2022; QY - 2024; April - 2025]

Sol. $A = \{1, 2, 3\}$, $B = \{2, 3, 5, 7\}$

$$A \times B = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$$

$$B \times A = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$$

3. If $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ find A and B . [QY - '19; April - 2023; July - 2024]**Sol.** Given $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ Here $B = \{-2, 0, 3\}$ [All the first elements of the order pair] and $A = \{3, 4\}$

[All the second elements of the order pair]

4. If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$, Show that $A \times A = (B \times B) \cap (C \times C)$. [FRT & Aug - 2022]**Sol.** $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$

$$A \times A = \{(5, 5), (5, 6), (6, 5), (6, 6)\} \quad \dots(1)$$

$$B \times B = \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\} \quad \dots(2)$$

$$C \times C = \{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)\} \quad \dots(3)$$

$$(B \times B) \cap (C \times C) = \{(5, 5), (5, 6), (6, 5), (6, 6)\} \quad \dots(4)$$

$$(1) = (4)$$

$$A \times A = (B \times B) \cap (C \times C). \quad \text{It is proved.}$$

5. Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$ and $D = \{1, 3, 5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true? [QY - '19 & 2024]**Sol.** LHS = $\{(A \cap C) \times (B \cap D)\}$

$$A \cap C = \{3\}$$

$$B \cap D = \{3, 5\}$$

$$(A \cap C) \times (B \cap D) = \{(3, 3), (3, 5)\} \quad \dots(1)$$

$$\text{RHS} = (A \times B) \cap (C \times D)$$

$$A \times B = \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5)\}$$

$$C \times D = \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$$

$$(A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\} \quad \dots(2)$$

 $\therefore (1) = (2) \therefore$ It is true.6. Let $A = \{x \in \mathbb{W} \mid x < 2\}$, $B = \{x \in \mathbb{N} \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

[PTA - 2; FRT - 2022; June & HY - 2023]

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

[PTA - 5; Sep - 2021; Mar - 2026]

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$ [QY - 2023]**Sol.** (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

$$A = \{x \in \mathbb{W} \mid x < 2\} = \{0, 1\}$$

[Whole numbers less than 2]

$$B = \{x \in \mathbb{N} \mid 1 < x \leq 4\} = \{2, 3, 4\}$$

[Natural numbers from 2 to 4]

$$C = \{3, 5\}$$

$$\text{LHS} = A \times (B \cup C)$$

$$B \cup C = \{2, 3, 4\} \cup \{3, 5\} = \{2, 3, 4, 5\}$$

$$A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\} \quad \dots(1)$$

$$\text{RHS} = (A \times B) \cup (A \times C)$$

$$(A \times B) = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$$

$$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$$

$$(A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\} \quad \dots(2)$$

(1) = (2), LHS = RHS Hence it is proved.

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$\text{LHS} = A \times (B \cap C)$$

$$(B \cap C) = \{3\}$$

$$A \times (B \cap C) = \{(0, 3), (1, 3)\} \quad \dots(1)$$

$$\text{RHS} = (A \times B) \cap (A \times C)$$

$$(A \times B) = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$$

$$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$$

$$(A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\} \quad \dots(2)$$

(1) = (2) $\Rightarrow$ LHS = RHS.

Hence it is verified.

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

LHS = $(A \cup B) \times C$

$A \cup B = \{0, 1, 2, 3, 4\}$

$(A \cup B) \times C = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\} \dots (1)$

RHS = $(A \times C) \cup (B \times C)$

$(A \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$(B \times C) = \{(2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$

$(A \times C) \cup (B \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\} \dots (2)$

$(1) = (2) \therefore$ LHS = RHS. Hence it is verified.

7. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

(i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

[Sep. - 2020; July - 2024; April - 2025]

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ [PTA - 1]

$A = \{1, 2, 3, 4, 5, 6, 7\}$ [May - 2022]

$B = \{2, 3, 5, 7\}$

$C = \{2\}$

[$\because$ 2 is the only even prime number]

Sol. (i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

LHS = $(A \cap B) \times C$

$A \cap B = \{2, 3, 5, 7\}$

$(A \cap B) \times C = \{(2, 2), (3, 2), (5, 2), (7, 2)\} \dots (1)$

RHS = $(A \times C) \cap (B \times C)$

$(A \times C) = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$

$(B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$

$(A \times C) \cap (B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\} \dots (2)$

$(1) = (2) \therefore$ LHS = RHS. Hence it is verified.

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

LHS = $A \times (B - C)$

$(B - C) = \{3, 5, 7\}$

$A \times (B - C) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\} \dots (1)$

RHS = $(A \times B) - (A \times C)$

$(A \times B) = \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7), (7, 2), (7, 3), (7, 5), (7, 7)\}$

$(A \times C) = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$

$(A \times B) - (A \times C) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7)\} \dots (2)$

$(1) = (2) \Rightarrow$ LHS = RHS. Hence it is verified.

EXERCISE 1.2

1. Let $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$, which of the following are relation from A to B?

(i) $R_1 = \{(2, 1), (7, 1)\}$

(ii) $R_2 = \{(-1, 1)\}$

(iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$ [FRT - 2022]

(iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$

Sol. Given $A = \{1, 2, 3, 7\}$ and $B = \{3, 0, -1, 7\}$

(i) $R_1 = \{(2, 1), (7, 1)\}$

2 and 7 cannot be related to 1 since $1 \notin B$

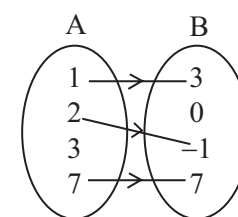
$\therefore R_1$ is not a relation.

(ii) $R_2 = \{(-1, 1)\}$

-1 cannot be related to 1 since $-1 \notin A$ and $1 \notin B$

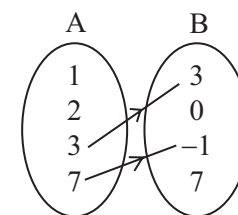
$\therefore R_2$ is not a relation.

(iii) $R_3 = \{(2, -1), (7, 7), (1, 3)\}$



R_3 is a relation since 2 is related to -1, 7 is related to 7 and 1 is related to 3.

(iv) $R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$



7 is related to -1
3 is related to 3

Since $0 \notin A$, 0 cannot be related to 3 and 7.

$\therefore R_4$ is not a relation.

2. Let $A = \{1, 2, 3, 4, \dots, 45\}$ and R be the relation defined as "square is of a number" on A. Write R as a subset of $A \times A$. Also, find the domain and range of R. [Sep - 2021]

Sol. Given $A = \{1, 2, 3, 4, \dots, 45\}$

$\therefore A \times A = \{(1, 1), (1, 2), (1, 3), \dots, (1, 45)$

$(2, 1), (2, 2), \dots, (2, 45), (45, 1), (45, 2), (45, 3), \dots, (45, 45)\} \dots (1)$

$\therefore R = \{(1,1) (2,4) (3,9) (4,16) (5, 25) (6,36)\} \dots (2)$
 $[\because 1 \text{ is the square of } 1, 2 \text{ is the square of } 4 \text{ and so on}]$

From (1) and (2), R is the subset of $A \times A$

$\therefore R \subset A \times A$

Domain of $R = \{1, 2, 3, 4, 5, 6\}$

[All the first elements of the order pair in (2)]

Range of $R = \{1, 4, 9, 16, 25, 36\}$

[All the second elements of the order pair in (2)]

3. A Relation $\mathbb{R}$ is given by the set $\{(x, y) / y = x + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$. Determine its domain and range. [PTA - 5; June - 2023; Mar - 2026]

Sol. Given $\mathbb{R} = \{(x, y) / y = x + 3\}$ and $x \in \{0, 1, 2, 3, 4, 5\}$

When $x = 0$, $y = 0 + 3 = 3$ [$\because y = x + 3$]

When $x = 1$, $y = 1 + 3 = 4$

When $x = 2$, $y = 2 + 3 = 5$

When $x = 3$, $y = 3 + 3 = 6$

When $x = 4$, $y = 4 + 3 = 7$

When $x = 5$, $y = 5 + 3 = 8$

$\therefore \mathbb{R} = \{(0, 3), (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)\}$

$\therefore$ Domain of $\mathbb{R} = \{0, 1, 2, 3, 4, 5\}$

[All the first element in $\mathbb{R}$]

Range of $\mathbb{R} = \{3, 4, 5, 6, 7, 8\}$

[All the second element in $\mathbb{R}$]

4. Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.

(i) $\{(x, y) / x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$

(ii) $\{(x, y) / y = x + 3, x, y \text{ are natural numbers } < 10\}$

[Aug - 2022]

Sol. (i) $R = \{(x, y) / x = 2y, x \in \{2, 3, 4, 5\} \text{ and } y \in \{1, 2, 3, 4\}\}$

When $x = 2$, $y = \frac{x}{2} = \frac{2}{2} = 1$

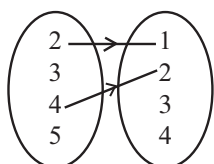
[$\because x = 2y \Rightarrow y = \frac{x}{2}$]

When $x = 3$, $y = \frac{3}{2}$

When $x = 4$, $y = \frac{4}{2} = 2$

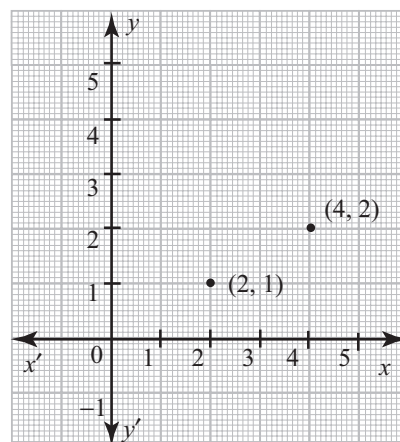
When $x = 5$, $y = \frac{5}{2}$

(a) an arrow diagram



3 cannot be related to $\frac{3}{2}$ and 5 cannot be related to $\frac{5}{2}$.

(b) a graph



(c) Roster form : $R = \{(2, 1), (4, 2)\}$

(ii) $R = \{(x, y) / y = x + 3,$

$x \text{ and } y \text{ are natural numbers } < 10\}$

$x = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$y = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

[$\because x \text{ and } y \text{ are natural numbers less than } 10$]

Given $y = x + 3$

When $x = 1$, $y = 1 + 3 = 4$

When $x = 2$, $y = 2 + 3 = 5$

When $x = 3$, $y = 3 + 3 = 6$

When $x = 4$, $y = 4 + 3 = 7$

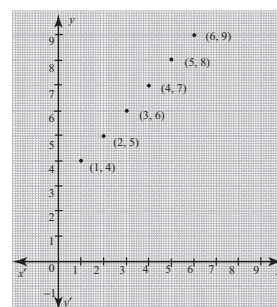
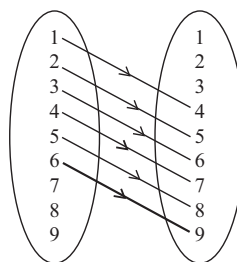
When $x = 5$, $y = 5 + 3 = 8$

When $x = 6$, $y = 6 + 3 = 9$

$\left\{ \begin{array}{l} \text{When } x = 7, \quad y = 7 + 3 = 10 \\ \text{When } x = 8, \quad y = 8 + 3 = 11 \\ \text{When } x = 9, \quad y = 9 + 3 = 12 \end{array} \right\} [10, 11, 12 \notin y]$

$R = \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$

(a) an arrow diagram (b) a graph



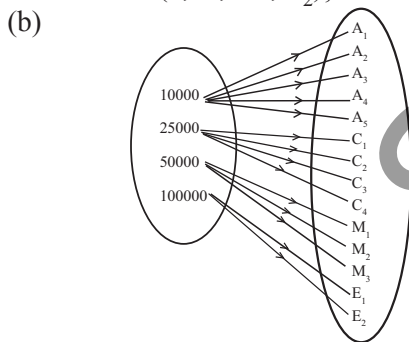
(c) Roster form :

$R = \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$

5. A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation $\mathbb{R}$ is defined by xRy , where x is the salary given to person y , express the relation $\mathbb{R}$ through an ordered pair and an arrow diagram. [FRT - 2022]

Sol. A – Assistants $\rightarrow A_1, A_2, A_3, A_4, A_5$
 C – Clerks $\rightarrow C_1, C_2, C_3, C_4$
 M – Managers $\rightarrow M_1, M_2, M_3$
 E – Executive officer $\rightarrow E_1, E_2$
 xRy is defined as x is the salary for assistants is ₹10,000, clerks is 25,000, Manger is ₹50,000 and for the executive officer ₹1,00,000.

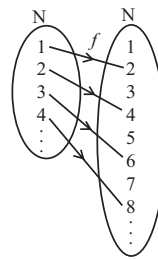
- (a) $\therefore R = \{(10,000, A_1), (10,000, A_2), (10,000, A_3), (10,000, A_4), (10,000, A_5), (25,000, C_1), (25,000, C_2), (25,000, C_3), (25,000, C_4), (50,000, M_1), (50,000, M_2), (50,000, M_3), (1,00,000, E_1), (1,00,000, E_2)\}$



EXERCISE 1.3

1. Let $f = \{(x, y) | x, y \in \mathbb{N} \text{ and } y = 2x\}$ be a relation on $\mathbb{N}$. Find the domain, co-domain and range. Is this relation a function?

Sol. Given $f = \{(x, y) | x, y \in \mathbb{N} \text{ and } y = 2x\}$
 When $x = 1$, $y = 2(1) = 2$
 When $x = 2$, $y = 2(2) = 4$
 When $x = 3$, $y = 2(3) = 6$
 When $x = 4$, $y = 2(4) = 8$ and so on.
 $R = \{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), \dots\}$
 Domain of $R = \{1, 2, 3, 4, \dots\}$,
 Range of $R = \{2, 4, 6, 8, \dots\}$



Since all the elements of domain are related to some elements of co-domain, this relation f is a function.

2. Let $X = \{3, 4, 6, 8\}$. Determine whether the relation $\mathbb{R} = \{(x, f(x)) | x \in X, f(x) = x^2 + 1\}$ is a function from X to $\mathbb{N}$?

Sol. $x = \{3, 4, 6, 8\}$
 $R = \{(x, f(x)) | x \in X, f(x) = x^2 + 1\}$

$$f(x) = x^2 + 1$$

$$f(3) = 3^2 + 1 = 10$$

$$f(4) = 4^2 + 1 = 17$$

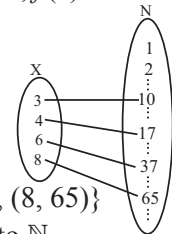
$$f(6) = 6^2 + 1 = 37$$

$$f(8) = 8^2 + 1 = 65$$

$$R = \{(3, 10), (4, 17), (6, 37), (8, 65)\}$$

Yes, R is a function from X to $\mathbb{N}$.

Since all the elements of X are related to some elements of $\mathbb{N}$.



3. Given the function $f: x \rightarrow x^2 - 5x + 6$, evaluate

(i) $f(-1)$

(ii) $f(2a)$

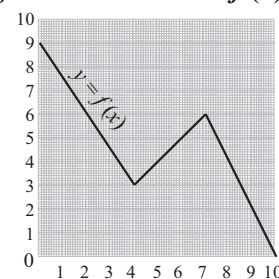
(iii) $f(2)$

(iv) $f(x-1)$

Give the function $f: x \rightarrow x^2 - 5x + 6$.

- (i) $f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = 12$
 (ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$
 (iii) $f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$
 (iv) $f(x-1) = (x-1)^2 - 5(x-1) + 6$
 $= x^2 - 2x + 1 - 5x + 5 + 6$
 $= x^2 - 7x + 12$

4. A graph representing the function $f(x)$ is given in figure it is clear that $f(9) = 2$.



- (i) Find the following values of the function
 (a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$
 (ii) For what value of x is $f(x) = 1$?
 (iii) Describe the following
 (i) Domain (ii) Range.
 (iv) What is the image of 6 under f ?

Sol. (i) From the graph

- (a) $f(0) = 9$ (c) $f(2) = 6$
 (b) $f(7) = 6$ (d) $f(10) = 0$

- (ii) If $f(x) = 1$ the value of x is 9.5
 (iii) Domain = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 $= \{x | 0 \leq x \leq 10, x \in \mathbb{R}\}$
 Range = $\{x | 0 \leq x \leq 9, x \in \mathbb{R}\}$
 $= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
 (iv) The image of 6 under f is 5. Since when you draw a line at $x = 6$, it meets the graph at 5.

5. Let $f(x) = 2x + 5$. If $x \neq 0$ then find $\frac{f(x+2) - f(2)}{x}$.

Sol. Given $f(x) = 2x + 5, x \neq 0$.

$$\begin{aligned} f(x) &= 2x + 5 \\ \Rightarrow f(x+2) &= 2(x+2) + 5 \\ &= 2x + 4 + 5 = 2x + 9 \\ \Rightarrow f(2) &= 2(2) + 5 = 4 + 5 = 9 \\ \therefore \frac{f(x+2) - f(2)}{x} &= \frac{2x + 9 - 9}{x} = \frac{2x}{x} = 2 \end{aligned}$$

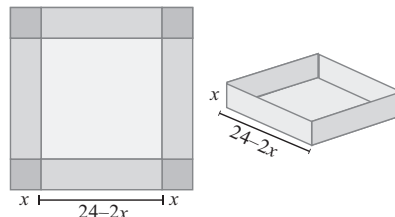
6. A function f is defined by $f(x) = 2x - 3$

- (i) find $\frac{f(0) + f(1)}{2}$
 (ii) find x such that $f(x) = 0$.
 (iii) find x such that $f(x) = x$.
 (iv) find x such that $f(x) = f(1 - x)$.

Sol. Given $f(x) = 2x - 3$

$$\begin{aligned} \text{(i)} \quad \frac{f(0) + f(1)}{2} &= \frac{f(0) + f(1)}{2} \\ f(0) &= 2(0) - 3 = -3 \\ f(1) &= 2(1) - 3 = -1 \\ \therefore \frac{f(0) + f(1)}{2} &= \frac{-3 - 1}{2} = \frac{-4}{2} = -2 \\ \text{(ii)} \quad f(x) = 0 &\Rightarrow 2x - 3 = 0 \\ 2x &= 3 \\ x &= \frac{3}{2} \\ \text{(iii)} \quad f(x) = x &\Rightarrow 2x - 3 = x \Rightarrow 2x - x = 3 \\ x &= 3 \\ \text{(iv)} \quad f(x) &= f(1 - x) \\ 2x - 3 &= 2(1 - x) - 3 \\ 2x - 3 &= 2 - 2x - 3 \\ 2x + 2x &= 2 - 3 + 3 \\ 4x &= 2 \\ x &= \frac{2}{4} \Rightarrow x = \frac{1}{2} \end{aligned}$$

7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown in figure. Express the volume V of the box as a function of x .



Sol. Volume of the box = Volume of the cuboid
 $= l \times b \times h$ cu. units

$$\text{Here } l = 24 - 2x$$

$$b = 24 - 2x$$

$$h = x$$

$$\begin{aligned} \therefore V &= (24 - 2x)(24 - 2x) \times x \\ &= (576 - 48x - 48x + 4x^2)x \\ V &= 4x^3 - 96x^2 + 576x \end{aligned}$$

8. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$. [QY & HY - 2023]

Sol. Given $f(x) = 3 - 2x$

Also, it is given that $f(x^2) = [f(x)]^2$

$$f(x^2) = 3 - 2x^2 \quad [\text{Replacing } x \text{ by } x^2]$$

... (1)

$$[f(x)]^2 = (3 - 2x)^2 = 9 - 12x + 4x^2$$

... (2)

$$[\because (a - b)^2 = a^2 - 2ab + b^2]$$

From (1) and (2),

$$\Rightarrow 9 - 12x + 4x^2 = 3 - 2x^2$$

$$\Rightarrow 9 - 12x + 4x^2 - 3 + 2x^2 = 0$$

$$\Rightarrow 6x^2 - 12x + 6 = 0$$

Dividing by 6, we get $x^2 - 2x + 1 = 0$

On factorizing we get, $(x - 1)(x - 1) = 0$

$$\Rightarrow x = 1$$

9. A plane is flying at a speed of 500 km per hour. Express the distance ' d ' travelled by the plane as function of time t in hours.

Sol. Speed = $\frac{\text{distance covered}}{\text{time taken}}$

$$\Rightarrow \text{distance} = \text{Speed} \times \text{time}$$

$$\Rightarrow d = 500 \times t \quad [\because \text{time} = t \text{ hrs}]$$

$$\Rightarrow d = 500t$$

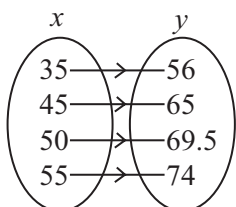
10. The data in the adjacent table depicts the length of a person's forearm and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forearm length (x) as $y = ax + b$, where a, b are constants. [PTA - 4]

Length ' x ' of forearm (in cm)	Height ' y ' (in inches)
35	56
45	65
50	69.5
55	74

- Check if this relation is a function.
- Find a and b .
- Find the height of a person whose forearm length is 40 cm.
- Find the length of forearm of a person if the height is 53.3 inches.

Sol. Given relation is $y = ax + b$... (1)

- The given ordered pairs are
 $R = \{ (35, 56) (45, 65) (50, 69.5) (55, 74) \}$



Since all the elements of x are related to some elements of y , the given relation is a function.

- Consider any two ordered pairs $(35, 56)$ and $(45, 65)$

Substitute $(35, 56)$ in $y = ax + b$ we get,
 $56 = a(35) + b$... (1)

Similarly substitute $(45, 65)$ in $y = ax + b$, we get
 $65 = a(45) + b$... (2)

$$(2) \rightarrow 65 = 45a + b \quad \dots (2)$$

$$(1) \rightarrow 56 = 35a + b \quad \dots (3)$$

Substituting, $9 = 10a$

$$\Rightarrow a = \frac{9}{10} = 0.9$$

Substituting $a = 0.9$ in (1) we get

$$56 = 35(0.9) + b$$

$$\Rightarrow 56 = 31.5 + b$$

$$\Rightarrow b = 56 - 31.5 = 24.5$$

$$\text{Since } y = ax + b$$

$$\text{We get } y = 0.9x + 24.5$$

- When the length of the forearm

$$x = 40 \text{ cm,}$$

$$y = 0.9(40) + 24.5$$

$$\Rightarrow y = 36 + 24.5 = 60.5 \text{ inches}$$

$\therefore$ The required height of the person is 60.5 inches.

- When the height of the person

$$y = 53.3 \text{ inches,}$$

$$53.3 = 0.9x + 24.5$$

$$[\because y = 0.9x + 24.5]$$

$$\Rightarrow 53.3 - 24.5 = 0.9x$$

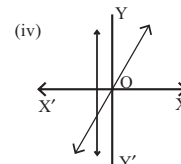
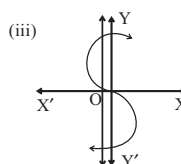
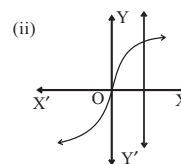
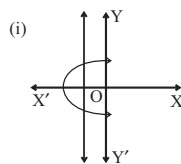
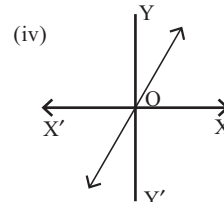
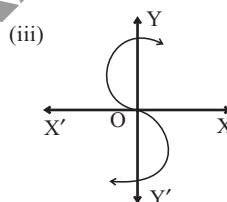
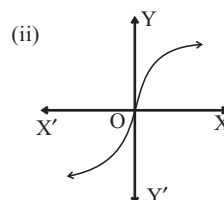
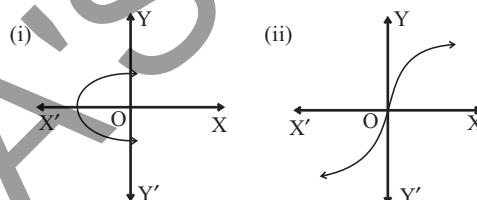
$$\Rightarrow 28.8 = 0.9x$$

$$\Rightarrow x = \frac{28.8 \times 10}{0.9 \times 10}$$

$$\Rightarrow x = \frac{288}{9} = 32 \text{ cm}$$

EXERCISE 1.4

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.



- It is not a function. The graph meets the vertical line at more than one points.
- It is a function as the curve meets the vertical line at only one point.
- It is not a function as it meets the vertical line at more than one points.
- It is a function as it meets the vertical line at only one point.

2. Let $f: A \rightarrow B$ be a function defined by $f(x) = \frac{x}{2} - 1$, where $A = \{2, 4, 6, 10, 12\}$, $B = \{0, 1, 2, 4, 5, 9\}$. Represent f by [Govt. MQP - '19; April - 2023 & 2025]

(i) set of ordered pairs; (ii) a table;
(iii) an arrow diagram; (iv) a graph

Sol. $f: A \rightarrow B$

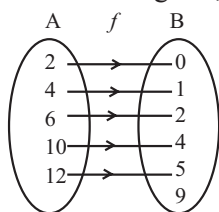
$$A = \{2, 4, 6, 10, 12\}, B = \{0, 1, 2, 4, 5, 9\}$$

$$\begin{array}{l|l} f(x) = \frac{x}{2} - 1, & f(2) = \frac{2}{2} - 1 = 0 \\ f(4) = \frac{4}{2} - 1 = 1 & f(6) = \frac{6}{2} - 1 = 2 \\ f(10) = \frac{10}{2} - 1 = 4 & f(12) = \frac{12}{2} - 1 = 5 \end{array}$$

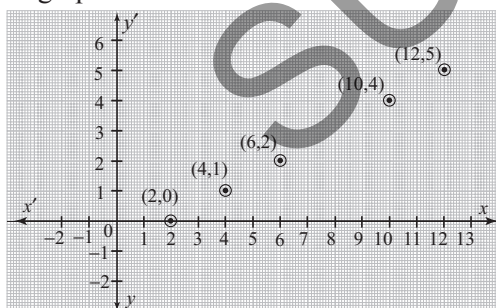
- (i) Set of ordered pairs
 $= \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$
(ii) a table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

- (iii) an arrow diagram;



- (iv) a graph

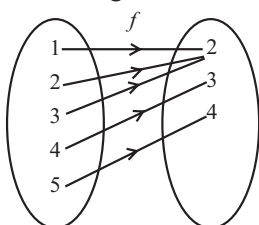


3. Represent the function $f = \{(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)\}$ through

(i) an arrow diagram (ii) a table form
(iii) a graph

Sol. $f = \{(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)\}$

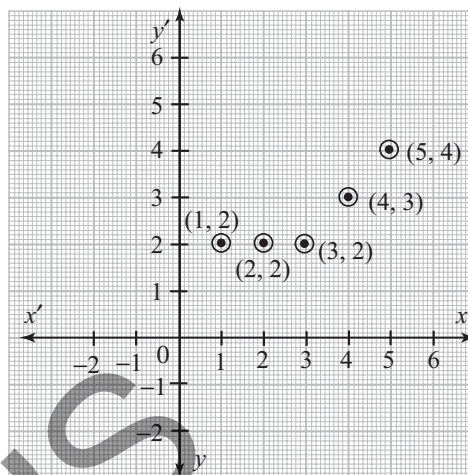
- (i) An arrow diagram.



- (ii) a table form

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

- (iii) A graph representation.



4. Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = 2x - 1$ is one - one but not onto.

Sol. $f: \mathbb{N} \rightarrow \mathbb{N}$

$$f(x) = 2x - 1$$

$$\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$$

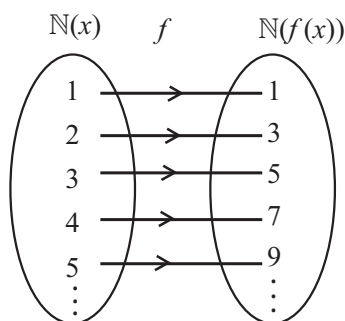
$$f(1) = 2(1) - 1 = 1$$

$$f(2) = 2(2) - 1 = 3$$

$$f(3) = 2(3) - 1 = 5$$

$$f(4) = 2(4) - 1 = 7$$

$$f(5) = 2(5) - 1 = 9$$



In the figure, for different elements in x , there are different images in $f(x)$.

Hence $f: \mathbb{N} \rightarrow \mathbb{N}$ is a one-one function.

A function $f: \mathbb{N} \rightarrow \mathbb{N}$ is said to be onto function if the range of f is equal to the co-domain of f .

$$\text{Range} = \{1, 3, 5, 7, 9, \dots\}$$

$$\text{Co-domain} = \{1, 2, 3, \dots\}$$

But here the range is not equal to co-domain. Therefore it is one-one but not onto function.

5. Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one - one function.

Sol. $f: \mathbb{N} \rightarrow \mathbb{N}$

$$f(m) = m^2 + m + 3$$

$$\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}, m \in \mathbb{N}$$

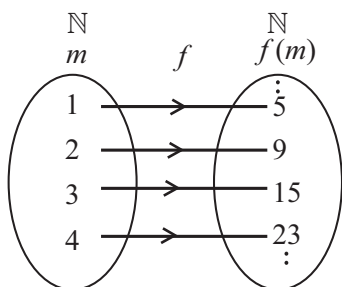
$$f(m) = m^2 + m + 3$$

$$f(1) = 1^2 + 1 + 3 = 5$$

$$f(2) = 2^2 + 2 + 3 = 9$$

$$f(3) = 3^2 + 3 + 3 = 15$$

$$f(4) = 4^2 + 4 + 3 = 23$$



In the figure, for different elements in the domain, there are different images in co-domain. Hence $f: \mathbb{N} \rightarrow \mathbb{N}$ is a one to one but not onto function as the range of f is not equal to co-domain.

Co-domain = $\mathbb{N}$

Range = $\{5, 9, 15, 23, \dots\}$

Hence it is proved.

6. Let $A = \{1, 2, 3, 4\}$ and $B = \mathbb{N}$. Let $f: A \rightarrow B$ be defined by $f(x) = x^3$ then, [HY - '19 & 2023]

(i) find the range of f

(ii) identify the type of function

Sol. $A = \{1, 2, 3, 4\}$

$B = \mathbb{N}$

$$f: A \rightarrow B, f(x) = x^3$$

$$f(1) = 1^3 = 1$$

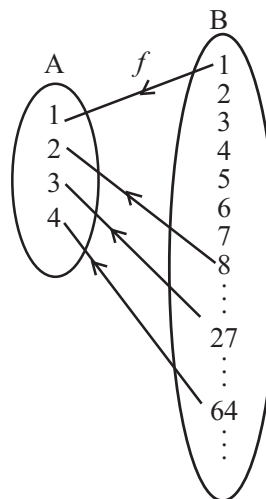
$$f(2) = 2^3 = 8$$

$$f(3) = 3^3 = 27$$

$$f(4) = 4^3 = 64$$

(i) The range of $f = \{1, 8, 27, 64, \dots\}$

(ii)



Here co-domain = $B = \{1, 2, 3, \dots\}$

Range = $\{1, 8, 27, 64\}$

Different elements have different images and co-domain $\neq$ Range.

$\therefore$ The given function is one-one and into function.

7. In each of the following cases state whether the function is bijective or not. Justify your answer.

(i) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2x + 1$

(ii) $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3 - 4x^2$

Sol. Given $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 2x + 1$

(i) When $x = 1$,

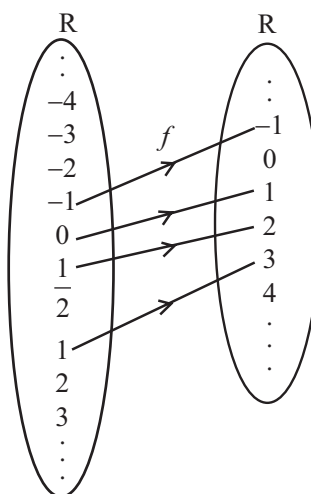
$$f(1) = 2(1) + 1 = 3$$

$$f(2) = 2(2) + 1 = 5$$

$$f(0) = 2(0) + 1 = 1$$

$$f(-1) = 2(-1) + 1 = -2 + 1 = -1$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 1 + 1 = 2 \text{ and so on}$$



Here, different element in domain have different images in B and Co-domain = Range = $\mathbb{R}$. $\therefore f$ is a bijective function.

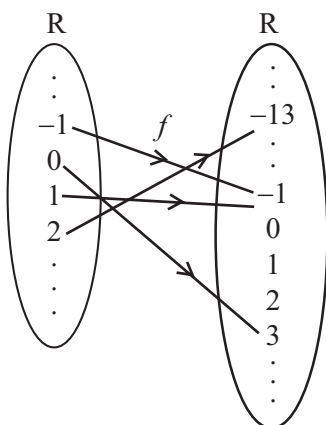
(ii) Given $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 3 - 4x^2$

$$f(1) = 3 - 4(1^2) = 3 - 4(1) = 3 - 4 = -1$$

$$f(2) = 3 - 4(2^2) = 3 - 4(4) = 3 - 16 = -13$$

$$f(0) = 3 - 4(0)^2 = 3 - 0 = 3$$

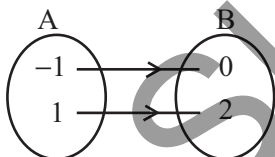
$$f(-1) = 3 - 4(-1)^2 = 3 - 4(1) = 3 - 4 = -1$$



Here, different element in domain do not have different images in B. Since 1 and -1 are related to -1. $\therefore f$ is not one - one. Hence, f is not a bijective function.

8. Let $A = \{-1, 1\}$ and $B = \{0, 2\}$. If the function $f: A \rightarrow B$ defined by $f(x) = ax + b$ is an onto function? Find a and b .

Sol. Given $A = \{-1, 1\}$, $B = \{0, 2\}$ and $f: A \rightarrow B$ is defined by $f(x) = ax + b$ is an onto function.



$$f(-1) = 0$$

$$\Rightarrow a(-1) + b = 0 \quad [\because \text{Sub } x = -1, y = 0 \text{ in } y = ax + b]$$

$$\Rightarrow -a + b = 0 \quad \dots (1)$$

$$\text{Also } f(1) = 2$$

$$\Rightarrow a(1) + b = 2 \quad [\because \text{Sub } x = 1, y = 2 \text{ in } y = ax + b]$$

$$\Rightarrow a + b = 2 \quad \dots (2)$$

$$(1) \Rightarrow -a + b = 0$$

$$\text{Adding, } 2b = 2$$

$$\Rightarrow b = \frac{2}{2} = 1$$

Substituting $b = 1$ in (2) we get

$$a + 1 = 2 \Rightarrow a = 2 - 1 = 1$$

$$\therefore a = 1, b = 1$$

9. If the function f is defined by

$$f(x) = \begin{cases} x+2; & x > 1 \\ 2; & -1 \leq x \leq 1 \\ x-1; & -3 < x < -1 \end{cases} \quad \text{find the values of}$$

(i) $f(3)$ (ii) $f(0)$

(iii) $f(-1.5)$ (iv) $f(2) + f(-2)$

[QY - 2024]

Sol. (i) $f(3) \Rightarrow f(x) = x + 2 \Rightarrow 3 + 2 = 5$ [$\because x = 3$]

(ii) $f(0) \Rightarrow 2$ [$\because 0 \in -1 \leq x \leq 1$]

(iii) $f(-1.5) = x - 1 = -1.5 - 1 = -2.5$

(iv) $f(2) + f(-2)$

$$f(2) = 2 + 2 = 4 \quad [\because f(x) = x + 2]$$

$$f(-2) = -2 - 1 = -3 \quad [\because f(x) = x - 1]$$

$$f(2) + f(-2) = 4 - 3 = 1$$

10. A function $f: [-5, 9] \rightarrow \mathbb{R}$ is defined as follows:

$$f(x) = \begin{cases} 6x+1; & -5 \leq x < 2 \\ 5x^2-1; & 2 \leq x < 6 \\ 3x-4; & 6 \leq x \leq 9 \end{cases} \quad \text{[July - 2024]}$$

Find (i) $f(-3) + f(2)$ (ii) $f(7) - f(1)$ [PTA - 4]

(iii) $2f(4) + f(8)$ (iv) $\frac{2f(-2) - f(6)}{f(4) + f(-2)}$ [PTA - 4]

Sol. $f: [-5, 9] \rightarrow \mathbb{R}$

(i) $f(-3) + f(2)$

$$f(-3) = 6x + 1 = 6(-3) + 1 = -17$$

$$f(2) = 5x^2 - 1 = 5(2^2) - 1 = 19$$

$$\therefore f(-3) + f(2) = -17 + 19 = 2$$

(ii) $f(7) - f(1)$

$$f(7) = 3x - 4 = 3(7) - 4 = 17$$

$$f(1) = 6x + 1 = 6(1) + 1 = 7$$

$$f(7) - f(1) = 17 - 7 = 10$$

(iii) $2f(4) + f(8)$

$$f(4) = 5x^2 - 1 = 5 \times 4^2 - 1 = 79$$

$$f(8) = 3x - 4 = 3 \times 8 - 4 = 20$$

$$\therefore 2f(4) + f(8) = 2 \times 79 + 20 = 178$$

(iv) $\frac{2f(-2) - f(6)}{f(4) + f(-2)}$

$$f(-2) = 6x + 1 = 6(-2) + 1 = -11$$

$$f(6) = 3x - 4 = 3(6) - 4 = 14$$

$$f(4) = 5x^2 - 1 = 5(4^2) - 1 = 79$$

$$f(-2) = 6x + 1 = 6(-2) + 1 = -11$$

$$\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{2(-11) - 14}{79 + (-11)} = \frac{-22 - 14}{68}$$

$$= \frac{-36}{68} = \frac{-9}{17}$$

11. The distance S an object travels under the influence of gravity in time t seconds is given

by $S(t) = \frac{1}{2}gt^2 + at + b$ where, (g is the acceleration due to gravity), $a > 0$, b are constants.

Verify whether the function $S(t)$ is one-one or not.

Sol. $S(t) = \frac{1}{2}gt^2 + at + b$ [PTA - 3]

Let t be 1, 2, 3, ..., seconds.

$$S(1) = \frac{1}{2}g(1^2) + a(1) + b = \frac{1}{2}g + a + b$$

$$S(2) = \frac{1}{2}g(2^2) + a(2) + b \\ = 2g + 2a + b$$

Yes, for every different values of t , there will be different values as images. And there will be different pre-images for the different values of the range. Therefore it is one-one function.

12. The function ' t ' which maps temperature in Celsius (C) into temperature in Fahrenheit

(F) is defined by $t(C) = F$ where $F = \frac{9}{5}C + 32$.

Find,

- (i) $t(0)$ [PTA - 1]
 (ii) $t(28)$ (iii) $t(-10)$
 (iv) the value of C when $t(C) = 212$ [PTA - 1]
 (v) the temperature when the Celsius value is equal to the Fahrenheit value. [PTA - 1]

Sol. (i) $t(0) = F$
 $F = \frac{9}{5}(C) + 32 = \frac{9}{5}(0) + 32 = 32^\circ F$

(ii) $t(28) = F = \frac{9}{5}(28) + 32 = \frac{252}{5} + 32$
 $= 50.4 + 32 = 82.4^\circ F$

(iii) $t(-10) = F = \frac{9}{5}(-10) + 32 = 14^\circ F$

(iv) $t(C) = 212$

i.e. $\frac{9}{5}(C) + 32 = 212 \Rightarrow \frac{9}{5}C = 212 - 32 = 180$

$$\frac{9}{5}C = 180 \Rightarrow C = \frac{180 \times 5}{9} = 100^\circ C$$

$C = 100^\circ C$.

(v) when $C = F$

$$\frac{9}{5}C + 32 = C$$

$$32 = C - \frac{9}{5}C$$

$$32 = C\left(1 - \frac{9}{5}\right)$$

$$32 = C\left(\frac{5-9}{5}\right)$$

$$32 = C\left(\frac{-4}{5}\right)$$

$$C = \frac{32 \times -5}{4}$$

$$C = -40^\circ$$

EXERCISE 1.5

1. Using the functions f and g given below, find fog and gof . Check whether $fog = gof$.

(i) $f(x) = x - 6$, $g(x) = x^2$ [June - 2023]

(ii) $f(x) = \frac{2}{x}$, $g(x) = 2x^2 - 1$

(iii) $f(x) = \frac{x+6}{3}$, $g(x) = 3 - x$

(iv) $f(x) = 3 + x$, $g(x) = x - 4$ [Govt. MQP - '19]

(v) $f(x) = 4x^2 - 1$, $g(x) = 1 + x$

Sol. (i) Given $f(x) = x - 6$, $g(x) = x^2$
 $fog(x) = f(g(x)) = f(x^2)$ [$\because g(x) = x^2$]
 $= x^2 - 6$

[In $f(x) = x - 6$, Replace x by x^2] ... (1)
 $gof(x) = g(f(x)) = g(x - 6)$
 $[\because f(x) = x - 6]$

$$= (x - 6)^2$$

[In $g(x) = x^2$, Replace x by $x - 6$]
 $= x^2 - 12x + 36$

[$\because (a - b)^2 = a^2 - 2ab + b^2$] ... (2)

From (1) and (2),

$$fog(x) \neq gof(x)$$

(ii) Given $f(x) = \frac{2}{x}$, $g(x) = 2x^2 - 1$

$fog(x) = f(g(x)) = f(2x^2 - 1)$
 $[\because g(x) = 2x^2 - 1]$
 $= \frac{2}{2x^2 - 1}$

[In $f(x) = \frac{2}{x}$, Replace x by $2x^2 - 1$] ... (1)

$gof(x) = g(f(x)) = g\left(\frac{2}{x}\right)$ [$\because f(x) = \frac{2}{x}$]
 $= 2\left(\frac{2}{x}\right)^2 - 1$

[In $g(x) = 2x^2 - 1$, Replace x by $\frac{2}{x}$]

$$= 2\left(\frac{4}{x^2}\right) - 1 = \frac{8}{x^2} - 1 \quad \dots (2)$$

From (1) and (2),

$$fog(x) \neq gof(x)$$

(iii) Given $f(x) = \frac{x+6}{3}$, $g(x) = 3-x$

$$f \circ g(x) = f(g(x)) = f(3-x) [\because g(x) = 3-x]$$

$$= \frac{3-x+6}{3}$$

$$[\text{In } f(x) = \frac{x+6}{3}, \text{ Replace } x \text{ by } 3-x]$$

$$= \frac{9-x}{3} \quad \dots(1)$$

$$g \circ f(x) = g(f(x)) = g\left(\frac{x+6}{3}\right)$$

$$[\because f(x) = \frac{x+6}{3}]$$

$$= 3 - \left(\frac{x+6}{3}\right)$$

$$[\text{In } g(x) = 3-x, \text{ Replace } x \text{ by } \frac{x+6}{3}]$$

$$= \frac{9-x-6}{3} = \frac{3-x}{3} \quad \dots(2)$$

From (1) and (2),

$$f \circ g(x) \neq g \circ f(x)$$

(iv) Given $f(x) = 3+x$, $g(x) = x-4$

$$f \circ g(x) = f(g(x)) = f(x-4) [\because g(x) = x-4]$$

$$= 3 + (x-4)$$

$$[\text{In } f(x) = 3+x, \text{ Replace } x \text{ by } x-4]$$

$$= 3 + x - 4 = x - 1 \quad \dots(1)$$

$$g \circ f(x) = g(f(x)) = g(3+x)$$

$$= 3 + x - 4 [\because f(x) = 3+x]$$

$$[\text{In } g(x) = x-4, \text{ Replace } x \text{ by } 3+x]$$

$$= x - 1 \quad \dots(2)$$

From (1) and (2),

$$f \circ g(x) = g \circ f(x)$$

(v) Given $f(x) = 4x^2 - 1$, $g(x) = 1+x$

$$f \circ g(x) = f(g(x)) = f(1+x) [\because g(x) = 1+x]$$

$$= 4(1+x)^2 - 1$$

$$[\text{In } f(x) = 4x^2 - 1, \text{ Replace } x \text{ by } 1+x]$$

$$= 4(1+x^2+2x) - 1 = 4 + 4x^2 + 8x - 1$$

$$= 4x^2 + 8x + 3 \quad \dots(1)$$

$$g \circ f(x) = g(f(x)) = g(4x^2 - 1)$$

$$[\because f(x) = 4x^2 - 1]$$

$$= 1 + 4x^2 - 1$$

$$[\text{In } g(x) = 1+x, \text{ Replace } x \text{ by } 4x^2 - 1]$$

$$= 4x^2 \quad \dots(2)$$

From (1) and (2),

$$f \circ g(x) \neq g \circ f(x)$$

2. Find the value of k , such that $f \circ g = g \circ f$

(i) $f(x) = 3x + 2$, $g(x) = 6x - k$ [HY - '19]

(ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

Sol. (i) Given $f(x) = 3x + 2$, $g(x) = 6x - k$ and

$$f \circ g = g \circ f$$

$$f \circ g(x) = f(g(x)) = f(6x - k)$$

$$[\because g(x) = 6x - k]$$

$$= 3(6x - k) + 2$$

$$[\text{In } f(x) = 3x + 2, \text{ Replace } x \text{ by } 6x - k]$$

$$= 18x - 3k + 2 \quad \dots(1)$$

$$\text{Now } g \circ f(x) = g(f(x)) = g(3x + 2)$$

$$[\because f(x) = 3x + 2]$$

$$= 6(3x + 2) - k$$

$$[\text{In } g(x) = 6x - k, \text{ Replace } x \text{ by } 3x + 2]$$

$$= 18x + 12 - k \quad \dots(2)$$

Also it is given that $f \circ g = g \circ f$

$$\Rightarrow 18x - 3k + 2 = 18x + 12 - k$$

$$[\text{Using (1) and (2)}]$$

$$\Rightarrow -3k + 2 = 12 - k \Rightarrow -3k + k = 12 - 2$$

$$\Rightarrow -2k = 12 - 2$$

$$\Rightarrow -2k = 10 \Rightarrow k = \frac{10}{-2} = -5 \therefore k = -5$$

(ii) $f(x) = 2x - k$, $g(x) = 4x + 5$

$$f \circ g(x) = f(g(x)) = f(4x + 5)$$

$$[\because g(x) = 4x + 5]$$

$$= 2(4x + 5) - k$$

$$[\because \text{In } f(x) = 2x - k, \text{ Replace } x \text{ by } 4x + 5]$$

$$= 8x + 10 - k \quad \dots(1)$$

$$g \circ f(x) = g(f(x)) = g(2x - k)$$

$$[\because f(x) = 2x - k]$$

$$= 4(2x - k) + 5$$

$$[\text{In } g(x) = 4x + 5, \text{ Replace } x \text{ by } 2x - k]$$

$$= 8x - 4k + 5 \quad \dots(2)$$

Given that

$$f \circ g(x) = g \circ f(x)$$

$$\Rightarrow 8x + 10 - k = 8x - 4k + 5$$

[From (1) and (2)]

$$\Rightarrow 10 - k = -4k + 5 \Rightarrow -k + 4k = 5 - 10$$

$$\Rightarrow 3k = -5 \Rightarrow k = \frac{-5}{3}$$

3. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$, show that $f \circ g = g \circ f = x$.

Sol. Given $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$

$$\text{S.T } f \circ g = g \circ f = x$$

$$f \circ g(x) = f(g(x)) = f\left(\frac{x+1}{2}\right) [\because g(x) = \frac{x+1}{2}]$$

$$= 2\left(\frac{x+1}{2}\right) - 1$$

$$[\because \text{In } f(x) = 2x - 1, \text{ Replace } x \text{ by } \frac{x+1}{2}]$$

$$= x + 1 - 1 = x \quad \dots(1)$$

Now,

$$gof(x) = g(f(x)) = g(2x - 1)$$

$$[\because f(x) = 2x - 1]$$

$$= \frac{2x - 1 + 1}{2}$$

$$[\text{In } g(x) = \frac{x+1}{2}, \text{ Replace } x \text{ by } 2x - 1]$$

$$= \frac{2x}{2} = x \quad \dots(2)$$

From (1) and (2)

$$fog(x) = gof(x) = x \quad \text{Hence proved.}$$

4. If $f(x) = x^2 - 1$, $g(x) = x - 2$ find a , if $gof(a) = 1$. [PTA - 2]

Sol. Given $f(x) = x^2 - 1$, $g(x) = x - 2$

$$gof(x) = g(f(x)) = g(x^2 - 1)$$

$$[\because f(x) = x^2 - 1]$$

$$= x^2 - 1 - 2$$

$$[\because \text{In } g(x) = x - 2, \text{ Replace } x \text{ by } x^2 - 1]$$

$$= x^2 - 3$$

$$\therefore gof(a) = a^2 - 3 \quad [\text{Replacing } x \text{ by } a]$$

$$\text{Given that } gof(a) = 1$$

$$\Rightarrow a^2 - 3 = 1 \Rightarrow a^2 = 4$$

$$\Rightarrow a = \pm \sqrt{4} \Rightarrow a = \pm 2$$

5. Let $A, B, C \subseteq \mathbb{N}$ and a function $f: A \rightarrow B$ be defined by $f(x) = 2x + 1$ and $g: B \rightarrow C$ be defined by $g(x) = x^2$. Find the range of fog and gof .

Sol. Given $f(x) = 2x + 1$ and

$$g(x) = x^2$$

$$fog(x) = f(g(x)) = f(x^2)$$

$$[\because g(x) = x^2]$$

$$= 2x^2 + 1$$

$$[\text{In } f(x) = 2x + 1, \text{ replace } x \text{ by } x^2]$$

$$\therefore fog(x) = 2x^2 + 1 \quad \dots(1)$$

$$gof(x) = g(f(x)) = g(2x + 1)$$

$$[\because f(x) = 2x + 1]$$

$$= (2x + 1)^2$$

$$[\text{In } g(x) = x^2, \text{ replace } x \text{ by } 2x + 1]$$

$$[\because (a + b)^2 = a^2 + 2ab + b^2]$$

Now $f: A \rightarrow B$, and $g: B \rightarrow C$

$\therefore fog: C \rightarrow A$ and $A, B, C \subseteq \mathbb{N}$

$\therefore$ Range of fog is

$$\{y/y = 2x^2 + 1, x \in \mathbb{C}\} \text{ and}$$

Range of gof is

$$\{y/y = (2x + 1)^2, x \in \mathbb{N}\} [\because gof: A \rightarrow C]$$

6. Let $f(x) = x^2 - 1$. Find (i) fof , (ii) $fofof$

[QY - 2024]

Sol. (i) fof

$$fof(x) = f(f(x)) = f(x^2 - 1)$$

$$[\because f(x) = x^2 - 1]$$

$$= (x^2 - 1)^2 - 1$$

$$[\text{In } f(x) = x^2 - 1, \text{ replace } x \text{ by } x^2 - 1]$$

$$= x^4 - 2x^2 + 1 - 1$$

$$[\because (a - b)^2 = a^2 - 2ab + b^2 \text{ Here } a = x^2, b = 1]$$

$$fof(x) = x^4 - 2x^2 \quad \dots(1)$$

$$(ii) fofof = fo[fof(x)]$$

$$= fo[x^4 - 2x^2] \quad [\text{Using (1)}]$$

$$= f(x^4 - 2x^2)$$

$$= (x^4 - 2x^2)^2 - 1$$

$$[\text{In } f(x) = x^2 - 1, \text{ replace } x \text{ by } x^4 - 2x^2]$$

$$\therefore fofof(x) = x^8 - 4x^6 + 4x^4 - 1$$

$$[\because (a - b)^2 = a^2 - 2ab + b^2 \text{ Here } a = x^4, b = +2x^2]$$

7. If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and fog is one-one? [PTA - 6]

Sol.

$$\text{Given } f(x) = x^5$$

$$\Rightarrow f(1) = 1$$

$$\Rightarrow f(-1) = -1$$

$$\Rightarrow f(2) = 32$$

$$\Rightarrow f(-2) = -32$$

$$\Rightarrow f(x) = x^5 \text{ is one - one function.}$$

$$\text{Given } g(x) = x^4$$

$$\Rightarrow g(2) = 16$$

$$\Rightarrow g(-2) = 16$$

$$\Rightarrow g(x) = x^4 \text{ is not one - one function}$$

$$fog(x) = f(g(x)) = f(x^4)$$

$$[\because g(x) = x^4]$$

$$= (x^4)^5$$

$$[\text{In } f(x) = x^5, \text{ replace } x \text{ by } x^4]$$

$$= x^{20}$$

$$\therefore fog(x) = x^{20}$$

$$\text{Now, } fog(1) = 1^{20} = 1$$

$$\text{and } fog(-1) = (-1)^{20} = 1 [\because 20 \text{ is an even number}]$$

$\therefore$ Two elements 1 and -1 have same image as 1.

$\therefore fog(x)$ is not one-one.

8. Consider the functions $f(x)$, $g(x)$, $h(x)$ as given below. Show that $(fog)oh = fo(goh)$ in each case.

(i) $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$
[QY - 2023; July - 2025; Mar - 2026]

(ii) $f(x) = x^2$, $g(x) = 2x$ and $h(x) = x + 4$

(iii) $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 3x - 5$
[PTA - 2]

(i) Given $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$

Sol. Consider $fog(x) = f(g(x)) = f(3x + 1)$
[$\because g(x) = 3x + 1$]

$$= 3x + 1 - 1$$

$$[In f(x) = x - 1, \text{ replace } x \text{ by } 3x + 1]$$

$$\therefore fog = 3x$$

$$LHS = (fog)h = fog(h(x))$$

$$= fog(x^2) \quad [\because h(x) = x^2]$$

$$= 3x^2 \quad [In fog = 3x, \text{ replace } x \text{ by } x^2]$$

$$RHS = fo(goh) = f(g(h(x)))$$

$$= f(g(x^2)) \quad [\because h(x) = x^2]$$

$$= f(3x^2 + 1)$$

$$[In g(x) = 3x + 1, \text{ replace } x \text{ by } x^2]$$

$$= 3x^2 + 1 - 1$$

$$[In f(x) = x - 1, \text{ replace } x \text{ by } 3x^2 + 1]$$

$$= 3x^2$$

$$\therefore LHS = RHS$$

[From (1) and (2)] Hence proved.

(ii) Given $f(x) = x^2$, $g(x) = 2x$, $h(x) = x + 4$

Consider $fog(x) = f(g(x)) = f(2x)$ [$\because g(x) = 2x$]

$$= (2x)^2 \quad [In f(x) = x^2, \text{ replace } x \text{ by } 2x]$$

$$= 4x^2$$

$$LHS (fog)oh = fog(h(x))$$

$$= fog(x + 4) \quad [\because h(x) = x + 4]$$

$$= 4(x + 4)^2$$

$$[In fog = 4x^2, \text{ replace } x \text{ by } x + 4]$$

$$= 4(x^2 + 8x + 16)$$

$$[\because (a + b)^2 = a^2 + 2ab + b^2. \text{ Here } a = x, b = 4]$$

$$= 4x^2 + 32x + 64 \quad \dots (1)$$

$$RHS = fo(goh) = f(g(h(x)))$$

$$= f(g(x + 4)) \quad [\because h(x) = x + 4]$$

$$= f(2(x + 4))$$

$$[In g(x) = 2x, \text{ replace } x \text{ by } x + 4]$$

$$= f(2x + 8) = (2x + 8)^2$$

$$[In f(x) = x^2, \text{ replace } x \text{ by } 2x + 8]$$

$$= 4x^2 + 32x + 64 \quad \dots (2)$$

$$[\because (a + b)^2 = a^2 + 2ab + b^2. \text{ Here } a = 2x, b = 8]$$

$$LHS = RHS$$

From (1) and (2) Hence proved.

(iii) $f(x) = x - 4$, $g(x) = x^2$, $h(x) = 3x - 5$

Consider

$$fog(x) = f(g(x)) = f(x^2) \quad [\because g(x) = x^2]$$

$$= x^2 - 4$$

$$[In f(x) = x - 4, \text{ replace } x \text{ by } x^2]$$

$$LHS = (fog)oh$$

$$= (fog)(h(x)) = fog(3x - 5) \quad [\because h(x) = 3x - 5]$$

$$= (3x - 5)^2 - 4$$

$$[In fog(x) = x^2 - 4, \text{ replace } x \text{ by } 3x - 5]$$

$$= 9x^2 - 30x + 25 - 4$$

$$[\because (a - b)^2 = a^2 - 2ab + b^2 \text{ Here } a = 3x, b = 5]$$

$$= 9x^2 - 30x + 21 \quad \dots (1)$$

$$RHS = fo(goh)(x) = f(g(h(x)))$$

$$= f(g(3x - 5)) \quad [\because h(x) = 3x - 5]$$

$$= f((3x - 5)^2)$$

$$[In g(x) = x^2, \text{ replace } x \text{ by } 3x - 5]$$

$$= (3x - 5)^2 - 4$$

$$[In f(x) = x - 4, \text{ replace } x \text{ by } (3x - 5)^2]$$

$$= 9x^2 - 30x + 25 - 4$$

$$[\because (a - b)^2 = a^2 - 2ab + b^2 \text{ Here } a = 3x, b = 5]$$

$$= 9x^2 - 30x + 21 \quad \dots (2)$$

From (1) and (2)

$$LHS = RHS \quad \text{Hence proved.}$$

9. Let $f = \{(-1, 3), (0, -1), (2, -9)\}$ be a linear function from $\mathbb{Z}$ into $\mathbb{Z}$. Find $f(x)$.

Sol. Given $f = \{(-1, 3), (0, -1), (2, -9)\}$ is a linear function from $\mathbb{Z}$ into $\mathbb{Z}$.

Since f is a linear function, let $y = ax + b$ be the linear function which is of degree one.

Sub $(-1, 3)$ in $y = ax + b$, we get

$$3 = a(-1) + b \Rightarrow -a + b = 3 \quad \dots (1)$$

Sub $(0, -1)$ in $y = ax + b$, we get

$$-1 = a(0) + b \Rightarrow -1 = b \Rightarrow b = -1$$

Sub $b = -1$ in (1) we get,

$$-a - 1 = 3 \Rightarrow -a = 3 + 1 = 4$$

$$\Rightarrow a = -4$$

Sub $a = -4$, $b = -1$ in $y = ax + b$ we get,

$$y = -4x - 1$$

$\therefore$ The required linear function is $-4x - 1$.

10. In electrical circuit theory, a circuit $C(t)$ is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a, b are constants. Show that the circuit $C(t) = 3t$ is linear.

Sol. Given $C(at_1 + bt_2) = a.c(t_1) + b.c(t_2)$

$$\text{Let } C(t) = 3t$$

$$LHS = C(at_1 + bt_2) = 3(at_1 + bt_2)$$

$$= 3at_1 + 3bt_2 \quad \dots (1)$$

$$RHS = a.c(t_1) + b.c(t_2) = a.3t_1 + b.3t_2$$

$$[\because c(t_1) = 3t_1 \text{ and } c(t_2) = 3t_2]$$

$$= 3at_1 + 3bt_2 \quad \dots (2)$$

$$LHS = RHS$$

From (1) and (2)

Hence $C(t) = 3t$ is linear function.

EXERCISE 1.6

Multiple choice questions.

1. If
- $n(A \times B) = 6$
- and
- $A = \{1, 3\}$
- then
- $n(B)$
- is

[Sep - 2021; QY - 2023; April - 2024]

- (A) 1 (B) 2 (C) 3 (D) 6

[Ans. (C) 3]

Hint:

If $n(A \times B) = 6$

$A = \{1, 3\}, n(A) = 2$

$$n(B) = \frac{6}{2} = 3$$

- 2.
- $A = \{a, b, p\}$
- ,
- $B = \{2, 3\}$
- ,
- $C = \{p, q, r, s\}$
- then
- $n[(A \cup C) \times B]$
- is [PTA - 3; April - 2023; July - 2025]

- (A) 8 (B) 20 (C) 12 (D) 16

[Ans. (C) 12]

Hint:

$A = \{a, b, p\}, B = \{2, 3\},$

$C = \{p, q, r, s\}$

$n((A \cup C) \times B)$ where $A \cup C = \{a, b, p, q, r, s\}$

$$(A \cup C) \times B = \{(a, 2), (a, 3), (b, 2), (b, 3), (p, 2), (p, 3), (q, 2), (q, 3), (r, 2), (r, 3), (s, 2), (s, 3)\}$$

$$n[(A \cup C) \times B] = 12$$

3. If
- $A = \{1, 2\}$
- ,
- $B = \{1, 2, 3, 4\}$
- ,
- $C = \{5, 6\}$
- and
- $D = \{5, 6, 7, 8\}$
- then state which of the following statement is true. [Sep. - 2020; FRT - 2022]

- (A)
- $(A \times C) \subset (B \times D)$
-
- (B)
- $(B \times D) \subset (A \times C)$
-
- (C)
- $(A \times B) \subset (A \times D)$
-
- (D)
- $(D \times A) \subset (B \times A)$

[Ans. (A) $(A \times C) \subset (B \times D)$]

Hint:

$A = \{1, 2\}, B = \{1, 2, 3, 4\},$

$C = \{5, 6\}, D = \{5, 6, 7, 8\}$

$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$

$$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$$

$$\therefore (A \times C) \subset B \times D \quad \text{It is true.}$$

4. If there are 1024 relations from a set
- $A = \{1, 2, 3, 4, 5\}$
- to a set
- B
- , then the number of elements in
- B
- is [PTA - 2; Aug - 2022; June - 2023]

- (A) 3 (B) 2 (C) 4 (D) 8

[Ans. (B) 2]

Hint:

$n(A) = 5$

$n(B) = x$

$P[n(A \times B)] = 1024 = 2^{10}$

$2^{5x} = 2^{10} \Rightarrow 5x = 10$

$$\Rightarrow x = 2$$

5. The range of the relation
- $R = \{(x, x^2) \mid x \text{ is a prime number less than } 13\}$
- is

[PTA - 4; HY - '19; FRT & Aug - 2022; April - 2025]

- (A)
- $\{2, 3, 5, 7\}$
- (B)
- $\{2, 3, 5, 7, 11\}$
-
- (C)
- $\{4, 9, 25, 49, 121\}$
- (D)
- $\{1, 4, 9, 25, 49, 121\}$

[Ans. (C) $\{4, 9, 25, 49, 121\}$]

Hint:

$R = \{(x, x^2) \mid x \text{ is a prime number } < 13\}$

The squares of 2, 3, 5, 7, 11 are

$\{4, 9, 25, 49, 121\}$

6. If the ordered pairs
- $(a + 2, 4)$
- and
- $(5, 2a + b)$
- are equal then
- (a, b)
- is

[May - 2022; FRT & QY - 2024]

- (A)
- $(2, -2)$
- (B)
- $(5, 1)$
-
- (C)
- $(2, 3)$
- (D)
- $(3, -2)$

[Ans. (D) $(3, -2)$]

Hint:

$(a + 2, 4), (5, 2a + b) \Rightarrow a + 2 = 5$

$a = 3 \Rightarrow 2a + b = 4$

$6 + b = 4 \Rightarrow b = -2$

7. Let
- $n(A) = m$
- and
- $n(B) = n$
- then the total number of non-empty relations that can be defined from
- A
- to
- B
- is [FRT - 2022; HY - 2023]

- (A)
- m^n
- (B)
- n^m
- (C)
- $2^{mn} - 1$
- (D)
- 2^{mn}

[Ans. (C) $2^{mn} - 1$]

Hint:

$n(A) = m$

$n(B) = n$

$P[n(A \times B)] = m \times n = mn$

No. of relations $= 2^{n(A \times B)} = 2^{mn}$

Non-empty relations $= 2^{mn} - 1$

8. If
- $\{(a, 8), (6, b)\}$
- represents an identity function, then the value of
- a
- and
- b
- are respectively

[PTA - 1; QY - 2023; July - 2024; Mar. - 2026]

- (A)
- $(8, 6)$
- (B)
- $(8, 8)$
- (C)
- $(6, 8)$
- (D)
- $(6, 6)$

[Ans. (A) $(8, 6)$]

Hint:

$\{(a, 8), (6, b)\} \Rightarrow a = 8 \Rightarrow b = 6$

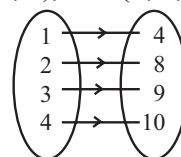
9. Let
- $A = \{1, 2, 3, 4\}$
- and
- $B = \{4, 8, 9, 10\}$
- . A function
- $f : A \rightarrow B$
- given by
- $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$
- is a [PTA - 4]

- (A) Many-one function
-
- (B) Identity function
-
- (C) One-to-one function
-
- (D) Into function

[Ans. (C) One-to one function]

Hint:

$A = \{1, 2, 3, 4\}, B = \{4, 8, 9, 10\}$



10. If $f(x) = 2x^2$ and $g(x) = \frac{1}{3x}$, then $f \circ g$ is

[HY - '19; FRT & July - 2024]

- (A) $\frac{3}{2x^2}$ (B) $\frac{2}{3x^2}$ (C) $\frac{2}{9x^2}$ (D) $\frac{1}{6x^2}$

Hint: $f(x) = 2x^2 \Rightarrow g(x) = \frac{1}{3x}$ [Ans. (C) $\frac{2}{9x^2}$]

$$\begin{aligned} f \circ g &= f(g(x)) = f\left(\frac{1}{3x}\right) = 2\left(\frac{1}{3x}\right)^2 \\ &= 2 \times \frac{1}{9x^2} = \frac{2}{9x^2} \end{aligned}$$

11. If $f: A \rightarrow B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to [PTA - 2; April - 2024]

- (A) 7 (B) 49 (C) 1 (D) 14

[Ans. (A) 7]

Hint: In a bijective function, $n(A) = n(B) \Rightarrow n(A) = 7$

12. Let f and g be two functions given by $f = \{(0, 1), (2, 0), (3, -4), (4, 2), (5, 7)\}$ $g = \{(0, 2), (1, 0), (2, 4), (-4, 2), (7, 0)\}$ then the range of $f \circ g$ is

- (A) $\{0, 2, 3, 4, 5\}$ (B) $\{-4, 1, 0, 2, 7\}$
(C) $\{1, 2, 3, 4, 5\}$ (D) $\{0, 1, 2\}$

Hint: $f \circ g = f(g(x))$ [Ans. (D) $\{0, 1, 2\}$]

$$f(g(0)) = f(2) = 0$$

$$f(g(1)) = f(0) = 1$$

$$f(g(2)) = f(4) = 2$$

$$f(g(-4)) = f(2) = 0$$

$$f(g(7)) = f(0) = 1$$

$$\therefore \text{Range of } f \circ g = \{0, 1, 2\}$$

13. Let $f(x) = \sqrt{1+x^2}$ then

- (A) $f(xy) = f(x) \cdot f(y)$ (B) $f(xy) \geq f(x) \cdot f(y)$
(C) $f(xy) \leq f(x) \cdot f(y)$ (D) None of these

[Ans. (C) $f(xy) \leq f(x) \cdot f(y)$]

Hint: $\sqrt{1+x^2} \cdot \sqrt{1+y^2} \leq \sqrt{(1+x^2)(1+y^2)}$

$$\Rightarrow f(xy) \leq f(x) \cdot f(y)$$

14. If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are [PTA - 6; July - 2025]

- (A) $(-1, 2)$ (B) $(2, -1)$
(C) $(-1, -2)$ (D) $(1, 2)$ [Ans. (B) $(2, -1)$]

Hint: $g(x) = y = \alpha x + \beta$... (1)

$$\text{Sub } x = 1, y = 1 \text{ in (1)}$$

$$1 = \alpha + \beta \quad \dots (2)$$

$$\text{Sub } x = 1, y = 3 \text{ in (1)}$$

$$3 = \alpha + \beta \quad \dots (3)$$

$$(3) - (2) \Rightarrow 2\alpha + \beta = 3$$

$$\alpha + \beta = 1$$

$$\begin{array}{r} (-) \quad (-) \quad (-) \\ \alpha + \beta = 1 \\ \alpha + \beta = 2 \end{array}$$

$$\alpha = 2$$

$$\text{Sub } \alpha = 2 \text{ in (2)}$$

$$\alpha + \beta = 1 \Rightarrow \beta = 1 - 2 = -1$$

$$\therefore \alpha = 2; \beta = -1$$

15. $f(x) = (x+1)^3 - (x-1)^3$ represents a function which is [PTA - 5; QY - 2019; 2023 & 2024; Mar. - 2026]

- (A) linear (B) cubic
(C) reciprocal (D) quadratic

[Ans. (D) quadratic]

Hint: $f(x) = (x+1)^3 - (x-1)^3$
 $= x^3 + 3x^2 + 3x + 1 - [x^3 - 3x^2 + 3x - 1]$
 $= x^3 + 3x^2 + 3x + 1 - x^3 + 3x^2 - 3x + 1 = 6x^2 + 2$
 It is a quadratic function.

Unit Exercise - 1

1. If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2, 5)$ are equal, then find x and y .

Sol: $(x^2 - 3x, y^2 + 4y) = (-2, 5)$

$$x^2 - 3x = -2$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x = 2, 1$$

$$y^2 + 4y = 5$$

$$y^2 + 4y - 5 = 0$$

$$(y+5)(y-1) = 0$$

$$y = -5, 1$$

$$\begin{array}{c} 2 \\ \swarrow \quad \searrow \\ -2 \quad -1 \end{array}$$

$$\begin{array}{c} -5 \\ \swarrow \quad \searrow \\ -5 \quad -1 \end{array}$$

2. The cartesian product $A \times A$ has 9 elements among which $(-1, 0)$ and $(0, 1)$ are found. Find the set A and the remaining elements of $A \times A$.

Sol: $A = \{-1, 0, 1\}, A = \{-1, 0, 1\}$

$$A \times A = \{(-1, -1), (-1, 0), (-1, 1), (0, -1), (0, 0), (0, 1), (1, -1), (1, 0), (1, 1)\}$$

$$\text{Remaining elements} = \{(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)\}$$

3. Given that $f(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ 4 & x < 1 \end{cases}$. Find

(i) $f(0)$ (ii) $f(3)$

(iii) $f(a+1)$ in terms of a . (Given that $a \geq 0$)

Sol: (i) $f(0) = 4$

$$(ii) f(3) = \sqrt{3-1} = \sqrt{2}$$

$$(iii) f(a+1) = \sqrt{a+1-1} = \sqrt{a}$$

4. Let $A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$ and let $f: A \rightarrow \mathbb{N}$ be defined by $f(n) =$ the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f .

Sol. $A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$
 $f: A \rightarrow \mathbb{N}$
 $f(n) =$ the highest prime factor of $n \in A$
 $f = \{(9, 3), (10, 5), (11, 11), (12, 3), (13, 13), (14, 7), (15, 5), (16, 2), (17, 17)\}$
Range = $\{3, 5, 11, 13, 7, 2, 17\} = \{2, 3, 5, 7, 11, 13, 17\}$

5. Find the domain of the function

$$f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$$

Sol. $f(x) = \sqrt{1 + \sqrt{1 - \sqrt{1 - x^2}}}$
Domain of $f(x) = \{-1, 0, 1\}$
($x^2 = 1, -1, 0$, because $\sqrt{1 - x^2}$ should be +ve, or 0)

6. If $f(x) = x^2$, $g(x) = 3x$ and $h(x) = x - 2$. Prove that $(f \circ g) \circ h = f \circ (g \circ h)$.

Sol.
 $f(x) = x^2$
 $g(x) = 3x$
 $h(x) = x - 2$
LHS = $f \circ (g \circ h)$
 $f \circ g = f(g(x)) = f(3x) = (3x)^2 = 9x^2$
 $(f \circ g) \circ h = (f \circ g) h(x) = (f \circ g)(x - 2)$
 $= 9(x - 2)^2 = 9(x^2 - 4x + 4)$
 $= 9x^2 - 36x + 36 \quad \dots(1)$
RHS = $f \circ (g \circ h)$
 $(g \circ h) = g(h(x)) = g(x - 2)$
 $= 3(x - 2) = 3x - 6$
 $f \circ (g \circ h) = f(3x - 6) = (3x - 6)^2$
 $= 9x^2 - 36x + 36 \quad \dots(2)$
 $(1) = (2)$
LHS = RHS
 $(f \circ g) \circ h = f \circ (g \circ h)$ is proved.

7. Let $A = \{1, 2\}$ and $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify whether $A \times C$ is a subset of $B \times D$?

Sol. $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$
 $C = \{5, 6\}$, $D = \{5, 6, 7, 8\}$
 $A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$
 $B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$
 $\Rightarrow (A \times C) \subset (B \times D)$ Hence it is proved.

8. If $f(x) = \frac{x-1}{x+1}$, $x \neq -1$ show that $f(f(x)) = -\frac{1}{x}$, provided $x \neq 0$.

Sol. $f(x) = \frac{x-1}{x+1}$, $x \neq -1$
 $f(f(x)) = f\left(\frac{x-1}{x+1}\right) = \frac{\left(\frac{x-1}{x+1}\right) - 1}{\left(\frac{x-1}{x+1}\right) + 1}$
 $= \frac{\cancel{x} - 1 - \cancel{x} - 1}{\cancel{x} + 1 + \cancel{x} + 1} = \frac{-2}{2x} = -\frac{1}{x}$

Hence it is proved.

9. The function f and g are defined by $f(x) = 6x + 8$; $g(x) = \frac{x-2}{3}$.

- (i) Calculate the value of $gg\left(\frac{1}{2}\right)$
(ii) Write an expression for $gf(x)$ in its simplest form.

Sol. $f(x) = 6x + 8$
 $g(x) = \frac{x-2}{3}$
(i) $gg(x) = g(g(x))$
 $= g\left(\frac{x-2}{3}\right) = \frac{\frac{x-2}{3} - 2}{3}$
 $= \frac{x-2-6}{3} \times \frac{1}{3} = \frac{x-8}{9}$
 $gog\left(\frac{1}{2}\right) = \frac{\frac{1}{2} - 8}{9} = \frac{1-16}{2} \times \frac{1}{9} = \frac{-15}{18} = -\frac{5}{6}$
(ii) $gof(x) = g(f(x)) = g(6x + 8)$
 $= \frac{6x+8-2}{3} = \frac{6x+6}{3}$
 $= \frac{\cancel{6}(2x+2)}{\cancel{3}} = 2x+2 = 2(x+1)$

10. Write the domain of the following real functions

- (i) $f(x) = \frac{2x+1}{x-9}$ [PTA - 6]
(ii) $p(x) = \frac{-5}{4x^2+1}$
(iii) $g(x) = \sqrt{x-2}$ [PTA - 6]
(iv) $h(x) = x + 6$

Sol. (i) $f(x) = \frac{2x+1}{x-9}$

The denominator should not be zero as the function is a real function.

$\therefore$ The domain = $\mathbb{R} - \{9\}$

(ii) $p(x) = \frac{-5}{4x^2+1}$

The domain is $\mathbb{R}$.

(iii) $g(x) = \sqrt{x-2}$

The domain = $[2, \infty)$

(iv) $h(x) = x+6$

The domain is $\mathbb{R}$.

PTA EXAM QUESTION & ANSWERS

1 MARK

1. If $n(A) = p$, $n(B) = q$ then the total number of relations that exist between A and B is

[PTA -1; April - 2023]

- (A) 2^p (B) 2^q (C) 2^{p+q} (D) 2^{pq}

Hint: $P(n(A \times B)) = 2^{n(A \times B)} = 2^{pq}$ [Ans. (D) 2^{pq}]

2. Given $f(x) = (-1)^x$ is a function from $\mathbb{N}$ to $\mathbb{Z}$. Then the range of f is

[PTA - 3; HY - 2023]

- (A) $\{1\}$ (B) $\mathbb{N}$ (C) $\{1, -1\}$ (D) $\mathbb{Z}$

[Ans. (C) $\{1, -1\}$]

Hint:

$$f(1) = (-1)^1 = -1$$

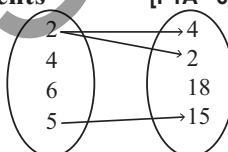
$$f(2) = (-1)^2 = 1$$

$\therefore$ Range of $f = \{1, -1\}$

3. The given diagram represents

[PTA - 6]

- (A) an onto function
(B) constant function
(C) an one-one function
(D) not a function



[Ans. (D) not a function]

Hint: 4 has no image

2 MARKS

1. A relation $f: X \rightarrow Y$ is defined by $f(x) = x^2 - 2$ where, $X = \{-2, -1, 0, 3\}$ and $Y = \mathbb{R}$ (i) List the elements of f (ii) Is f a function? [PTA - 1; QY - '19]

Sol. $f(x) = x^2 - 2$ where $X = \{-2, -1, 0, 3\}$

(i) $f(-2) = (-2)^2 - 2 = 2;$

$f(-1) = (-1)^2 - 2 = -1$

$f(0) = 0^2 - 2 = -2$

$f(3) = 3^2 - 2 = 9 - 2 = 7$

$\therefore f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$

- (ii) We note that each element in the domain of f has a unique image.

Therefore f is a function.

2. A relation R is given by the set $\{(x, y) | y = x^2 + 3, x \in \{0, 1, 2, 3, 4, 5\}\}$ Determine its domain and range. [PTA - 2; QY - 2023]

Sol. Domain = $\{0, 1, 2, 3, 4, 5\}$
 $x = 0, y = 0^2 + 3 = 3$
 $x = 1, y = 1^2 + 3 = 4$
 $x = 2, y = 2^2 + 3 = 7$
 $x = 3, y = 3^2 + 3 = 12$
 $x = 4, y = 4^2 + 3 = 19$
 $x = 5, y = 5^2 + 3 = 28$
 Range = $\{3, 4, 7, 12, 19, 28\}$

3. Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$.

[PTA - 4; April - 2023; FRT - 2024]

Sol. Given, $f(k) = 2k - 1$
 $f \circ f(k) = f(f(k))$
 $= 2(2k - 1) - 1 = 4k - 2 - 1 = 4k - 3$
 Given, $f \circ f(k) = 5$
 $\therefore 4k - 3 = 5 \Rightarrow 4k = 5 + 3 \Rightarrow 4k = 8$
 $\therefore k = \frac{8}{4} \Rightarrow k = 2$

4. Let $A = \{1, 2, 3, \dots, 100\}$ and R be the relation defined as "is cube of" on A . Find the domain and range of R . [PTA - 4]

Sol. $R = \{(1, 1), (2, 8), (3, 27), (4, 64)\}$
 Domain = $\{1, 2, 3, 4, \dots, 100\}$
 Range = $\{1, 8, 27, 64\}$

5. Let $A = \{1, 2, 3, 4\}$ and $B = \mathbb{N}$. Let $f: A \rightarrow B$ be defined by $f(x) = x^2$ (i) the range of f (ii) identify the type of function. [PTA - 5]

Sol. $f(1) = 1; f(2) = 4; f(3) = 9; f(4) = 16$

(i) Range = $\{1, 4, 9, 16\}$

(ii) One - one and into function

6. Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = 3x - 5$ Find the values of a and b given that $(a, 4)$ and $(1, b)$ belong to f . [PTA - 6]

Sol. $f(x) = 3x - 5$ can be written as
 $f = \{(x, 3x - 5) | x \in \mathbb{R}\}$

$(a, 4)$ means the image of a is 4.

That is, $f(a) = 4$

$3a - 5 = 4 \Rightarrow a = 3$

$(1, b)$ means the image of 1 is b .

That is, $f(1) = b$

$3(1) - 5 = b \Rightarrow b = -2$

7. $R = \{(x, -2), (-5, y)\}$ represents the identity function, find the values x and y . [PTA - 6]

Sol.
 $x = -2$
 $y = -5$

5 MARKS

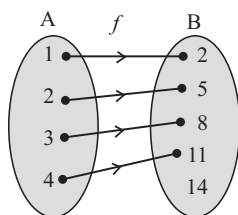
1. Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 5, 8, 11, 14\}$ be two sets. Let $f: A \rightarrow B$ be a function given by $f(x) = 3x - 1$ Represent this function. [PTA - 3]

- (i) by arrow diagram [Sep - 2020; FRT - 2024]
 (ii) in a table form [July - 2025]
 (iii) as a set of ordered pairs
 (iv) in a graphical form

Sol. Let $A = \{1, 2, 3, 4\}$; $B = \{2, 5, 8, 11, 14\}$;
 $f(x) = 3x - 1$
 $f(1) = 3(1) - 1 = 3 - 1 = 2$; $f(2) = 3(2) - 1 = 6 - 1 = 5$
 $f(3) = 3(3) - 1 = 9 - 1 = 8$; $f(4) = 3(4) - 1 = 12 - 1 = 11$

- (i) **Arrow diagram**

Let us represent the function $f: A \rightarrow B$ by an arrow diagram



- (ii) **Table form**

The given function f can be represented in a tabular form as given below

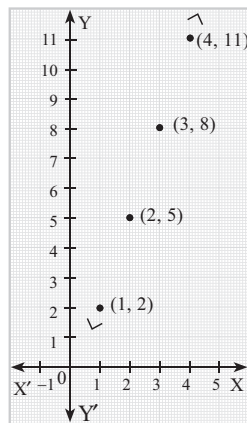
x	1	2	3	4
$f(x)$	2	5	8	11

- (iii) **Set of ordered pairs**

The function f can be represented as a set of ordered pairs as

$$f = \{(1, 2), (2, 5), (3, 8), (4, 11)\}$$

- (iv) **Graphical form**



In the adjacent xy -plane the points $(1, 2)$, $(2, 5)$, $(3, 8)$, $(4, 11)$ are plotted

2. Let $A = \{x \in \mathbb{W} / 0 < x < 5\}$, $B = \{x \in \mathbb{W} / 0 \leq x \leq 2\}$, $C = \{x \in \mathbb{W} / x < 3\}$ then verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ [PTA - 3]

Sol.
 $A = \{1, 2, 3, 4\}$
 $B = \{0, 1, 2\}$
 $C = \{0, 1, 2\}$
 $B \cap C = \{0, 1, 2\} \cap \{0, 1, 2\} = \{0, 1, 2\}$
 $A \times (B \cap C) = \{1, 2, 3, 4\} \times \{0, 1, 2\}$
 $= \{(1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2)\}$... (1)
 $A \times B = \{1, 2, 3, 4\} \times \{0, 1, 2\}$
 $= \{(1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2)\}$
 $A \times C = \{1, 2, 3, 4\} \times \{0, 1, 2\}$
 $= \{(1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2)\}$
 $(A \times B) \cap (A \times C) = \{(1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (4, 0), (4, 1), (4, 2)\}$... (2)
 $(1) = (2)$ Hence it is proved.

3. If $f(x) = 2x + 3$, $g(x) = 1 - 2x$ and $h(x) = 3x$, prove that $f \circ (g \circ h) = (f \circ g) \circ h$. [PTA - 5; HY - 2023]

Sol.
 $f(x) = 2x + 3$, $g(x) = 1 - 2x$,
 $h(x) = 3x$
 Now, $(f \circ g)(x) = f(g(x)) = f(1 - 2x)$
 $= 2(1 - 2x) + 3 = 5 - 4x$

Then,

$$(f \circ g) \circ h(x) = (f \circ g)(h(x)) = (f \circ g)(3x)$$

$$= 5 - 4(3x) = 5 - 12x \quad \dots (1)$$

$$(g \circ h)(x) = g(h(x)) = g(3x) = 1 - 2(3x)$$

$$= 1 - 6x$$

$$\text{So, } f \circ (g \circ h)(x) = f(1 - 6x) = 2(1 - 6x) + 3$$

$$= 5 - 12x \quad \dots (2)$$

From (1) and (2), we get

$$(f \circ g) \circ h = f \circ (g \circ h)$$

GOVT. EXAM QUESTION & ANSWERS

1 MARK

Multiple choice questions.

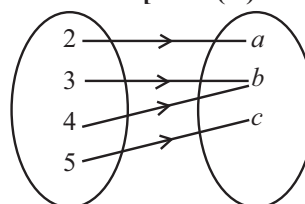
1. $f = \{(2, a), (3, b), (4, b), (5, c)\}$ is a _____.

[Govt. MQP - '19; QY - 2024]

- (A) identity function (B) one-one function
 (C) many-one function (D) constant function

[Ans. (C) many-one function]

Hint:



2. Let $f(x) = x^2 - x$, then $f(x-1) - f(x+1)$ is :

[Sep - 2020]

- (A) $4x$ (B) $2-2x$ (C) $2-4x$ (D) $4x-2$

[Ans. (C) $2-4x$]

[Hint:

$$\begin{aligned} f(x-1) &= (x-1)^2 - (x-1) \\ &= x^2 - 2x + 1 - (x-1) \\ &= x^2 - 2x + 1 - x + 1 = x^2 - 3x + 2 \end{aligned}$$

$$\begin{aligned} f(x+1) &= (x+1)^2 - (x+1) \\ &= x^2 + 2x + 1 - x - 1 = x^2 + x \end{aligned}$$

$$\begin{aligned} \therefore f(x-1) - f(x+1) &= (x^2 - 3x + 2) - (x^2 + x) \\ &= x^2 - 3x + 2 - x^2 - x = -4x + 2 \end{aligned}$$

3. If $n(A) = p$ and $n(B) = q$ then $n(A \times B) =$ _____

[QY - '19]

- (A) $p+q$ (B) $p-q$ (C) $p \times q$ (D) $\frac{p}{q}$

[Ans. (C) $p \times q$]

[Hint:

$$n(A \times B) = n(A) \times n(B) = p \times q$$

4. For any two sets P and Q, $P \cap Q$ is [FRT - 2022]

- (A) $\{x : x \in P \text{ or } x \in Q\}$
 (B) $\{x : x \in P \text{ and } x \notin Q\}$
 (C) $\{x : x \in P \text{ and } x \in Q\}$
 (D) $\{x : x \notin P \text{ and } x \in Q\}$

[Ans. (C) $\{x : x \in P \text{ and } x \in Q\}$]

5. Let $n(A) = m$ and $n(B) = n$, then the total number of functions that exist from B to A is : [April - 2025]

- (A) m^n (B) n^m (C) $2^{mn}-1$ (D) 2^{mn}

[Ans. (A) m^n]

2 MARKS

1. Define a function. [Govt. MQP - '19]

Sol. A relation f between two non-empty sets X and Y is called a function from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x, y) \in f$. That is, $f = \{(x, y) \mid \text{for all } x \in X, y \in Y\}$

2. Let f be a function $f : \mathbb{N} \rightarrow \mathbb{N}$ be defined by $f(x) = 3x + 2, x \in \mathbb{N}$. [Govt. MQP - '19]

- (i) Find the images of 1, 2, 3 [QY - 2023]
 (ii) Find the pre-images of 29, 53 [QY - 2023]
 (iii) Identify the type of function

Sol. The function $f : \mathbb{N} \rightarrow \mathbb{N}$ is defined by $f(x) = 3x + 2$,

(i) $x = 1, f(1) = 3(1) + 2 = 3 + 2 = 5$

$x = 2, f(2) = 3(2) + 2 = 6 + 2 = 8$

$x = 3, f(3) = 3(3) + 2 = 9 + 2 = 11$

The images of 1, 2, 3 are 5, 8, 11 respectively.

- (ii) If x is the pre-image of 29, then $f(x) = 29$.

$$\Rightarrow 3x + 2 = 29$$

$$3x = 27 \Rightarrow x = 9.$$

Similarly, if x is the pre-image of 53, then

$$f(x) = 53 \Rightarrow 3x + 2 = 53$$

$$3x = 51 \Rightarrow x = 17.$$

$\therefore$ The pre-images of 29 and 53 are 9 and 17 respectively.

- (iii) Since different elements of $\mathbb{N}$ have different images in the co-domain, the function f is one - one function.

The co-domain of f is $\mathbb{N}$.

But the range of $f = \{5, 8, 11, 14, 17, \dots\}$ is a proper subset of $\mathbb{N}$.

$\therefore f$ is not an onto function. That is, f is an into function.

Thus f is one - one and into function.

3. Let $A = \{1, 2, 3, 4, 5\}$, $B = W$ and $f : A \rightarrow B$ is defined by $f(x) = x^2 - 1$ find the range of f . [QY - '19]

Sol. $f(1) = 0; f(2) = 3; f(3) = 8; f(4) = 15; f(5) = 24$
 Range of $f = \{0, 3, 8, 15, 24\}$

4. If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$ then show that $n(A \times B) = n\{B\}$. [Sep - 2021; April - 2024]

Sol. $A = \{1, 3, 5\}$ $B = \{2, 3\}$

$$n(A) = 3; n(B) = 2$$

$$\therefore n(A) \times n(B) = 3 \times 2 = 6 \quad \dots(1)$$

$$A \times B = \{(1, 2)(1, 3)(3, 2)(3, 3)(5, 2)(5, 3)\}$$

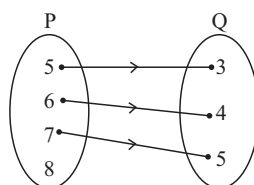
$$\therefore n(A \times B) = 6 \quad \dots(2)$$

From (1) and (2)

$$n(A \times B) = n(A) \times n(B)$$

5. The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form. [May - 2022; July - 2025]

Sol.



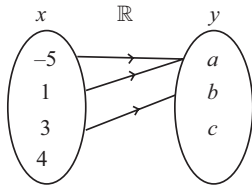
- (i) Set builder form of $\mathbb{R} = \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

- (ii) Roster form $R = \{(5, 3), (6, 4), (7, 5)\}$

6. Let $X = \{-5, 1, 3, 4\}$ and $Y = \{a, b, c\}$. Determine whether the relation $\mathbb{R} = \{(-5, a), (1, a), (3, b)\}$ is a function from X to Y? [Mar - 2026]

Sol. $\mathbb{R} = \{(-5, a), (1, a), (3, b)\}$ we may represent the relation $\mathbb{R}$ in an arrow diagram.

$\therefore \mathbb{R}$ is not a function as $4 \in x$ does not have an image in y .



7. If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$, then find A and B. [FRT - 2022 & 2024; Aug - 2022]

Sol. Refer Text Book Example 1.2

8. For the given relation $R = \{(1, 3), (2, 5), (4, 7), (5, 9), (3, 1)\}$, write the domain and range.

Sol. Domain = $\{1, 2, 3, 4, 5\}$ [FRT - 2022]
Range = $\{1, 3, 5, 7, 9\}$

9. If the function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by,

$$f(x) = \begin{cases} 2x+7; & x < -2 \\ x^2 - 2; & -2 \leq x < 3 \\ 3x - 2; & x \geq 3 \end{cases} \text{ then find the values}$$

of (i) $f(4)$ (ii) $f(-2)$ (iii) $f(4) + 2f(1)$

(iv) $\frac{f(1) - 3f(4)}{f(-3)}$ [QY - 2023]

Sol. Refer Text Book Example 1.18

10. Given $f(x) = 2x - x^2$ Find (i) $f(1)$ (ii) $f(x+1)$

Sol. Refer Text Book Example 1.9 [July - 2024]

11. If $A = \{-2, -1, 0, 1, 2\}$ and $f: A \rightarrow B$ is an onto function defined by $f(x) = x^2 + x + 1$ then find B.

Sol. Refer Text Book Example 1.14 [QY - 2024]

12. Find $f \circ g$ and $g \circ f$ when $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. [April - 2025]

Sol. Refer Text Book Example 1.19

5 MARKS

1. If $f(x) = 3x - 2$, $g(x) = 2x + k$ and if $f \circ g = g \circ f$, then find the value of k. [QY - '19; April - 2024]

Sol. $f(x) = 3x - 2$, $g(x) = 2x + k$
 $f \circ g(x) = f(g(x)) = f(2x + k)$
 $= 3(2x + k) - 2$
 $= 6x + 3k - 2$

Thus, $f \circ g(x) = 6x + 3k - 2$
 $g \circ f(x) = g(3x - 2)$
 $= 2(3x - 2) + k$

Thus, $g \circ f(x) = 6x - 4 + k$

Given that $f \circ g = g \circ f$
 $\therefore 6x + 3k - 2 = 6x - 4 + k$

$6x - 6x + 3k - k = -4 + 2 \Rightarrow k = -1$

2. Let $A = \{x \in \mathbb{N} / 1 < x < 4\}$, $B = \{x \in \mathbb{W} / 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} / x < 3\}$. Then verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. [HY - '19]

Sol. $A = \{x \in \mathbb{N} / 1 < x < 4\} = \{2, 3\}$
 $B = \{x \in \mathbb{W} / 0 \leq x < 2\} = \{0, 1\}$
 $C = \{x \in \mathbb{N} / x < 3\} = \{1, 2\}$
To prove $A \times (B \cap C) = (A \times B) \cap (A \times C)$
 $B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$
 $A \times (B \cap C) = \{2, 3\} \times \{1\} = \{(2, 1), (3, 1)\}$
 $A \times B = \{2, 3\} \times \{0, 1\}$
 $= \{(2, 0), (2, 1), (3, 0), (3, 1)\}$
 $A \times C = \{2, 3\} \times \{1, 2\}$
 $= \{(2, 1), (2, 2), (3, 1), (3, 2)\}$
 $(A \times B) \cap (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$
 $\cap \{(2, 1), (2, 2), (3, 1), (3, 2)\}$
 $= \{(2, 1), (3, 1)\}$

$A \times (B \cap C) = (A \times B) \cap (A \times C)$ is verified.

3. Let $A = \{x \in \mathbb{W} / x < 3\}$, $B = \{x \in \mathbb{N} / 1 < x \leq 5\}$ and $C = \{3, 5, 7\}$ verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$. [April - 2023]

Sol. $A = \{x \in \mathbb{W} / x < 3\} = \{0, 1, 2\}$
[Whole numbers less than 3]
 $B = \{x \in \mathbb{N} / 1 < x \leq 5\} = \{2, 3, 4, 5\}$
[Natural numbers from 2 to 5]
 $C = \{3, 5, 7\}$

Verify : $A \times (B \cup C) = (A \times B) \cup (A \times C)$

LHS = $A \times (B \cup C)$
 $B \cup C = \{2, 3, 4, 5, 7\}$
 $A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (0, 7), (1, 2), (1, 3), (1, 4), (1, 5), (1, 7), (2, 3), (2, 2), (2, 3), (2, 4), (2, 5), (2, 7)\} \dots (1)$
RHS = $(A \times B) \cup (A \times C)$
 $(A \times B) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5)\}$
 $(A \times C) = \{(0, 3), (0, 5), (0, 7), (1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7)\}$
 $(A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (0, 7), (1, 2), (1, 3), (1, 4), (1, 5), (1, 7), (2, 2), (2, 3), (2, 4), (2, 5), (2, 7)\} \dots (2)$
(1) = (2), LHS = RHS Hence it is proved.

4. Find x if $g \circ f(x) = f \circ g(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$. [FRT - 2024]

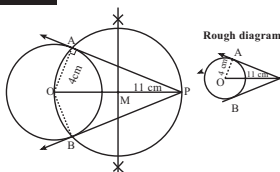
Sol. Refer Text Book Example 1.24

5. Let $A = \{x \in \mathbb{N} / 1 < x < 4\}$, $B = \{x \in \mathbb{W} / 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} / x < 3\}$. Then verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ [April - 2024]

Sol. Refer Text Book Example 1.3 (i).

8 MARKS

1. Let $X = \{1, 2, 3, 4, 5\}$,
 $Y = \{1, 3, 5, 7, 9\}$, which
 of the following are
 relation from X to Y ?



- (i) $R_1 = \{(1, 3) (2, 4) (3, 5) (4, 6) (5, 7)\}$
 (ii) $R_2 = \{(1, 1) (2, 1) (3, 3) (4, 3) (5, 5)\}$
 (iii) $R_3 = \{(1, 1) (1, 3) (3, 5) (3, 7) (5, 7)\}$
 (iv) $R_4 = \{(1, 3) (2, 5) (4, 7) (5, 9) (3, 1)\}$ [FRT - 2022]

Sol. (i) $R_1 = \{(1, 3) (2, 4) (3, 5) (4, 6) (5, 7)\}$

Here 1 is related to 3, 3 is related to 5, 5 is related to 7.

But $4 \notin y$ and $6 \notin y$, we can say that 2 cannot be related to 4 and 4 cannot be related to 6.

Hence R_1 is not a relation from X to Y .

- (ii) $R_2 = \{(1, 1) (2, 1) (3, 3) (4, 3) (5, 5)\}$

R_2 is a relation since $\{1, 3, 5\}$, belongs to Y and $\{1, 2, 3, 4, 5\}$ belongs to X .

- (iii) $R_3 = \{(1, 1) (1, 3) (3, 5) (3, 7) (5, 7)\}$

R_3 is a relation

- (iv) $R_4 = \{(1, 3) (2, 5) (4, 7) (5, 9) (3, 1)\}$

R_4 is a relation

2. Let $A = \{0, 1, 2, 3\}$ and $B = \{1, 3, 5, 7, 9\}$ be two sets. Let $f: A \rightarrow B$ be a function given by $f(x) = 2x + 1$. Represent this function as (i) a set of ordered pairs (ii) a table (iii) an arrow and (iv) a graph. [April - 2024]

Sol. $A = \{0, 1, 2, 3\}$, $B = \{1, 3, 5, 7, 9\}$

$$f(x) = 2x + 1$$

$$f(0) = 2(0) + 1 = 1$$

$$f(1) = 2(1) + 1 = 3$$

$$f(2) = 2(2) + 1 = 5$$

$$f(3) = 2(3) + 1 = 7$$

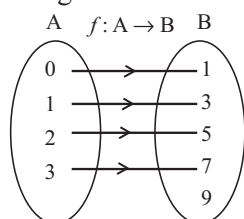
- (i) A set of ordered pairs.

$$f = \{(0, 1), (1, 3), (2, 5), (3, 7)\}$$

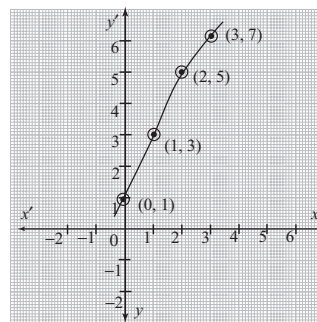
- (ii) A table

x	0	1	2	3
$f(x)$	1	3	5	7

- (iii) An arrow diagram



- (iv) A Graph $f = \{(x, f(x)) / x \in A\}$
 $= \{(0, 1), (1, 3), (2, 5), (3, 7)\}$



ADDITIONAL QUESTION & ANSWERS

2 MARKS

1. Let $A = \{1, 2, 3, 4\}$ and $B = \{-1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. Let $R = \{(1, 3), (2, 6), (3, 10), (4, 9)\} \subseteq A \times B$ be a relation. Show that R is a function and find its domain, co-domain and the range of R .

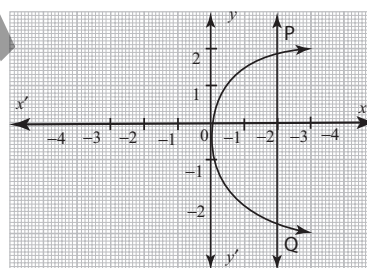
Sol. Domain of $R = \{1, 2, 3, 4\}$

Co-domain $B = \{-1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

Range of $R = \{3, 6, 10, 9\}$

Here R is a function, because each element in the domain has a unique image in the codomain.

- 2.

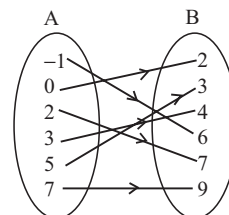


State whether the graph represents a function. Use the vertical line test.

Sol. It is not a function as the vertical line PQ cuts the graph at two points.

3. Let $f = \{(2, 7), (3, 4), (7, 9), (-1, 6), (0, 2), (5, 3)\}$ be a function from $A = \{-1, 0, 2, 3, 5, 7\}$ to $B = \{2, 3, 4, 6, 7, 9\}$. Is this (i) a one-one function (ii) an onto function, (iii) both one-one and onto function?

Sol. It is both one-one and onto function.



All the elements in A have their separate images in B . All the elements in B have their pre-image in A . Therefore it is one-one and onto function.

4. $f(x) = (1 + x)$

$g(x) = (2x - 1)$ Show that $f \circ g(x) = g \circ f(x)$

Sol.

$$f(x) = 1 + x$$

$$g(x) = (2x - 1)$$

$$\begin{aligned} f \circ g(x) &= f(g(x)) = f(2x - 1) \\ &= 1 + 2x - 1 = 2x \quad \dots(1) \end{aligned}$$

$$\begin{aligned} g \circ f(x) &= g(f(x)) = g(1 + x) = 2(1 + x) - 1 \\ &= 2 + 2x - 1 = 2x + 1 \quad \dots(2) \end{aligned}$$

$$(1) \neq (2)$$

$$\therefore f \circ g(x) \neq g \circ f(x)$$

It is verified.

5. Let $A = \{1, 2, 3, 4, 5\}$, $B = \mathbb{N}$ and $f: A \rightarrow B$ be defined by $f(x) = x^2$. Find the range of f . Identify the type of function.

Sol.

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{1, 2, 3, 4, \dots\}$$

$$f: A \rightarrow B, f(x) = x^2$$

$$\therefore f(1) = 1^2 = 1$$

$$f(2) = 2^2 = 4$$

$$f(3) = 3^2 = 9$$

$$f(4) = 4^2 = 16$$

$$f(5) = 5^2 = 25$$

$$\therefore \text{Range of } f = \{1, 4, 9, 16, 25\}$$

Elements in A have been connected with different elements in B . Therefore it is one-one function. But not all the elements in B have pre-images in A . Therefore it is not on-to function.

6. The following table represents a function from $A = \{5, 6, 8, 10\}$ to $B = \{19, 15, 9, 11\}$, where $f(x) = 2x - 1$. Find the values of a and b .

Sol.

x	5	6	8	10
$f(x)$	a	11	b	19

$$A = \{5, 6, 8, 10\}, B = \{19, 15, 9, 11\}$$

$$f(x) = 2x - 1$$

$$f(5) = 2(5) - 1 = 9$$

$$f(8) = 2(8) - 1 = 15$$

$$\therefore a = 9, b = 15$$

7. If $R = \{(a, -2), (-5, b), (8, c), (d, -1)\}$ represents the identity function, find the values of a, b, c and d .

Sol. $R = \{(a, -2), (-5, b), (8, c), (d, -1)\}$ represents the identity function.

$$a = -2, b = -5, c = 8, d = -1.$$

5 MARKS

1. A function $f: [-7, 6) \rightarrow \mathbb{R}$ is defined as follows.

$$f(x) = \begin{cases} x^2 + 2x + 1 & -7 \leq x < -5 \\ x + 5 & -5 \leq x \leq 2 \\ x - 1 & 2 < x < 6 \end{cases}$$

- Find (i) $2f(-4) + 3f(2)$ (ii) $f(-7) - f(-3)$

Sol. $f(x) = \begin{cases} x^2 + 2x + 1 & -7 \leq x < -5 \\ x + 5 & -5 \leq x < 2 \\ x - 1 & 2 < x < 6 \end{cases}$

- (i) $2f(-4) + 3f(2)$

$$f(-4) = x + 5 = -4 + 5 = 1$$

$$2f(-4) = 2 \times 1 = 2$$

$$f(2) = x + 5 = 2 + 5 = 7$$

$$3f(2) = 3(7) = 21$$

$$\therefore 2f(-4) + 3f(2) = 2 + 21 = 23$$

- (ii)

$$f(-7) = x^2 + 2x + 1$$

$$= (-7)^2 + 2(-7) + 1$$

$$= 49 - 14 + 1 = 36$$

$$f(-3) = x + 5 = -3 + 5 = 2$$

$$f(-7) - f(-3) = 36 - 2 = 34$$

2. A function $f: [1, 6) \rightarrow \mathbb{R}$ is defined as follows:

$$f(x) = \begin{cases} 1 + x, & 1 \leq x < 2 \\ 2x - 1, & 2 \leq x < 4 \\ 3x^2 - 10, & 4 \leq x < 6 \end{cases}, x \in \mathbb{R}: 1 \leq x < 6$$

- Find the value of (i) $f(5)$, (ii) $f(3)$, (iii) $f(2) - f(4)$.

Sol. $f(x) = \begin{cases} 1 + x & : 1 \leq x < 2 \\ 2x - 1 & : 2 \leq x < 4 \\ 3x^2 - 10 & : 4 \leq x < 6 \end{cases}$

(i) $f(5) = 3x^2 - 10$

$$= 3(5^2) - 10 = 75 - 10 = 65$$

(ii) $f(3) = 2x - 1$

$$= 2(3) - 1 = 6 - 1 = 5$$

(iii) $f(2) - f(4)$

$$f(2) = 2x - 1 = 2(2) - 1 = 3$$

$$f(4) = 3x^2 - 10$$

$$= 3(4^2) - 10 = 38$$

$$\therefore f(2) - f(4) = 3 - 38 = -35$$

Unit Test

Time : 45 Minutes

Marks : 25

Section - A

$5 \times 1 = 5$

- If $n(A \times B) = 6$ and $A = \{1, 3\}$ then $n(B)$ is
(A) 1 (B) 2 (C) 3 (D) 6
- $A = \{a, b, p\}$, $B = \{2, 3\}$, $C = \{p, q, r, s\}$ then $n[(A \cup C) \times B]$ is
(A) 8 (B) 20 (C) 12 (D) 16
- Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 8, 9, 10\}$. A function $f: A \rightarrow B$ given by $f = \{(1, 4), (2, 8), (3, 9), (4, 10)\}$ is a
(A) Many-one function
(B) Identity function
(C) One-to-one function
(D) Into function
- If $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ is a function given by $g(x) = \alpha x + \beta$ then the values of α and β are
(A) $(-1, 2)$ (B) $(2, -1)$
(C) $(-1, -2)$ (D) $(1, 2)$
- The range of the relation $R = \{(x, x^2) | x \text{ is a prime number less than } 13\}$ is
(A) $\{2, 3, 5, 7\}$ (B) $\{2, 3, 5, 7, 11\}$
(C) $\{4, 9, 25, 49, 121\}$ (D) $\{1, 4, 9, 25, 49, 121\}$

Section - B

$5 \times 2 = 10$

- Let $A = \{1, 2, 3, 4, \dots, 45\}$ and R be the relation defined as "square is of a number" on A . Write R as a subset of $A \times A$. Also, find the domain and range of R .
- Let $f = \{(x, y) | x, y \in \mathbb{N} \text{ and } y = 2x\}$ be a relation on $\mathbb{N}$. Find the domain, co-domain and range. Is this relation a function?
- A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

- Let $A, B, C \subseteq \mathbb{N}$ and a function $f: A \rightarrow B$ be defined by $f(x) = 2x + 1$ and $g: B \rightarrow C$ be defined by $g(x) = x^2$. Find the range of $f \circ g$ and $g \circ f$.
- Let $A = \{-1, 1\}$ and $B = \{0, 2\}$. If the function $f: A \rightarrow B$ defined by $f(x) = ax + b$ is an onto function? Find a and b .

Section - C

$2 \times 5 = 10$

- Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = 2x - 1$ is one - one but not onto.
- Represent the function $f = \{(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)\}$ through
(i) an arrow diagram
(ii) a table form
(iii) a graph

Answers

SECTION - A

- (C) 3
- (C) 12
- (C) One-to-one function
- (B) $(2, -1)$
- (C) $\{4, 9, 25, 49, 121\}$

Section - B

- Refer Sura's Guide Exercise 1.2; Q.No.2
- Refer Sura's Guide Exercise 1.3; Q. No.1
- Refer Sura's Guide Exercise 1.3; Q. No.8
- Refer Sura's Guide Exercise 1.5; Q. No.5
- Refer Sura's Guide Exercise 1.4; Q. No.8

Section - C

- Refer Sura's Guide Exercise 1.4; Q.No.4
- Refer Sura's Guide Exercise 1.4; Q.No.3



NUMBERS AND SEQUENCES

FORMULAE TO REMEMBER

- ❑ Euclid's Division Lemma Let a and b ($a > b$) be any two positive integers. Then, there exist unique integers q and r such that $a = bq + r$, $0 \leq r < b$.
- ❑ If a, b are two positive integers with $a > b$ then G.C.D of $(a, b) = \text{G.C.D of } (a - b, b)$.
- ❑ Number of terms in A.P., $n = \frac{l - a}{d} + 1$
- ❑ If the sum of three consecutive terms of an A.P is given, then they can be taken as $a - d$, a and $a + d$.
- ❑ If a prime number p divides ab then either p divides a or p divides b , that is p divides at least one of them.
- ❑ Two integers a and b are congruence modulo n if they differ by an integer multiple of n . That is $b - a = kn$ for some integer k . This can also be written as $a \equiv b \pmod{n}$.
- ❑ A real valued sequence is a function defined on the set of natural numbers and taking real values.
- ❑ Let a and d be real numbers. Then the numbers of the form $a, a + d, a + 2d, a + 3d, a + 4d, \dots$ is said to form an Arithmetic progression.

EXERCISE 2.1

1. Find all positive integers, when divided by 3 leaves remainder 2.

Sol. The positive integers when divided by 3 leaves remainder 2.

By Euclid's division lemma

$$a = bq + r, 0 \leq r < b.$$

Here $a = 3q + 2$, where $0 \leq q < 3$.

When $q = 0$, $a = 3(0) + 2 = 2$

When $q = 1$, $a = 3(1) + 2 = 5$

When $q = 2$, $a = 3(2) + 2 = 8$

When $q = 3$, $a = 3(3) + 2 = 11$

The required positive numbers are 2, 5, 8, 11,...

2. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.

[April - 2025]

Sol. By Euclid's division algorithm,
 $a = bq + r, 0 \leq r < b$.

Here $532 = 21q + r \dots (1)$

$\Rightarrow 532 = 21(25) + 7 \dots (2)$

$\therefore q = 25$, and $r = 7$

[Comparing (1) and (2)]

$\therefore$ Number of completed rows = 25 and the leftover flower pots = 7.

3. Prove that the product of two consecutive positive integers is divisible by 2.

Sol. Let $n - 1$ and n be two consecutive positive integers. Then their product is $(n - 1)n$.

$$(n - 1)(n) = n^2 - n.$$

We know that any positive integer is of the form $2q$ or $2q + 1$ for some integer q . So, following cases arise.

Case I. When $n = 2q$. In this case, we have

$$n^2 - n = (2q)^2 - 2q = 4q^2 - 2q = 2q(2q - 1)$$

$$\Rightarrow n^2 - n = 2r, \text{ where } r = q(2q - 1)$$

$\Rightarrow n^2 - n$ is divisible by 2.

Case II : When $n = 2q + 1$. In this case, we have

$$n^2 - n = (2q + 1)^2 - (2q + 1)$$

$$= (2q + 1)(2q + 1 - 1) = 2q(2q + 1)$$

$$\Rightarrow n^2 - n = 2r, \text{ where } r = q(2q + 1).$$

$\Rightarrow n^2 - n$ is divisible by 2.

Hence, $n^2 - n$ is divisible by 2 for every positive integer n .

Hence it is proved.

4. When the positive integers a , b and c are divided by 13, the respective remainders are 9, 7 and 10. Show that $a + b + c$ is divisible by 13. [HY - 2023]

Sol. When a is divided by 13, the remainder is 9.

By Euclid's lemma, $a = bq + r, 0 \leq r < b$

$$\Rightarrow a = 13q + 9 \dots (1)$$

Similarly when the positive integers b and c are divided by 13, the remainders are 7 and 10.

$$\therefore b = 13q + 7 \dots (2)$$

$$\text{and } c = 13q + 10 \dots (3)$$

Adding (1), (2) and (3) we get,

$$\begin{aligned} a + b + c &= 13q + 9 + 13q + 7 + 13q + 10 \\ &= 39q + 26 = 13(3q + 2) \end{aligned}$$

Which is divisible by 13.

$\therefore a + b + c$ is divisible by 13.

5. Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.

Sol. Let x be any integer.

The square of x is x^2 .

Case (i) : Let x be an even integer.

$$\Rightarrow x = 2q \quad [\because x \text{ is even}]$$

Where q is some integer

$$\Rightarrow x^2 = (2q)^2 = 4q^2$$

$$\Rightarrow x^2 = 4(q^2)$$

$\Rightarrow x^2$ is divisible by 4.

$\therefore$ When x is an even integer, x^2 is divisible by 4.

$\Rightarrow x^2$ leaves the remainder 0 when divided by 4.

Case (ii) : Let x be an odd integer.

$$\therefore x = 2k + 1 \text{ for some integer } k.$$

$$\therefore x^2 = (2k + 1)^2 = 4k^2 + 4k + 1$$

$$= 4k(k + 1) + 1$$

$$= 4q + 1 \text{ where } q = k(k + 1)$$

$\Rightarrow x^2 = 4q + 1$ leaves the remainder 1 when divided by 4.

From Case (i) and Case (ii), the square of any integer leaves the remainder either 0 or 1 when divided by 4.

6. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of

(i) 340 and 412 (ii) 867 and 255

(iii) 10224 and 9648

(iv) 84, 90 and 120

[FRT - 2022]

Sol. (i) To find the HCF of 340 and 412. Using Euclid's division algorithm.

$$\text{We get } 412 = 340 \times 1 + 72$$

$$\text{The remainder } 72 \neq 0$$

Again applying Euclid's division algorithm

$$340 = 72 \times 4 + 52$$

The remainder $52 \neq 0$.

Again applying Euclid's division algorithm

$$72 = 52 \times 1 + 20$$

The remainder $20 \neq 0$.

Again applying Euclid's division algorithm,

$$52 = 20 \times 2 + 12$$

The remainder $12 \neq 0$.

Again applying Euclid's division algorithm.

$$20 = 12 \times 1 + 8$$

The remainder $8 \neq 0$.

Again applying Euclid's division algorithm

$$12 = 8 \times 1 + 4$$

The remainder $4 \neq 0$.

Again applying Euclid's division algorithm

$$8 = 4 \times 2 + 0$$

The remainder is zero.

Therefore HCF of 340 and 412 is 4.

- (ii) To find the HCF of 867 and 255, using Euclid's division algorithm.

$$867 = 255 \times 3 + 102$$

The remainder $102 \neq 0$.

Again using Euclid's division algorithm

$$255 = 102 \times 2 + 51$$

The remainder $51 \neq 0$.

Again using Euclid's division algorithm

$$102 = 51 \times 2 + 0$$

The remainder is zero.

Therefore the HCF of 867 and 255 is 51.

- (iii) To find HCF 10224 and 9648. Using Euclid's division algorithm.

$$10224 = 9648 \times 1 + 576$$

The remainder $576 \neq 0$.

Again using Euclid's division algorithm

$$9648 = 576 \times 16 + 432$$

Remainder $432 \neq 0$.

Again applying Euclid's division algorithm

$$576 = 432 \times 1 + 144$$

Remainder $144 \neq 0$.

Again using Euclid's division algorithm

$$432 = 144 \times 3 + 0$$

The remainder is zero.

Therefore the HCF of 10224 and 9648 is 144.

- (iv) To find HCF of 84, 90 and 120.

Using Euclid's division algorithm

$$90 = 84 \times 1 + 6$$

The remainder $6 \neq 0$.

Again using Euclid's division algorithm

$$84 = 6 \times 14 + 0$$

The remainder is zero.

$\therefore$ The HCF of 84 and 90 is 6. To find the HCF of 6 and 120 using Euclid's division algorithm.

$$120 = 6 \times 20 + 0$$

The remainder is zero.

Therefore HCF of 120 and 6 is 6

$\therefore$ HCF of 84, 90 and 120 is 6.

7. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.

[Aug - 2022]

Sol. The required number is the HCF of the numbers.

$$1230 - 12 = 1218,$$

$$1926 - 12 = 1914$$

First we find the HCF of 1218 & 1914 by Euclid's division algorithm.

$$1914 = 1218 \times 1 + 696$$

The remainder $696 \neq 0$.

Again using Euclid's algorithm

$$1218 = 696 \times 1 + 522$$

The remainder $522 \neq 0$.

Again using Euclid's algorithm.

$$696 = 522 \times 1 + 174$$

The remainder $174 \neq 0$.

Again by Euclid's algorithm

$$522 = 174 \times 3 + 0$$

The remainder is zero.

$\therefore$ The HCF of 1218 and 1914 is 174.

$\therefore$ The required number is 174.

8. If d is the Highest Common Factor of 32 and 60, find x and y satisfying $d = 32x + 60y$.

Sol. Applying Euclid's division lemma to 32 and 60, we get

$$60 = 32 \times 1 + 28 \quad \dots(i)$$

The remainder is $28 \neq 0$.

Again applying division lemma

$$32 = 28 \times 1 + 4 \quad \dots(ii)$$

The remainder $4 \neq 0$.

Again applying division lemma

$$28 = 4 \times 7 + 0 \quad \dots(iii)$$

The remainder zero.

$\therefore$ HCF of 32 and 60 is 4.

From (ii), we get

$$32 = 28 \times 1 + 4$$

$$\Rightarrow 4 = 32 - 28 \times 1$$

$$\Rightarrow 4 = 32 - (60 - 32 \times 1) \times 1$$

$$[\because 28 = (60 - 32) \times 1]$$

$$\Rightarrow 4 = 32 - 60 + 32$$

$$\Rightarrow 4 = 32 \times \underline{2} + (-1) \times \underline{60}$$

$$\therefore x = 2 \text{ and } y = -1$$